

# 10 Length, Area, and Volume

## Dear Student,

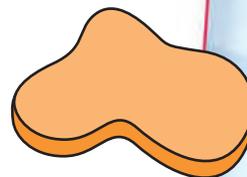
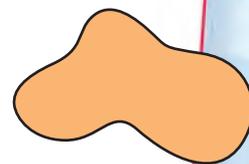
In this chapter, you will be measuring length, area, and volume. These are all measurements of space.

When you use a measuring tape or ruler to measure distance along a straight line or a wiggly path, you are measuring length. When a path loops to make a closed figure like the one shown, you can still measure its length. That length is called the perimeter of the figure. You can also measure something new: the area inside the loop.

Now imagine that you've drawn a loop on the top of a block of wood and cut around it with a saw. You can still measure the length of the loop (perimeter of the figure you've drawn) and the area inside that figure, but the amount of space this lump of wood takes up depends on its thickness. When you measure that amount of space, you are measuring volume.

You'll get a chance to measure the length, area, and volume of many things during this chapter!

Mathematically yours,  
The authors of *Think Math!*



# Fancy Fish

## FACT • ACTIVITY 1

**M**any people keep tropical fish as pets. Tropical fish are colorful, fun to watch, and they don't need to be taken for a walk!



Goldfish



Royal Gramma Basslet



Betta  
(Siamese fighting fish)

**For 1–3, use a ruler to measure the length of each fish to the nearest quarter inch.**

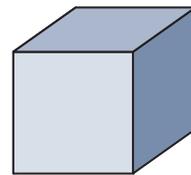
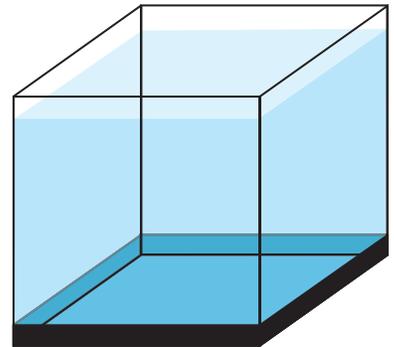
- 1 Goldfish
- 2 Betta
- 3 Royal Gramma Basslet
- 4 Find an object in the classroom that is about  $2\frac{1}{2}$  inches long.
- 5 Write the lengths of the fish and the object you found in order from least to greatest.

## FACT ACTIVITY 2

**H**ave you ever wondered how many fish can live together in a tank? This actually depends on the type of fish. A Betta is an aggressive fish and is usually kept alone in a 10-gallon tank or bigger. A Betta can live with other fish in the same tank as long as the other fish are peaceful.

**For 1–4, you may use cubes to help.**

- 1 Suppose you have a Betta fish, and you put it in a tank that is 2 feet wide and 2 feet long. What is the perimeter of the tank's bottom?
- 2 Suppose your tank is 2 feet wide, 2 feet long, and 2 feet tall. Explain how you can find the volume of the tank in cubic feet.
- 3 Suppose you want to keep a goldfish and a Basslet together with your Betta in a larger tank. The new tank is 4 feet wide, 5 feet long, and 3 feet high. Find the volume of the tank.



**Hint:**  
Remember that one cube represents one cubic foot for these problems.

## CHAPTER PROJECT

### Materials: cubes

Design your own large fish tank. Your tank needs to have a volume of 48 cubic feet.

- Use cubes to build 3 different rectangular fish tank designs.
- Find the perimeter of your tank's bottom.
- Find the area of your tank's bottom.
- If the bottom of your fish tank has an area of 4 square feet, what is the tallest your tank could be? Explain how you can use cubes to find out.
- Describe how you could find the total area of the glass sides of any of your tanks. Do not include the top.



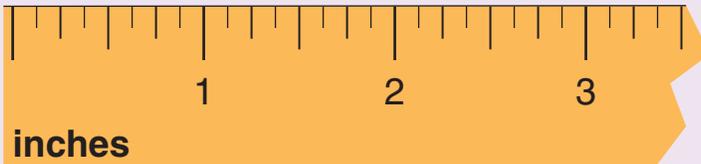
### ALMANAC Fact

Americans own about 50 million pet fish, making fish the third most popular pet after cats and dogs.

## REVIEW MODEL

Measuring to the Nearest Inch,  $\frac{1}{2}$  Inch, and  $\frac{1}{4}$  Inch

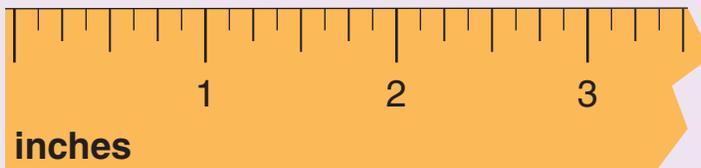
You can use a ruler to measure objects to the nearest inch, half inch, and quarter inch.



To the nearest inch, this crayon measures 3 inches.

To the nearest half inch, this crayon measures  $2\frac{1}{2}$  inches.

To the nearest quarter inch, this crayon also measures  $2\frac{1}{2}$  inches.



To the nearest quarter inch, this crayon measures  $2\frac{3}{4}$  inches.

### ✓ Check for Understanding

- 1 What is the measure of the ribbon to the nearest half inch?



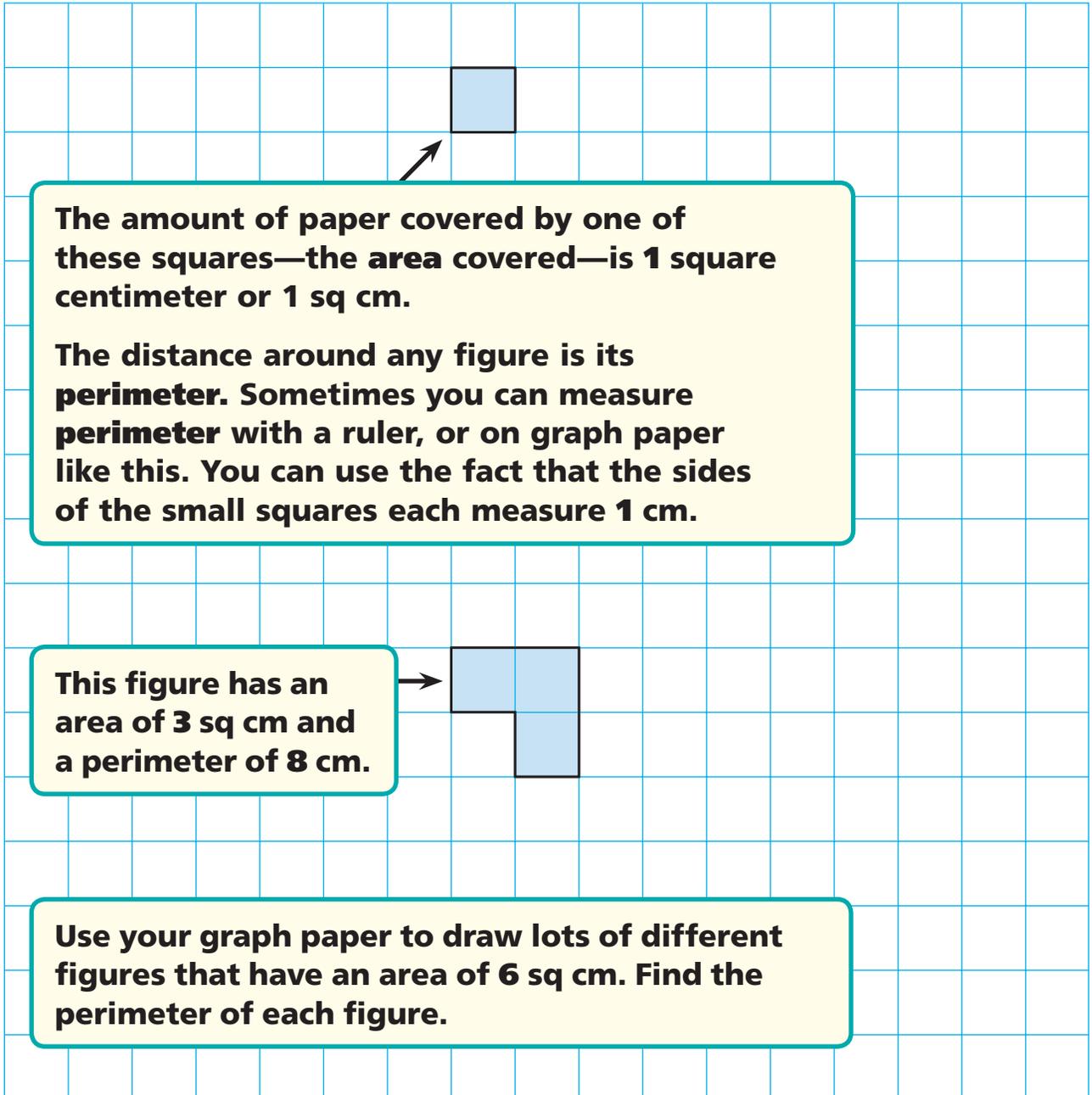
- 2 What is the measure of the ribbon to the nearest quarter inch?



# EXPLORE

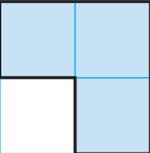
## Measuring Area and Perimeter

Each small square on this page measures 1 cm on each of its four sides.



The amount of paper covered by one of these squares—the **area** covered—is 1 square centimeter or 1 sq cm.

The distance around any figure is its **perimeter**. Sometimes you can measure **perimeter** with a ruler, or on graph paper like this. You can use the fact that the sides of the small squares each measure 1 cm.



This figure has an area of 3 sq cm and a perimeter of 8 cm.

Use your graph paper to draw lots of different figures that have an area of 6 sq cm. Find the perimeter of each figure.

Do all the figures have the same perimeter?  
If not, find as many different perimeters as you can.

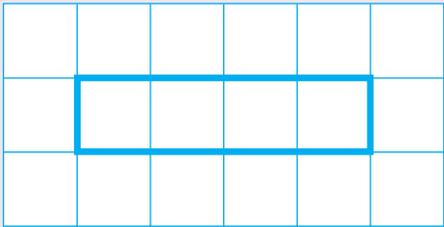
## REVIEW MODEL

## Measuring Perimeter and Area

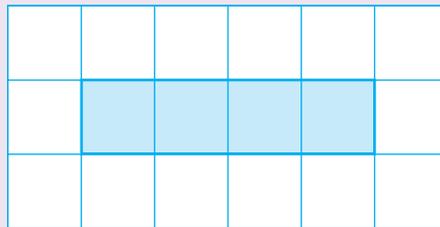
**Perimeter (P)** is the distance around a figure.  
**Area (A)** is the number of square units needed to cover a flat surface.

You can measure the perimeter of a figure in centimeters (cm) and the area of a figure in square centimeters (sq cm).

The perimeter of the figure is 10 cm.



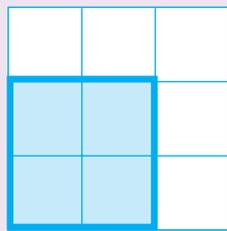
The area of the figure is 4 sq cm.



Two figures with the same area can have different perimeters.

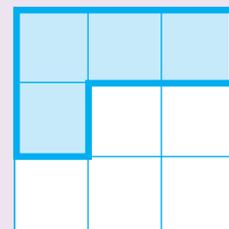
$$P = 8 \text{ cm}$$

$$A = 4 \text{ sq cm}$$



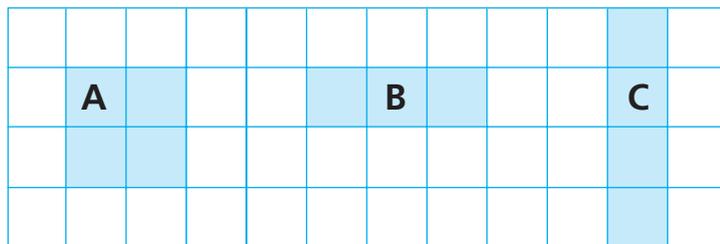
$$P = 10 \text{ cm}$$

$$A = 4 \text{ sq cm}$$



### ✓ Check for Understanding

- 1 Which two figures have the same perimeter?

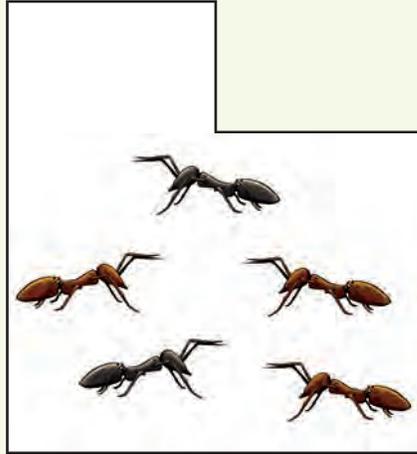


- 2 Which two figures have the same area?

# EXPLORE

## An Ant Corral

Imagine that you are an ant wrangler, and you're making a tiny corral for your herd of ants.



You have 16 cm of fencing for the corral.

**Use a piece of centimeter graph paper to try some designs. Make the fence follow the lines of the graph paper. Use all 16 cm of fence for each design.**

1 Record the area and perimeter of each design.

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2 What is the smallest area of the corrals you designed?

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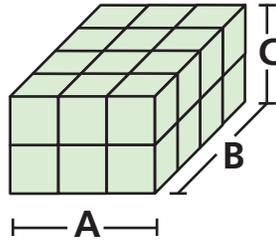
3 What is the largest area of the corrals you designed?

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4 If you had 18 cm of fencing instead of 16 cm, could you enclose a larger area? What would be the largest possible area?

## Building with Cubes

- 1 Using units from a set of base-ten blocks, build this box:

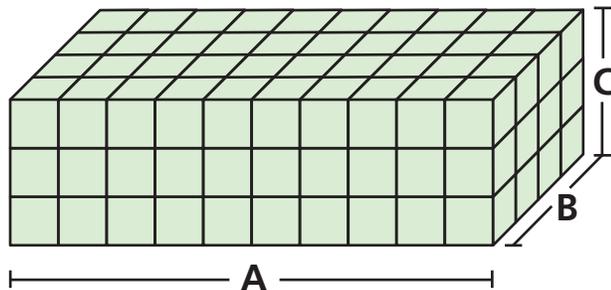


- 2 Use a centimeter ruler to measure your box.

- A From left to right
- B From front to back
- C From top to bottom

- 3 How many units did you use to build the box?

- 4 Using rods from a set of base-ten blocks, build this box:



- 5 Use a centimeter ruler to measure your box.

- A From left to right
- B From front to back
- C From top to bottom

- 6 Although you actually used rods to build this box, if you built it with units instead, how many units would you need?

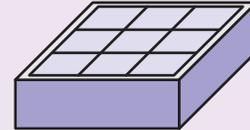
# REVIEW MODEL

## Measuring Volume

**Volume is the amount of space a three-dimensional figure takes up. This is 1 cubic unit.**  **It is used to measure volume.**

You can find volume by counting the number of cubic units needed to fill an object.

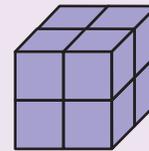
$$\text{Volume} = 9 \text{ cubic units}$$



You can find volume by counting the number of cubes in each layer of a three-dimensional figure.

$$2 \text{ layers} \times 4 \text{ cubes in each layer}$$

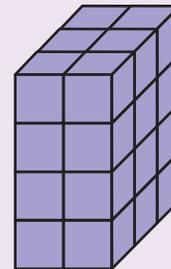
$$\text{Volume} = 8 \text{ cubic units}$$



You can find volume of a rectangular box by multiplying the length, width, and height.

$$3 \text{ cubes} \times 2 \text{ cubes} \times 4 \text{ cubes}$$

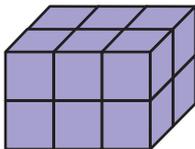
$$\text{Volume} = 24 \text{ cubic units}$$



### Check for Understanding

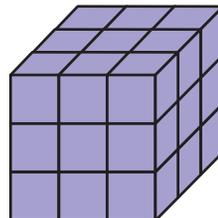
Write the volume for each figure in cubic units.

1



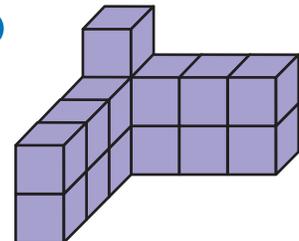
■ cubic units

2



■ cubic units

3



■ cubic units

## REVIEW MODEL

## Problem Solving Strategy

## Draw a Picture

Katie drew a rectangular figure with an area of 20 sq cm and a perimeter of 24 cm. What are the length and width of the figure she drew?

**Strategy:** Draw a Picture**Read to Understand**

What do you know from reading the problem?

The rectangular figure has an area of 20 sq cm and a perimeter of 24 cm.

**Plan**

How can you solve the problem?

You can draw a picture to solve the problem.

**Solve**

How can you draw a picture to solve the problem?

You can draw different rectangles that have an area of 20 sq cm. Then you can find the one that has a perimeter of 24 cm. Find the length and width of this figure.

**Check**

Look back at the problem. Did you answer the question that was asked? Does the answer make sense?

## Problem Solving Practice

### Draw a picture to solve.

- 1 Francisco drew a rectangle with a perimeter of 20 cm. The length of the rectangle is 9 cm. What is the area of the rectangle?
- 2 Celine is setting up for the book fair. She has 28 books to display on a table. If she places the same number of books in each row, how many different ways could she arrange the books?

## Mixed Strategy Practice

### Use any strategy to solve. Explain.

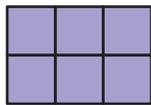
- 3 Erika has 140 pictures in her photo albums. Her first album has 20 more pictures in it than her second album. How many pictures are in each album?
- 4 Jackie's karate class begins at 4:45. It takes her 10 minutes to get from home to her class. She spends 20 minutes getting ready for the class. At what time should Jackie begin getting ready for karate?
- 5 Jared wrote the numbers 3, 6, 8, 11, 13, 16, and 18 on the board. What are the next two numbers in his pattern?
- 6 Luis is studying for the Spelling Bee. He studied 12 words each night for the last 5 nights. How many words has he studied so far?
- 7 Nathan is taller than Julio. Emily is shorter than Julio. What is the order of the children from shortest to tallest?
- 8 Alely used quarters and nickels to pay 80¢ for her snack. She used 2 quarters. How many nickels did she use?

## Problem Solving Strategies

- ✓ Act It Out
- ✓ Draw a Picture
- ✓ Guess and Check
- ✓ Look for a Pattern
- ✓ Make a Graph
- ✓ Make a Model
- ✓ Make an Organized List
- ✓ Make a Table
- ✓ Solve a Simpler Problem
- ✓ Use Logical Reasoning
- ✓ Work Backward
- ✓ Write a Number Sentence

Choose the best vocabulary term from Word List A for each sentence.

- 1 The middle mark between 0 and 1 inches measures a(n)    ?   .
- 2 The dimensions of a box are    ?   ,    ?   , and    ?   .
- 3 The    ?    is the distance around something.
- 4 The    ?    of this figure is the number of squares that can cover it.



- 5 You can use    ?    to measure volume.

Complete each analogy. Use the best term from Word List B.

- 6 Inch is to foot as    ?    is to meter.
- 7 Length is to centimeter as volume is to    ?   .

### Word List A

area  
centimeter  
cubic  
centimeters  
half inch  
height  
length  
perimeter  
quarter inch  
volume  
width

### Word List B

area  
centimeter  
cubic  
centimeter  
perimeter

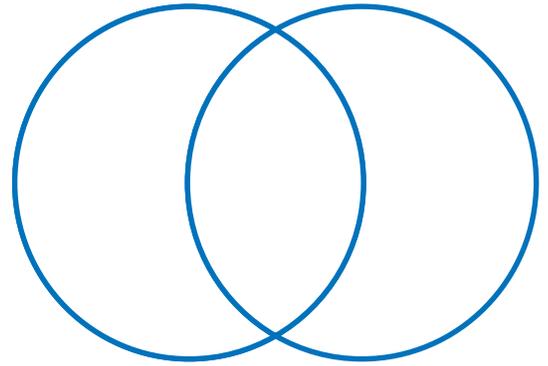
### Talk Math

Discuss with a partner what you have learned about perimeter, area, and volume. Use the vocabulary terms *length*, *width*, and *height*.

- 8 How can you measure the perimeter of a sheet of paper?
- 9 How can you measure the area of a sheet of paper?
- 10 How can you measure the volume of a box?

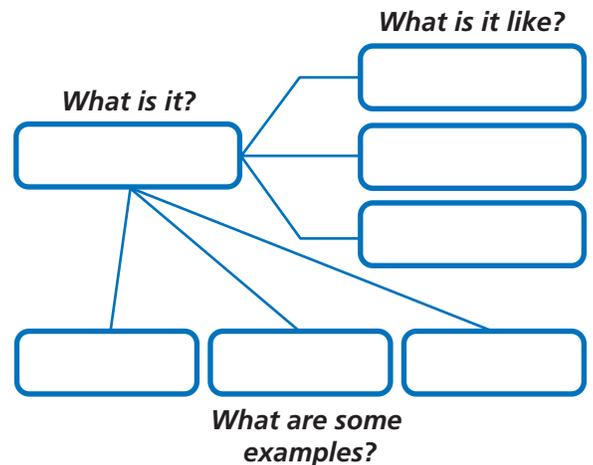
## Venn Diagram

- 11 Create a Venn diagram for the terms *height*, *length*, *width*, *area*, *perimeter*, *volume*, *square centimeter*, *cubic centimeter*, *square inch*, and *cubic inch*. Call one circle of the Venn diagram "Measurements of a Square." Call the other circle "Measurements of a Box."



## Word Definition Map

- 12 Create a word definition map for the term *volume*.



### What's in a Word?



**RANGE** The word *range* in the song *Home on the Range* means a wide-open area where animals can roam and feed. Another type of *range* is a cooking stove with burners on top and an oven. Both of those meanings of *ranges* are nouns. *Range* can also be a verb. "To *range* through the park" means "to walk around and explore the park."

In mathematics, *range* is a noun. It is the difference between the largest number and smallest number in a set of numbers.



#### Technology

Multimedia Math Glossary

[www.harcourtschool.com/thinkmath](http://www.harcourtschool.com/thinkmath)

# GAME

## Ruler Game

### Game Purpose

To practice identifying fractional parts of inches

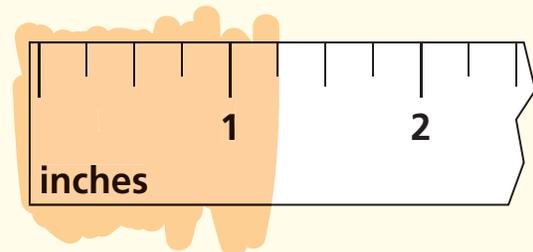
### Materials

- Activity Master 104: Ruler Game
- Paper clip and pencil
- Crayons or markers

### How to Play the Game

- 1 Play this game with a partner. To use the spinner on Activity Master 104, lay a paper clip flat. Put the pencil point through one end of it and onto the dot in the center of the spinner. The paper clip will spin around the pencil point.
- 2 Spin the spinner. The player who spins the greater fraction goes first. That player will choose a ruler on Activity Master 104.
- 3 The first player spins the spinner and shades his or her ruler for the fraction of an inch shown by the spin.
- 4 The other player spins and shades his or her ruler the fraction of an inch shown by the spin.
- 5 Take turns. On each turn, add the fraction of an inch shown by your spin to your shaded section.

**Example:** On your first two turns, you spin  $\frac{3}{4}$  inch and  $\frac{1}{2}$  inch. So, your ruler looks like this:



- 6 Play until one player gets to 5 inches or more on the ruler. That player wins.

# GAME

## Perimeter Golf

### Game Purpose

To draw figures with a given area and the smallest perimeter possible

### Materials

- 2 number cubes, colored pencils or markers
- Activity Master 111: Centimeter Grid Paper

### How to Play the Game

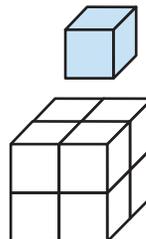
- 1 Play this game with a partner. Each player tosses the number cubes. The player with the larger sum goes first.
- 2 The first player tosses the 2 number cubes and finds the sum. The player must draw a figure on the grid paper that has an area in square centimeters equal to their sum. The player tries to get the smallest perimeter possible.
- 3 The player's score for the "hole" is the perimeter of the figure drawn.
- 4 Take turns tossing the number cubes and drawing a figure. The player with the lower score after 18 holes of *Perimeter Golf* is the winner.



# CHALLENGE

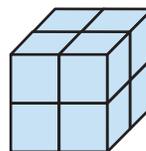
You can build models of cubes using smaller cubes.

- 1 A cube has 6 faces. If you paint the outside of 1 cube blue, all 6 faces will be blue.



- A How many small cubes do you need to build the next larger cube?

- 2 Suppose you paint the outside of the next larger cube blue and take it apart.



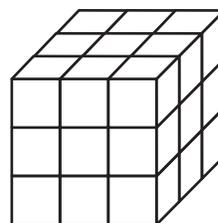
- A How many cubes will have 0 blue faces?

- B How many cubes will have 1 blue face?

- C How many cubes will have 2 blue faces?

- D How many cubes will have 3 blue faces?

- E How many small cubes do you need to build the next larger cube?



- 3 Suppose you paint the outside of the next larger cube blue and take it apart.

- A How many cubes will have 0 blue faces?

- B How many cubes will have 1 blue face?

- C How many cubes will have 2 blue faces?

- D How many cubes will have 3 blue faces?

