

5 Understanding Addition and Subtraction Algorithms

Dear Student,

An algorithm is a step-by-step process to solve a problem. You have already learned algorithms to add and subtract large numbers, and you have experience using them. This chapter will give you a closer look at how these algorithms work.

Look at these puzzles:

200	50	4	254
700	40	6	746
900	90	10	■

600	140	6	746
200	50	4	254
400	90	2	■

How are they like the problems shown below?
How are they different from the problems below?

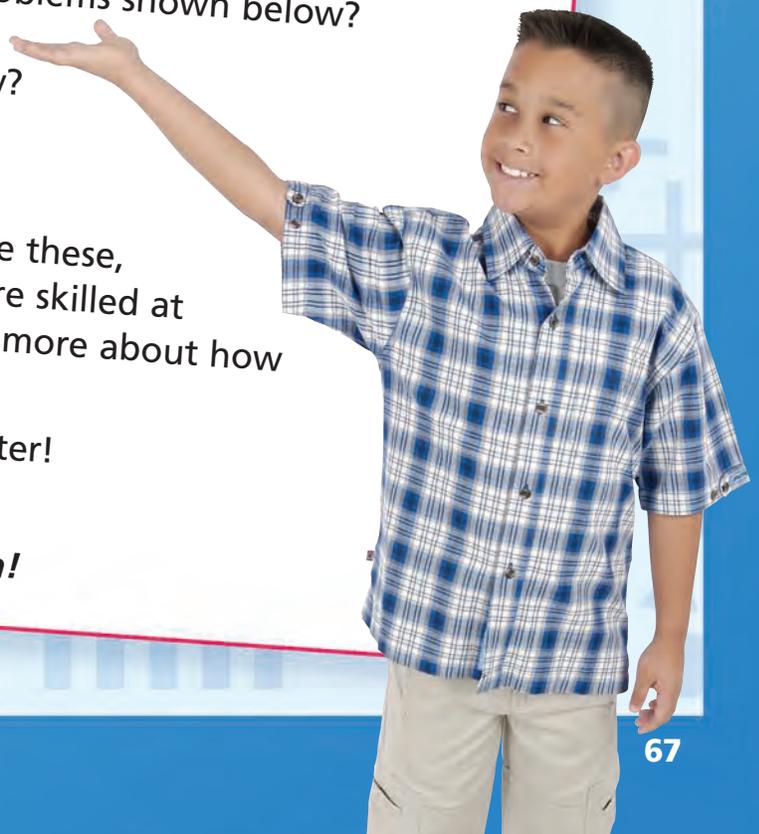
$$\begin{array}{r} 254 \\ + 746 \\ \hline \end{array}$$

$$\begin{array}{r} 746 \\ - 254 \\ \hline \end{array}$$

By exploring questions like these, you will become even more skilled at computing, and will learn more about how arithmetic works.

Best wishes for a fun chapter!

Mathematically yours,
The authors of *Think Math!*



All Kinds of Puzzles



Puzzles challenge us to think and use our brains. Crossword puzzles teach us about words. Jigsaw puzzles help us tell different shapes apart. Did we forget to mention that puzzles are fun?

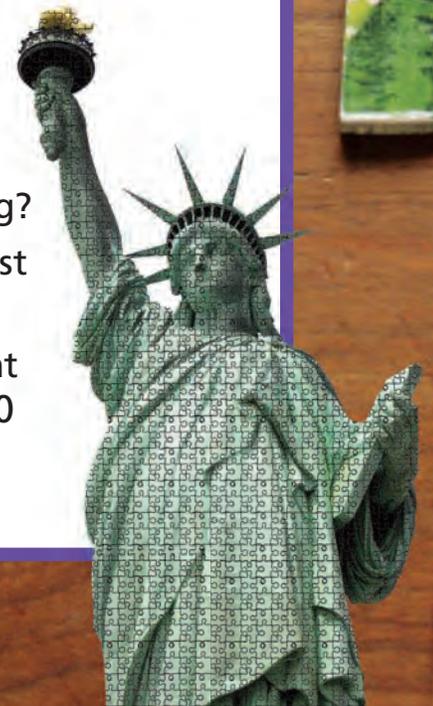
FACT ACTIVITY 1

Melissa likes jigsaw puzzles, especially ones that are unusual shapes. All of Melissa's shaped puzzles have about 1,000 pieces. The chart shows the exact number of pieces in each of her shape puzzles.

Shape Puzzles	
Shape	Number of Pieces
Bear	899
Frog	975
Ladybug	918
Snowman	981
Space Shuttle	995
Statue of Liberty	958

Use the chart to answer the questions.

- Which is the puzzle with the least number of pieces? Explain how you could represent the number using the fewest base-ten blocks.
- If the puzzle with the most pieces takes you the longest to assemble, which puzzle would take you the most time to complete?
- Which puzzle has more pieces, Snowman or Ladybug?
- Write the number of puzzle pieces in order from least to greatest.
- Melissa wants to organize her puzzles into those that are about 900 pieces and those that are about 1,000 pieces. Use rounding to tell which puzzles belong in each category.



FACT ACTIVITY 2

Some puzzles come in books. Melissa has puzzle books and other types of books. She takes the following five books with her on a long trip.

For 1–4, use the table.

- 1 How much longer is the Crossword Puzzle Book than the Number Puzzle Book?
- 2 If Melissa reads the Funny Poems book and Princess Tale book by the end of her trip, how many pages will she have read?
- 3 Melissa has read 47 pages of the animal stories book. How many more pages must she read to finish the book?
- 4 What is a good estimate for the total number of pages in all of the books? Explain.

Melissa's Books	
Title	Number of Pages
Number Puzzle Book	32
Crossword Puzzle Book	80
Animal Stories	96
Funny Poems	128
The Princess Tale	112

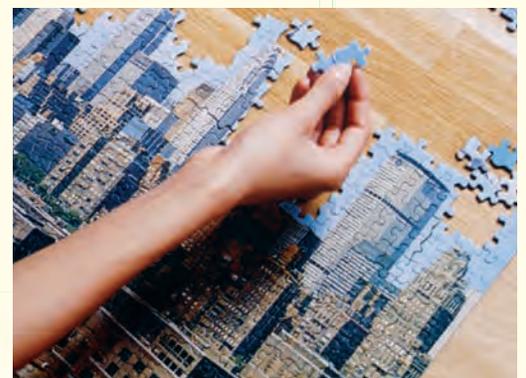
CHAPTER PROJECT

What kinds of books do you like? Some kinds of books are puzzle books, animal books, fantasy books, joke books, science books, and reference books.

- Use books from your home, school, or library. List 2 books in each of 3 different categories, such as 2 joke books, 2 science books, and 2 adventure books. You can use books that you have read or books that you would like to read.
- Make a poster with the name of each category, the titles of the books in each category, and the number of pages in each book.
- Find the total number of pages for the books in each category.
- Describe how you might find the total number of pages in all 6 books.

ALMANAC Fact

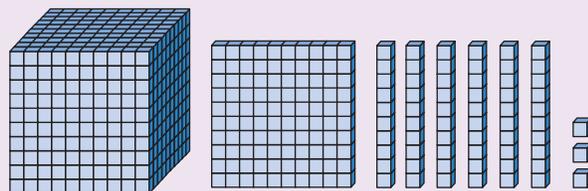
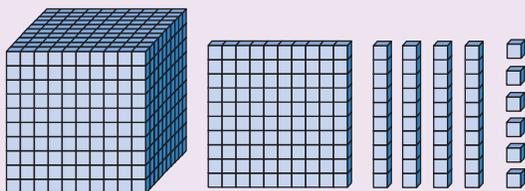
Some jigsaw puzzles have as many as 18,000 pieces.



Comparing Numbers

You can use base-ten blocks to help you compare numbers.

Example Compare 1,146 and 1,163.



Step 1 Use base-ten blocks to show each number. Then compare from left to right.

Step 2 Look at the thousands. They are the same, so continue to compare.

Step 3 Look at the hundreds. They are the same, so continue to compare.

Step 4 Look at the tens. 6 tens is greater than 4 tens. So, 1,163 is the greater number. (You do not need to look at the ones, because the tens are different.)

Step 5 Use these symbols: $<$ (less than) and $>$ (greater than)

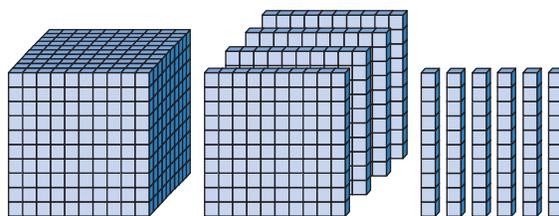
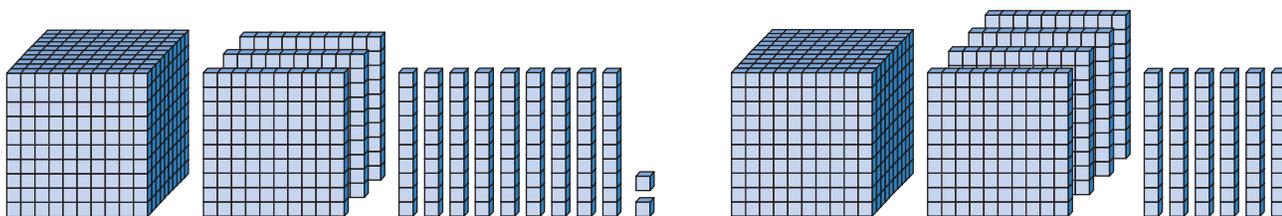
$$1,146 < 1,163$$

$$1,163 > 1,146$$

✓ Check for Understanding

Compare the numbers. Write $<$ or $>$.

1

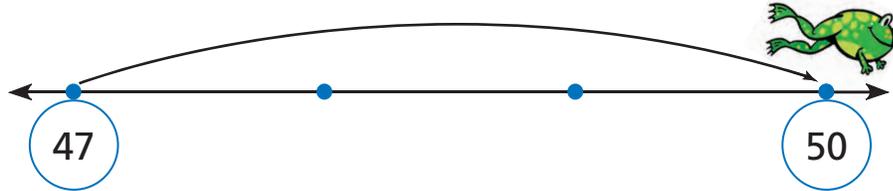


$$1,392 \bullet 1,460$$

EXPLORE

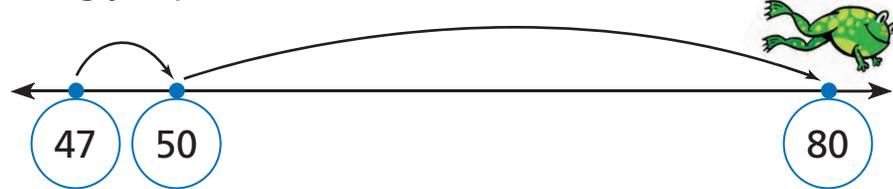
Jumping on the Number Line

- 1 A frog jumped from 47 to 50 on the number line.



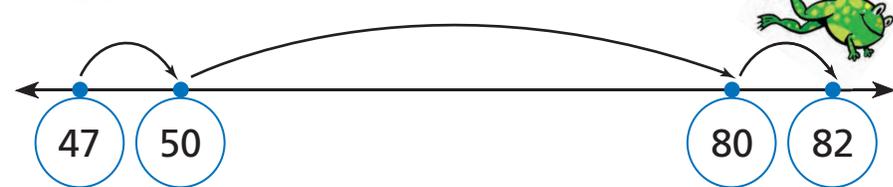
How many spaces did the frog jump?

- 2 Next, the frog jumped from 50 to 80.



How far was the second jump?

- 3 Then the frog jumped from 80 to 82.



How far was the third jump?

- 4 How far is it to jump all the way from 47 to 82?
-

- 5 Solve $82 - 47 = \blacksquare$
-

- 6 Draw a picture showing one or more jumps to go from 66 to 104 on the number line. Find the distance for each jump you make and the total distance for all the jumps.

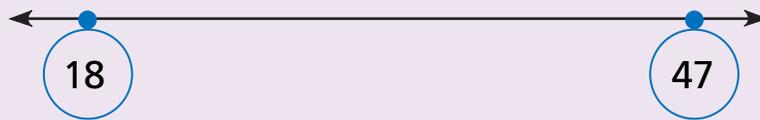
REVIEW MODEL

Using the Number Line to Find Differences

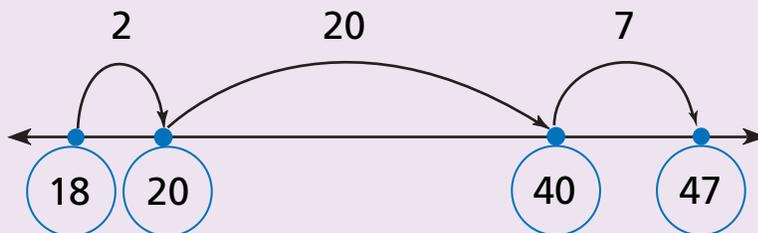
You can use the number line to help you subtract.
The distance between the two numbers is their difference.

Example $47 - 18 = \blacksquare$

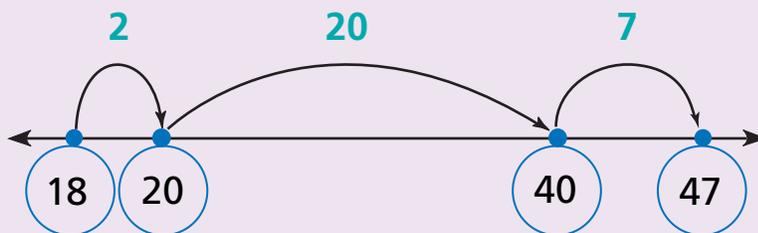
Step 1 Draw the section of the number line that starts with the smaller number and ends with the larger number.



Step 2 Jump from the smaller number to the larger number. Use landing places that are easy to work with, such as multiples of ten.



Step 3 Find the total distance jumped.



$$2 + 20 + 7 = 29$$

$$\text{So, } 47 - 18 = 29.$$

✓ Check for Understanding

Find the difference. Draw the number line on your own paper.

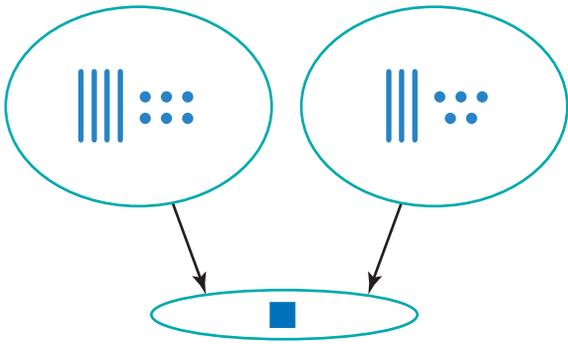
1 $93 - 55 = \blacksquare$

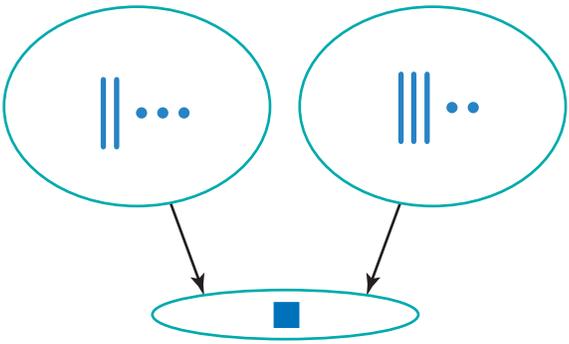
2 $138 - 119 = \blacksquare$

EXPLORE

Predicting Digits

Predict the number of base-ten rods you will need to show each sum with fewest blocks.

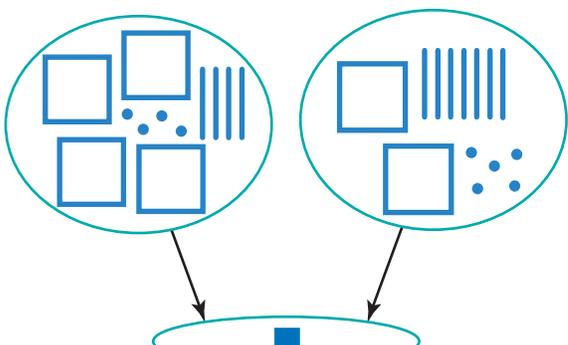
1 

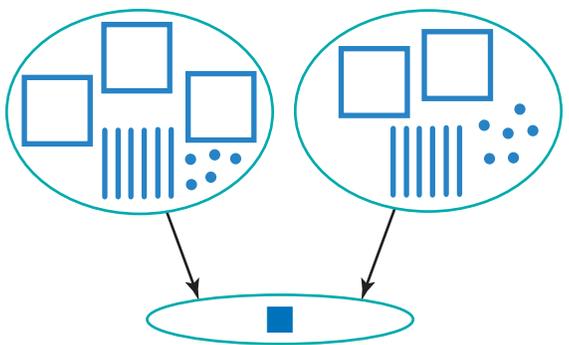
2 

Write the answer to each question.

- 3** Will $28 + 45$ be in the **sixties** or the **seventies**?
- 4** Will $16 + 78$ be in the **eighties** or the **nineties**?

Predict the number of base-ten flats you will need to show each sum with fewest blocks.

5 

6 

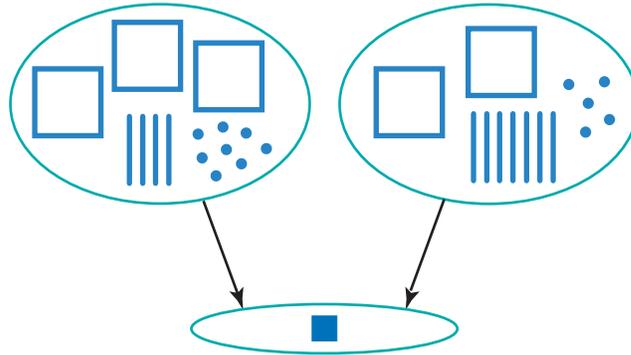
Write the answer to each question.

- 7** Will $356 + 482$ be in the **700s** or the **800s**?
- 8** Will $238 + 319$ be in the **500s** or the **600s**?

EXPLORE

Predicting Sums

Imagine combining the two piles of base-ten blocks and making trades until you have the fewest blocks for the sum.



- 1 How many flats will be in the sum?
- 2 How many rods will be in the sum?
- 3 How many units will be in the sum?
- 4 Add to check your predictions.

$$\begin{array}{r} 348 \\ + 275 \\ \hline \end{array}$$

Without calculating of the sum.

$$\begin{array}{r} 183 \\ + 594 \\ \hline \end{array}$$

, predict each digit

- 5 the hundreds digit
- 6 the tens digit
- 7 the ones digit
- 8 Add to check your predictions.

$$\begin{array}{r} 183 \\ + 594 \\ \hline \end{array}$$

REVIEW MODEL

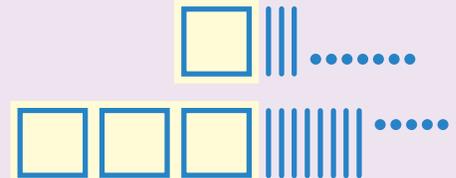
Addition with Regrouping

You can use base-ten block pictures to find a sum. Regroup to find the fewest blocks.

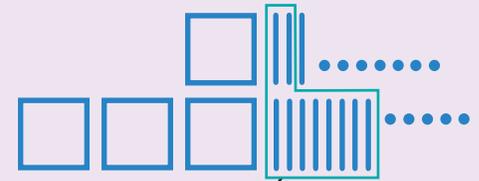
Step 1

Look at the hundreds.

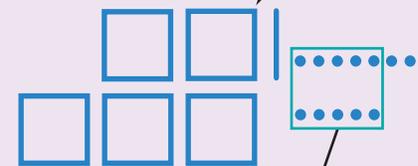
$$\begin{array}{r} 137 \\ + 385 \\ \hline \end{array}$$

**Step 2**

See if you have 10 tens you can regroup to make another hundred.

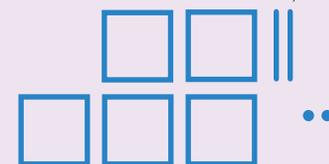
**Step 3**

See if you have 10 ones you can regroup to make another ten.

**Step 4**

Count the remaining blocks. There are 5 hundreds, 2 tens, and 2 ones. So,

$$\begin{array}{r} 137 \\ + 385 \\ \hline 522 \end{array}$$



✓ Check for Understanding

Find the sum. Draw the base-ten blocks on your own paper.

1
$$\begin{array}{r} 214 \\ + 269 \\ \hline \end{array}$$

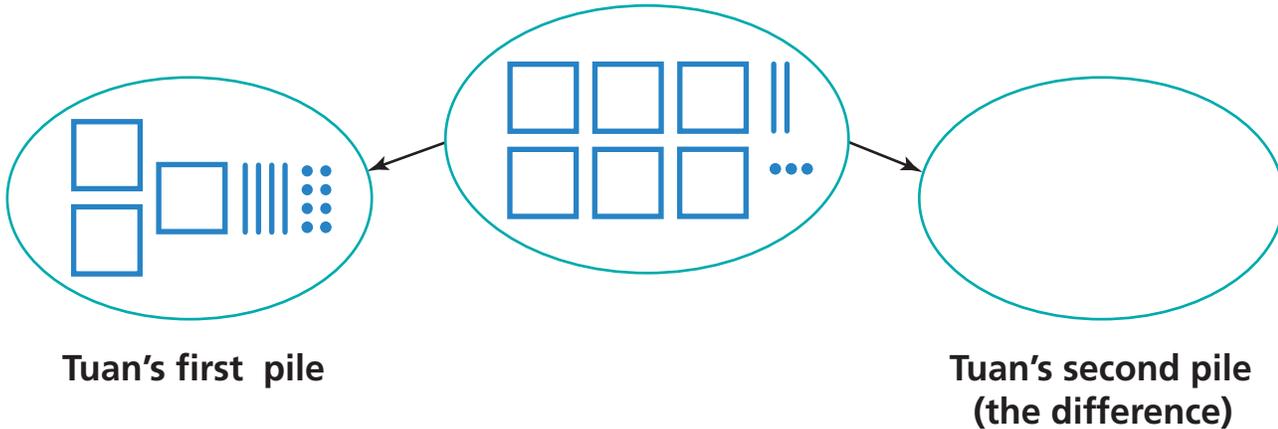
2
$$\begin{array}{r} 452 \\ + 178 \\ \hline \end{array}$$

EXPLORE

Predicting Differences

Tuan needed to split his collection of base-ten blocks into two piles. In order to put the blocks he wanted into the first pile, he had to trade some of his original blocks for smaller ones. Show what is left for the other pile.

Tuan's original collection



Tuan's first pile

Tuan's second pile
(the difference)

- 1 How many flats are in the difference?
- 2 How many rods are in the difference?
- 3 How many units are in the difference?

- 4 Subtract to check your predictions.

$$\begin{array}{r} 623 \\ - 348 \\ \hline \end{array}$$

Without calculating of the difference.

$$\begin{array}{r} 777 \\ - 594 \\ \hline \end{array}$$

, predict each digit

- 5 the hundreds digit
- 6 the tens digit
- 7 the ones digit

- 8 Subtract to check your predictions.

$$\begin{array}{r} 777 \\ - 594 \\ \hline \end{array}$$

REVIEW MODEL

What Is a Cross Number Puzzle?

Cross Number Puzzles are a tool for adding and subtracting multi-digit numbers.

Three-digit numbers are separated into hundreds, tens, and ones, so you can add or subtract each place value.

Addition Puzzle

	Hundreds	Tens	Ones	
345 + 564	300	40	5	345
	500	60	4	564
	800	100	9	909

Subtraction Puzzle

	Hundreds	Tens	Ones	
891 - 704	800	80	11	891
	700	0	4	704
	100	80	7	187

Use 80 and 11, so 4 can be combined with something (7) to get 11, rather than using 90 and 1 and trying to combine 4 with something to get 1.

Amounts on either side of a heavy line must be the same.

Addition Puzzle

300	40	5	345	$300 + 40 + 5 = 345$
500	60	4	564	$500 + 60 + 4 = 564$
800	100	9	909	$800 + 100 + 9 = 909$

300	40	5	345
+ 500	+ 60	+ 4	+ 564
800	100	9	909

Subtraction Puzzle

800	80	11	891	$800 + 80 + 11 = 891$
700	0	4	704	$700 + 0 + 4 = 704$
100	80	7	187	$100 + 80 + 7 = 187$

700	0	4	704
+ 100	+ 80	+ 7	+ 187
800	80	11	891

Check for Understanding

Copy the Cross Number Puzzle on your own paper. Then complete the puzzle.

1

400	20	3	■
100	50	9	■
■	■	■	■

2

500	140	■	647
■	50	6	256
■	■	■	■

REVIEW MODEL

Problem Solving Strategy

Solve a Simpler Problem

The volunteers made 430 care packages to send overseas. They mailed 249 packages on Monday. Do they have enough packages left to meet their goal of mailing at least 200 packages on Tuesday?

Strategy: Solve a Simpler Problem

Read to Understand

What do you know from reading the problem?

There were 430 care packages made and 249 packages were mailed on Monday. You need to find out if there are 200 packages left.

Plan

How can you solve this problem?

You can solve a simpler problem.

Solve

How can you solve a simpler problem?

You do not need to subtract the ones, tens, and hundreds to find the exact difference. You just need to find how many hundreds are left.

$$\begin{array}{r} 430 \\ - 249 \\ \hline \end{array}$$

Write a subtraction problem and work from left to right to find the number of hundreds in the difference. Look at the tens digits to see if you need to regroup a hundred to subtract the tens. You cannot subtract 4 tens from 3 tens, so you will need to trade 1 hundred for 10 tens. If you subtract 2 hundreds from the 3 hundreds left, you see they will not meet their goal of mailing 200 packages on Tuesday.

Check

Look back at the problem. Did you answer the question that was asked? Does the answer make sense?

Problem Solving Practice

Use the strategy *solve a simpler problem*.

- 1 It takes Yolanda 1 minute to copy 10 pages. She made 46 copies of one page and 52 copies of another page. Did Yolanda finish making the copies in 10 minutes?
- 2 Jeremy has \$2.00. He wants to buy a cup of yogurt for 89¢ and 2 pieces of fruit for 48¢ each. Does Jeremy have enough money?

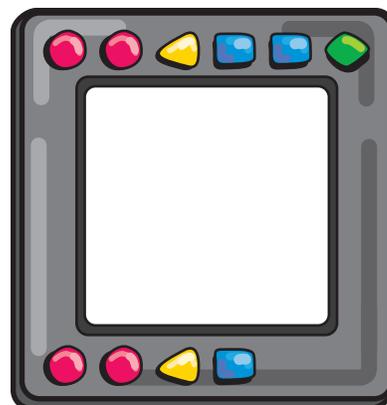
Problem Solving Strategies

- ✓ Act It Out
- ✓ Draw a Picture
- ✓ Guess and Check
- ✓ Look for a Pattern
- ✓ Make a Graph
- ✓ Make a Model
- ✓ Make an Organized List
- ✓ Make a Table
- ✓ **Solve a Simpler Problem**
- ✓ Use Logical Reasoning
- ✓ Work Backward
- ✓ Write a Number Sentence

Mixed Strategy Practice

Use any strategy to solve. Explain.

- 3 An artist can make 6 clay pots in 2 days. How many clay pots can he make in 8 days?
- 4 There are 36 birds in a special exhibit at the zoo. There are 8 more females than males. How many birds are male?
- 5 A road map shows the Baker family must travel 267 miles south and then 156 miles west to get to the campground. How far must they travel?
- 6 Josh brought home 3 watermelons left at the end of the picnic. There were 14 watermelons eaten at the picnic. Tory took 2 watermelons home. How many watermelons were there at the beginning of the picnic?
- 7 Marissa is making a picture frame. She glues stones across the top of the frame. She continues the pattern along the bottom. Look at the picture. What color stone will Marissa place next?



Choose the best vocabulary term from Word List A for each sentence.

- 1 A(n) ? is a line labeled with evenly spaced numbers.
- 2 To find a(n) ? of a sum, add rounded numbers instead of the exact numbers.
- 3 To find the ? between two numbers, subtract one number from the other.
- 4 The number 15 is a ? of 3.
- 5 Trade means the same as ?.
- 6 To find the ? of two numbers, add the numbers.
- 7 To ? a number to the nearest hundred, decide which multiple of 100 the number is closer to.

Word List A

difference
estimate
exchange
group
multiple
number line
predict
round
sum

Complete each analogy using the best term from Word List B.

- 8 Flat is to 100 as ? is to 10.
- 9 Addition is to ? as subtraction is to difference.

Word List B

unit
regroup
rod
sum

Talk Math

Discuss with a partner what you have learned about addition and subtraction. Use the vocabulary terms *group* and *regroup*.

- 10 How can you use flats, rods, and units to find the total for 2 three-digit numbers?
- 11 How do you know when you need to trade to find a difference?

Word Line

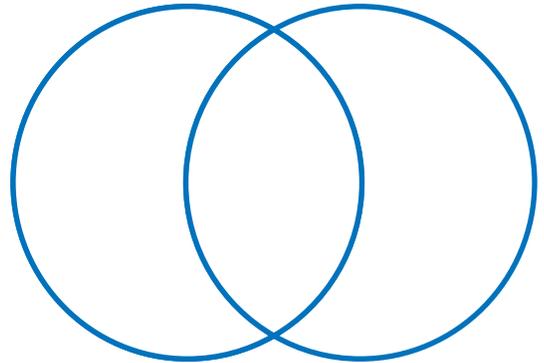
- 12 Create a word line for the words *unit*, *flat*, and *rod*.

Words:

Sequence:

Venn Diagram

- 13 Create a Venn diagram for addition and subtraction. Include these terms: *base-ten blocks*, *Cross Number Puzzle*, *difference*, *estimate*, *number line*, *regroup*, *sum*, and *total*.



What's in a Word?



TRADE The word *trade* has many different meanings. *Trade* can describe a job. "He is a car mechanic by *trade*." It can also be used to mean "business." "She did an excellent *trade* in souvenirs for the fair." It can be a naming word for an exchange. "He made a good *trade*." It can be an action word for an exchange. "She and her friend *traded* jackets." In math, you can *trade* hundreds for tens or tens for ones when subtracting. If you use base-ten blocks, you can *trade* or exchange 1 rod for 10 units.



Technology

Multimedia Math Glossary

www.harcourtschool.com/thinkmath

GAME

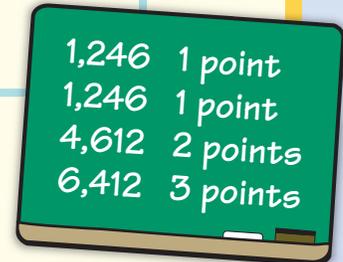
Ordering Numbers

Game Purpose

To practice comparing and ordering four-digit numbers

Materials

- 4 number cubes (labeled 1–6)



1,246	1 point
1,246	1 point
4,612	2 points
6,412	3 points

How to Play the Game

- 1** This is a game for 4 to 6 players. The goal is to score points by making up numbers from a toss of the number cubes.
- 2** The group tosses all 4 number cubes. Each player uses the numbers tossed to secretly write a four-digit number.
- 3** Everyone shows their numbers. Work together to put the numbers in order from smallest to largest.
- 4** This is how you earn points:
 - 2 points if no one else has the number
 - 1 point for the smallest number
 - 1 point for the largest number

Example: Number cubes



Player	Number	Points Earned	Total Points
Carli	5,265	No one else has it: 2 points Smallest number: 1 point	3
Lamar	5,562	No one else has it: 2 points	2
Royce	6,552	Largest number: 1 point	1
Becka	6,552	Largest number: 1 point	1

- 5** Keep playing until time is called. The player with the most points wins. Ties are possible.

GAME

Least to Greatest

Game Purpose

To practice estimating and ordering sums

Materials

- Activity Masters 33 and 49:
Least to Greatest Cards
- Stopwatch or a watch with
a second hand



How to Play the Game

- 1 Play this game with a partner. Cut out the cards from Activity Masters 33 and 49. Mix them up. Put them face down in a pile.
- 2 Choose one player to be the card Placer. The other player will be the Timer.
- 3 The Timer says, "Go." The Placer turns over one card at a time and puts it where it belongs in a line from the least sum to the greatest sum. You can pass on a card by setting it aside.
- 4 The Timer says "Stop" after 60 seconds and checks the order of the cards. The Timer says whether there are mistakes but not what the mistakes are.
- 5 The Placer may remove cards from the line to correct the order. When the Timer agrees that the order is correct, the Placer gets 1 point for each card.

Example: These four cards are placed correctly.

$\begin{array}{r} 18 \\ +36 \\ \hline \end{array}$	$\begin{array}{r} 14 \\ +68 \\ \hline \end{array}$	$\begin{array}{r} 481 \\ + 23 \\ \hline \end{array}$	$\begin{array}{r} 360 \\ +386 \\ \hline \end{array}$
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- 6 Switch roles. Play until time is called. The player with more points wins!

CHALLENGE

Here is a math trick that will let you add 3 three-digit numbers in your head very quickly.

Practice this trick on your own. Then try it on friends and family members.

Step 1 Ask someone to name a three-digit number. Suppose the person says 534. This number is the key to the answer. Write the number.

534

Step 2 Ask for a second three-digit number. Suppose the person says 741. Write it below the first one.

741

Step 3 Then write the third number. You want the second and third numbers to have a sum of 999.

Think: $7 + 2 = 9$, $4 + 5 = 9$, and $1 + 8 = 9$. So, you write **258**.

258

The addition looks like this:

$$\begin{array}{r} 534 \text{ First number} \\ 741 \text{ Second number} \\ + 258 \text{ Third number} \\ \hline \end{array}$$

Step 4 Now you can write the sum without adding the columns. Here's how.

Think: $741 + 258 = 999$, which is **1 less than 1,000**.

Think: $1,000 + 534$, or 1,534. That is why the first number is the key to the answer. The sum is **1,000 + the first number - 1**. And you can do that using mental math!

1,533

Practice the trick. Find the third number and the sum for each set of numbers without adding the columns. Then use a calculator to check.

1 428, 375

2 856, 602

3 787, 529