

6 Rules and Patterns

Dear Student,




The number of clouds on the back of this card was found by applying a rule to the number of clouds on the front. What do you think the rule could be?

Some ideas for the rule might be:

- Multiply the number of clouds on the front by 2.
- Add 1 cloud to the amount on the front.
- Add 3 clouds, and then subtract 2 clouds from the clouds on the front.

You cannot be sure of the rule until you see the other cards in the set that follow the same rule. In this chapter, you will be working with sets of cards like these as you think about patterns. Have fun finding a rule for all the cards!

Mathematically yours,
The authors of **Think Math!**

FRONT	BACK
	 
FAR Set I	



Beading Fun

For thousands of years, beads have been used for jewelry, decorations, and trade. Beads can be made from shells, gems, glass, and even from seeds.

FACT-ACTIVITY 1

Two girls are making necklaces. Alona uses 3 white beads for every red bead she puts in her necklace. Cara uses 3 more small beads than the number of large beads in her necklace.



Alona's Necklace	
Red	White
1	3
2	6
3	9
4	■
5	■

Cara's Necklace	
Large	Small
2	5
4	7
6	9
8	■
10	■

Use the tables to answer the questions.

- 1 What is a rule for finding the number of white beads in Alona's necklace for a particular number of red beads? How many white beads are needed if there are 4 red beads? 5 red beads?
- 2 Look at the table for Cara's Necklace. What is a rule for finding the number of small beads in Cara's necklace for a particular number of large beads? What are the missing numbers in the table?

FACT • ACTIVITY 2

Janet is stringing beads to make a necklace. The table below shows the relationship between the number of long purple beads and round yellow beads.

	2	4		8			
	3	6	9				

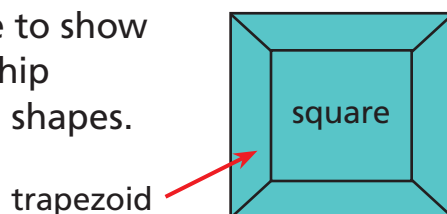
Use the table to answer the questions.

- Janet begins with 2 purple beads and 3 yellow beads. When there are 4 purple beads, how many yellow beads are there?
- When there are 9 yellow beads, how many purple beads are there?
- Complete the table.
- Draw a picture of what Janet's necklace might look like. Describe the pattern.

CHAPTER PROJECT

Look in magazines or catalogs for floor tiles that have simple geometric shapes. You may find something like the design below.

- Select a tile or draw a design that has a clear number of two or more geometric shapes on it.
- Suppose you have 2, 3, 4, or 5 of the tiles. Write a rule that gives the number of each shape when you know how many tiles you have.
- Make a table to show the relationship between the shapes.



ALMANAC Fact

The Bead Museum in Washington, D.C., has exhibits showing how beads have been used throughout history and amazing beaded crafts from all over the world. The largest bead in the collection is 6 inches long and weighs 6 pounds.

REVIEW MODEL

Recording Data

You can record information from Find a Rule cards onto a graph.

The numbers on the front of the FAR cards refer to the number of stickers. The numbers on the back of the cards refer to the cost of the stickers.

FRONT	BACK
1	10¢

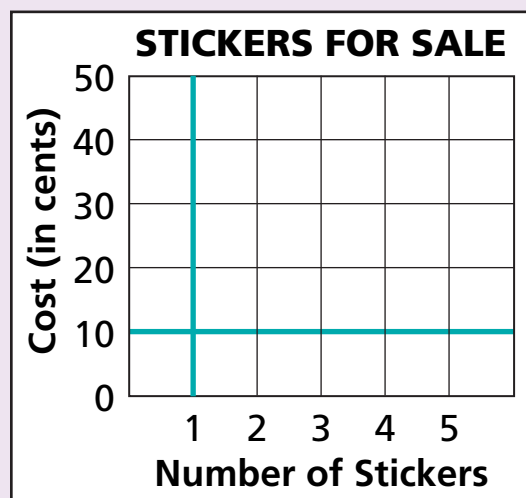
FRONT	BACK
2	20¢

Step 1

Look at the first card to find the first number of stickers. On the graph, find the line for 1 sticker.

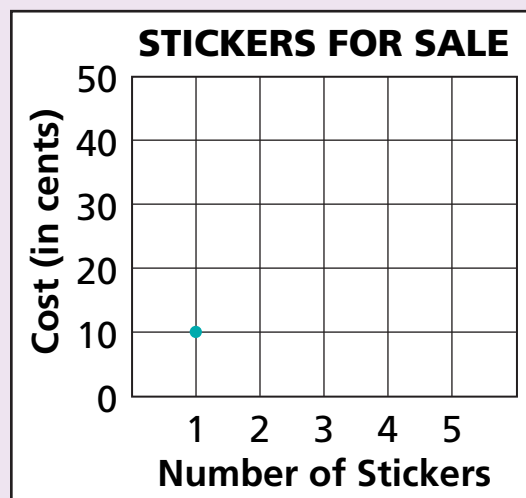
Step 2

Look at the first card to find the cost of the sticker. On the graph, find the line labeled 10¢.



Step 3

Place a point where the two lines intersect.



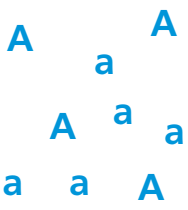
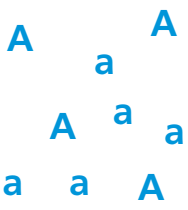


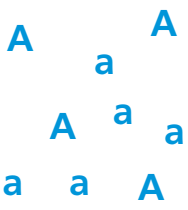




Check for Understanding

- 1 Use the second FAR card at the top of the page. Explain how you would record the information on the graph above.

EXPLORE

Exploring FAR Cards

These FAR cards have two rules.

A	<table> <tr> <th colspan="2">FRONT</th><th colspan="2">BACK</th></tr> <tr> <td colspan="2">  </td><td>Rule I 4</td><td>Rule II 9</td></tr> </table>	FRONT		BACK				Rule I 4	Rule II 9	B	<table> <tr> <th colspan="2">FRONT</th><th colspan="2">BACK</th></tr> <tr> <td colspan="2">  </td><td>Rule I 8</td><td>Rule II 14</td></tr> </table>	FRONT		BACK				Rule I 8	Rule II 14
FRONT		BACK																	
		Rule I 4	Rule II 9																
FRONT		BACK																	
		Rule I 8	Rule II 14																
C	<table> <tr> <th colspan="2">FRONT</th><th colspan="2">BACK</th></tr> <tr> <td colspan="2">  </td><td>Rule I 8</td><td>Rule II 9</td></tr> </table>	FRONT		BACK				Rule I 8	Rule II 9										
FRONT		BACK																	
		Rule I 8	Rule II 9																

This table shows the numbers on the backs of cards A, B, and C.

Card	Rule I	Rule II
A	4	9
B	8	14
C	8	9

On your own paper, continue the table for cards D, E, and F. The fronts of those cards are shown below.

D

FRONT	
a	A
a	a
A	A
a	a
a	A

E

FRONT	
A	A
A	A

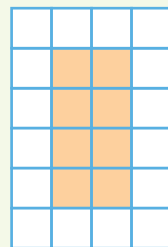
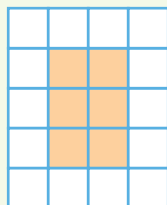
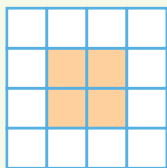
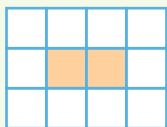
F

FRONT	
a	A
a	a
A	A
a	A
A	A

EXPLORE

Exploring a Pattern

Study the figures below. Look for a pattern.



- 1 How many tiles are in each figure?

- 2 How many orange tiles are in each figure?

- 3 How many white tiles are in each figure?

- 4 How does the total number of tiles change from one figure to the next figure?

- 5 How does the number of orange tiles change from one figure to the next figure?

- 6 How does the width of the figure change?

- 7 How does the length of the figure change?

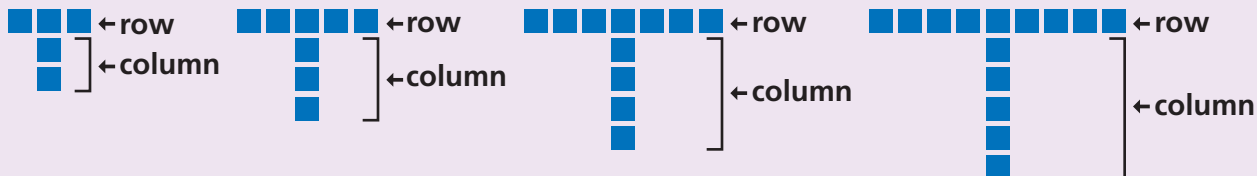
- 8 Draw the next figure.

REVIEW MODEL

Finding a Rule

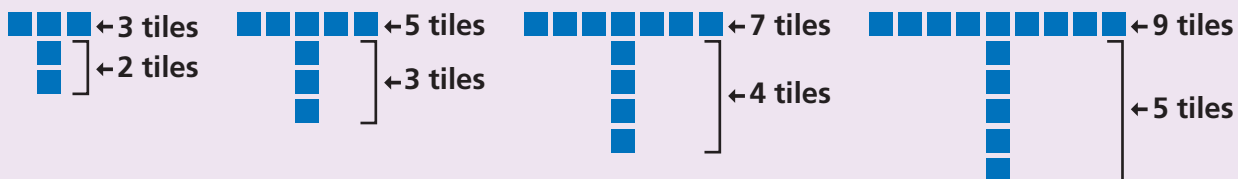
You can use a rule to describe the pattern in a sequence.

Step 1 Think about how each figure in the sequence is the same.



Each figure has a row with an odd number of tiles across the top and a column of tiles down the middle.

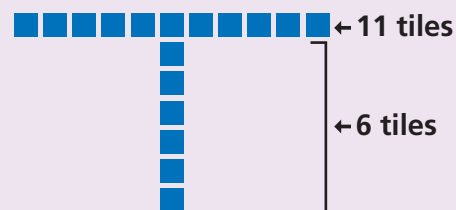
Step 2 Think about how each figure is different from the previous figure.



Each figure has two more tiles in the row across the top.
Each figure has one more tile in the middle column.

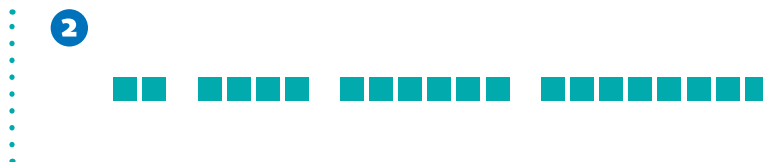
Step 3 State the rule and draw the next figure.

Add two more tiles to the top row. Add one more tile to the middle column.



✓ Check for Understanding

Draw the next figure following the pattern.



EXPLORE

Exploring the Number Line Hotel

A part of the Number Line Hotel is shown below. The entire hotel contains the number line from 0 to 99.

40	41	42	43	44	45	46	47	48	49
30	31	32	33	34	35	36	37	38	39
20	21	22	23	24	25	26	27	28	29
10	11	12	13	14	15	16	17	18	19
0	1	2	3	4	5	6	7	8	9

- 1 Look at the column of numbers above 6. What changes, and what stays the same for the numbers within the column?
- 2 What kind of jump would change the ones digit but not the tens digit?
- 3 How could you use the Number Line Hotel to help find $11 + 37$?
- 4 How could you use the Number Line Hotel to help find $25 + 16$?
- 5 How could you use the Number Line Hotel to help find $28 - 13$?
- 6 How could you use the Number Line Hotel to help find $32 - 26$?

REVIEW MODEL

Adding and Subtracting on a Grid

You can add and subtract by using a grid.

40	41	42	43	44	45	46	47	48	49
30	31	32	33	34	35	36	37	38	39
20	21	22	23	24	25	26	27	28	29
10	11	12	13	14	15	16	17	18	19
0	1	2	3	4	5	6	7	8	9

You can use an arrow to represent a move of 1 square on the grid.

↑: up ↓: down →: right ←: left

To Add

Move up to add tens.
Move to the right to add ones.

1 move up (↑) adds 10.

So, $26 + 10 = 36$.

3 moves right (→→→) adds 3.

So, $14 + 3 = 17$.

Add tens and ones by using a combination of moves.

Start on 13. Move up 3 (↑↑↑).

Move 2 to the right (→→).

You land on 45.

So, $13 + 32 = 45$.

To Subtract

Move down to subtract tens.
Move to the left to subtract ones.

4 moves down (↓↓↓↓) subtracts 40.

So, $45 - 40 = 5$.

1 move left (←) subtracts 1.

So, $14 - 1 = 13$.

Subtract tens and ones by using a combination of moves.

Start on 37. Move down 2 (↓↓).

Move 3 to the left (←←←).

You land on 14.

So, $37 - 23 = 14$.

✓ Check for Understanding

Write the landing number. Then write an addition or subtraction sentence to match.

1 $15 \uparrow \uparrow \rightarrow$ ■

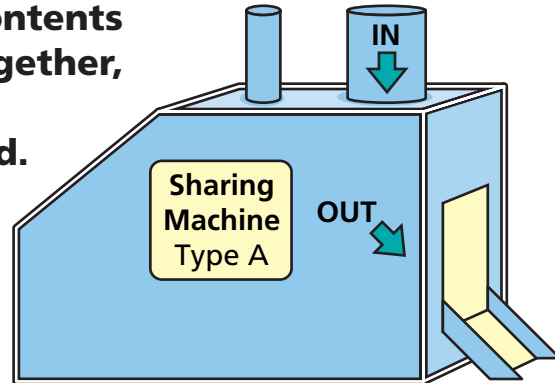
2 $7 \uparrow \uparrow \uparrow \rightarrow \rightarrow$ ■

3 $43 \downarrow \leftarrow \leftarrow \leftarrow$ ■

EXPLORE**Exploring Sharing Machine A**

This machine takes a package as input. It outputs two smaller packages that share the contents of the input equally between them. Together, these two smaller packages contain everything the input package contained.

Aki put a package containing 2 quarters, 4 dimes, and 2 pennies into the machine. Use coins to act out what the machine will do.



- 1** What was in each package that came out of the machine?

- 2** How much money did the input package contain?

- 3** How much money did each output package contain?

- 4** Aki then tried an input package of 6 dimes, 2 nickels, and 4 quarters. What came out of the machine?

- 5** What did Aki put in the machine if the output was two packages, each containing 2 quarters and 4 pennies?

- 6** Aki input a package of 18 marbles. What came out of the machine?

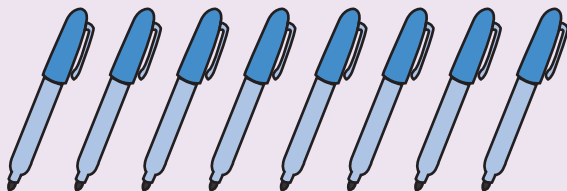
- 7** The machine returns all packages that it cannot share evenly without cutting an object. What might you put in the machine that it would return?

REVIEW MODEL

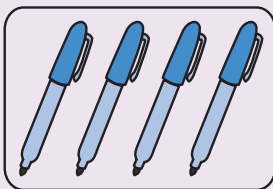
Writing Division Sentences

You can write division number sentences to represent sharing situations.

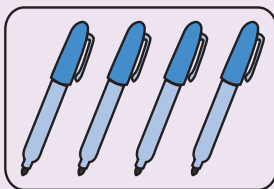
There are 8 markers.



The markers are shared equally between 2 students.



1



2

Each student gets 4 markers.
So, $8 \div 2 = 4$.

There are 15 trading cards.



The cards are shared equally among 3 friends.



Each person gets 5 cards.
So, $15 \div 3 = 5$.

✓ Check for Understanding

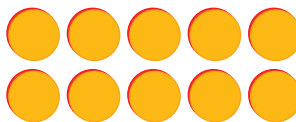
Write a division sentence for each situation.

1



There are 9 star stickers. They are shared equally among 3 third graders.

2



There are 10 counters. They are shared equally between 2 groups.

3



There are 12 quarters. They are shared equally among 3 brothers.

REVIEW MODEL

Problem Solving Strategy
Look for a Pattern

Leo makes a table to show how many stickers each of his brothers will get if he shares different numbers of stickers equally among them. How many stickers will each brother get if Leo shares 36 stickers?

Number of Stickers	12	4	8	28	20
Number Each Brother Gets	3	1	2	7	5

Strategy: Look for a Pattern**Read to Understand**

What do you need to find out?

You need to find out how many stickers each brother will get if Leo shares 36 stickers.

Plan

How can you solve the problem?

You can look for a pattern.

Solve

How can you look for a pattern to solve the problem?

Look for a pattern in the table. The rule is divide by 4; $36 \div 4 = 9$; So, each brother will get 9 stickers.

Check

Look back at the problem. Did you answer the question that was asked? Does the answer make sense?

Problem Solving Practice

Look for a pattern to solve.

- 1 Robert uses a pattern to stack boxes for a store display. He puts 11 boxes in the first row. He puts 9 boxes in the second row and 7 boxes in the third row. How many rows of boxes will be in the display when Robert is finished, if the top row has only 1 box?
- 2 Noriko uses a pattern to write the following numbers: 5 10 20 40
What is a rule for Noriko's pattern?

Problem Solving Strategies

- ✓ Act It Out
- ✓ Draw a Picture
- ✓ Guess and Check
- ✓ **Look for a Pattern**
- ✓ Make a Graph
- ✓ Make a Model
- ✓ Make an Organized List
- ✓ Make a Table
- ✓ Solve a Simpler Problem
- ✓ Use Logical Reasoning
- ✓ Work Backward
- ✓ Write a Number Sentence

Mixed Strategy Practice

Use any strategy to solve. Explain.

- 3 Jenna is choosing an outfit to wear. She can wear a long-sleeve shirt or a short-sleeve shirt. She can wear a skirt, short pants, or long pants. How many different outfits are possible?
- 4 Tyrone earned some money doing chores. He put half of what he earned in the bank. Then he paid his sister the \$2 he owed her. He has \$5 left to spend. How much did Tyrone earn doing chores?
- 5 There are 6 people in the Ling family. At the mall, each person in the family bought either a pizza for \$4 or a hot dog for \$3. They spent \$22 in all. How many pizzas and hot dogs did the Ling family buy?
- 6 Eduardo makes snack bags for a class trip. He has 34 pretzels. If he puts 3 pretzels in each bag, how many bags can Eduardo fill?
- 7 Jacob and Gabe plan to go fishing every week this summer. The table shows how many fish the boys have caught each week so far. If the pattern continues, how many fish will they catch in Week 5?

FISH CAUGHT					
Week	1	2	3	4	5
Number of Fish	9	12	15	18	

Choose the best vocabulary term from Word List A for each sentence.

- 1 $6 \div 3 = 2$ is a ____?
- 2 A(n) ____ can be made from two numbers and an operation symbol.
- 3 The number that is being divided is called the ____?
- 4 A(n) ____ is a picture of information.
- 5 In $45 \div 5 = 9$, the number 5 is the ____?
- 6 The number on the bottom of a fraction is called the ____?
- 7 A table can be used to show the ____ for an input.

Word List A

column
denominator
dividend
division
sentence
divisor
expression
graph
input
numerator
output
pattern
row
rule

Complete each analogy. Use the best term from Word List B.

- 8 Numerator is to denominator as ____ is to whole.
- 9 Addition is to sum as division is to ____.

Word List B

dividend
package
part
quotient

Talk Math

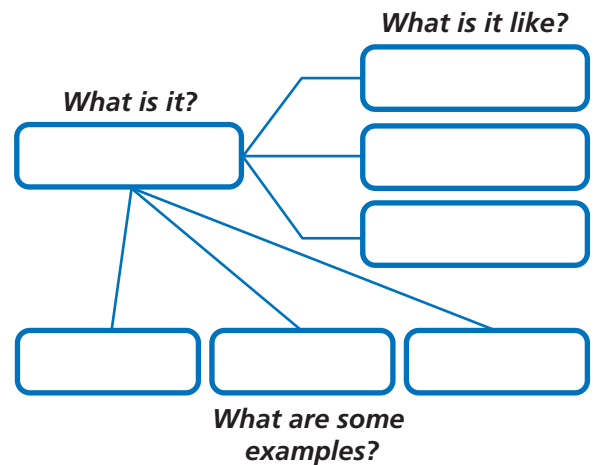
Discuss with a partner what you know about rules and patterns. Use the vocabulary terms *rule* and *pattern*.

- 10 You want to write a rule for a table. How do you know which operation to use?
- 11 How can you find a rule from a graph?

Word Definition Map

- 12 Create a word definition map for the word **graph**.

- A What is it?
- B What is it like?
- C What are some examples?



Tree Diagram

- 13 Create a tree diagram using the word **fraction**. Use what you know about the words **numerator**, **denominator**, **top number**, **bottom number**, **part**, and **whole**.

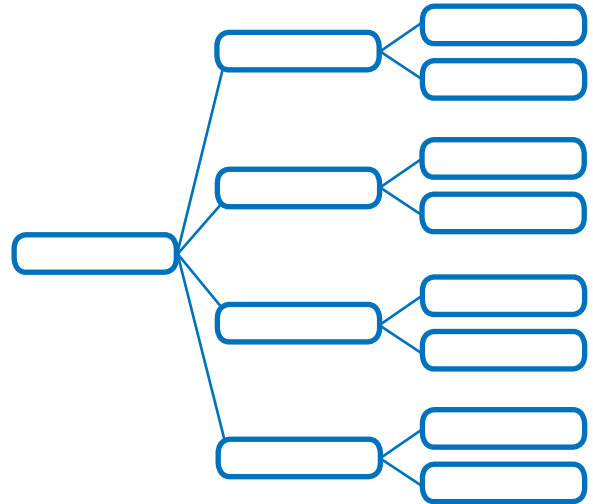


TABLE A *table* is a piece of furniture with a smooth flat top fixed on legs. Some people use a table as a desk or a workbench. A *table of contents* in a book is a short list of what is in the book and where to find it. In math, a table uses columns and rows to organize and display information like a table of contents. The information can be numbers, but it does not have to be.



Technology

Multimedia Math Glossary

www.harcourtschool.com/thinkmath

GAME

Find a Rule

Game Purpose

To practice using two-number inputs and outputs to find a rule

Materials

- Activity Master 68: Find a Rule

How to Play the Game

- 1 Play this game with a partner. Each player thinks of a rule for two input numbers and keeps it a secret.
- 2 Your partner names pairs of inputs. Write them in the first column of your table. Then use mental math or paper and pencil along with your rule to find the outputs.
- 3 When you have filled in the rows of the table, your partner guesses the rule and scores 1 point if correct.
- 4 Switch roles and continue playing. The winner is the player with more points when time is called.

Find a Rule		
Two Numbers	Rule A	Rule B
2, 3	6	
10, 9	90	
4, 2	8	
7, 4	28	
3, 11	33	
8, 8	64	

GAME

Make a Rule

Game Purpose

To practice finding a rule for a set of numbers

Materials

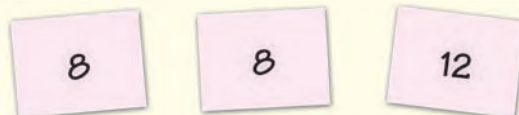
- Index cards



How to Play the Game

- 1 Play this game with a partner. Make 2 sets of number cards, each numbered 1 through 12. Mix up all the cards. Place them face down in a stack.
- 2 Take turns turning over a card from the top of the deck. Both players look for 3 cards on the table that follow a rule.
 - Rules such as “greater than” and “less than” are not allowed.
 - A rule can be used only once. For example, if the rule for a set of cards is “numbers that are even,” that rule cannot be used for another set of cards.
- 3 The first player to correctly name a rule for 3 cards takes the cards. Keep a record of the rules you use.
- 4 If neither player can name a rule, the cards remain on the table.

Example: The number cards are:



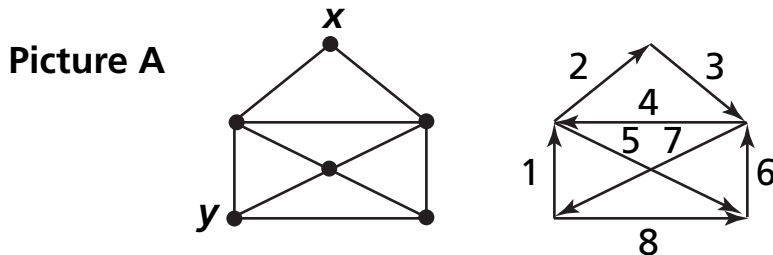
Possible rules are:

- even numbers
- one more than an odd number
- multiples of 4
- one less than an odd number

- 5 Play until all the cards in the stack have been turned over and neither player can find a rule for the cards that are left. The winner is the player with more cards.

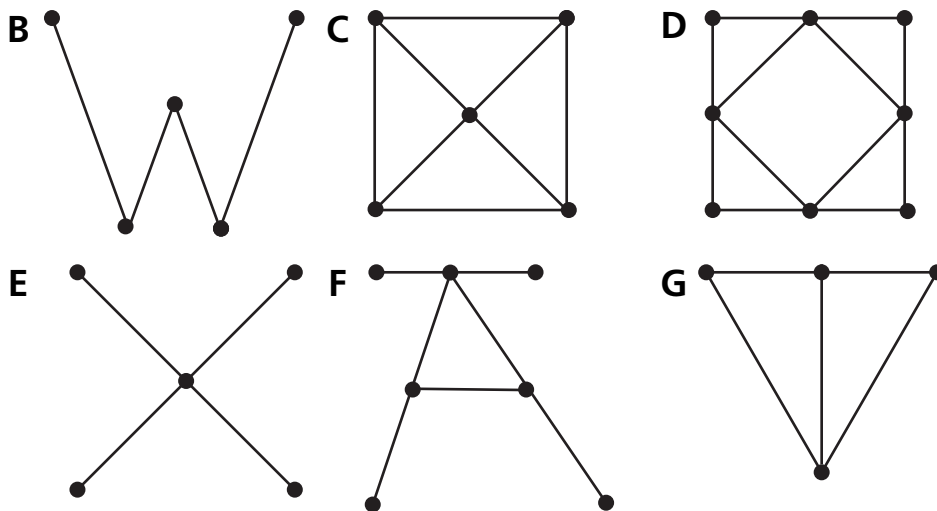
CHALLENGE

Put your finger on one corner of Picture A. Can you trace a path over the picture without going back over any line? Try it. A path that works is shown on the right.



Pictures like this one are called *networks*. They have even and odd corners. Corner x is even, because it has an even number of lines meeting at the dot. Corner y is odd, because it has an odd number of lines meeting at the dot.

Trace over each picture. Try not to go back over a line.



Use the table to help you look for a pattern.

Picture	A	B	C	D	E	F	G
How many odd corners?	2	■	■	■	■	■	■
Can you trace over it?	yes	■	■	■	■	■	■

You can trace over a picture if it has ■ or ■ odd corners.