Chapter 1 Magic Squares

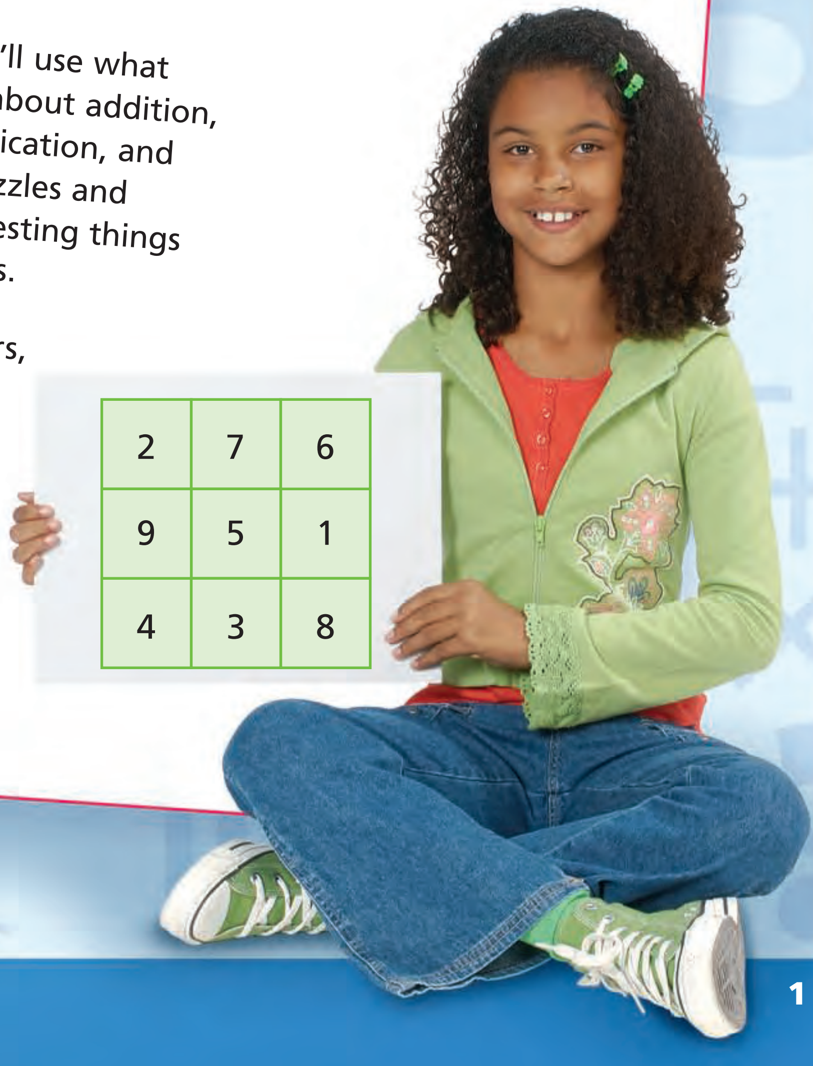
Dear Student,

As you can tell from the title of this chapter, "Magic Squares," you are about to spend some time exploring magic squares. Have you seen this type of math puzzle before?

A magic square is a grid of numbers arranged in a special way. What do you get if you add up the three numbers that make up the top row of the grid? Now try the same thing with the second row and the third row. Find the sums of the numbers in each column and each diagonal. What do you notice? Can you guess the special rule that makes this a magic square?

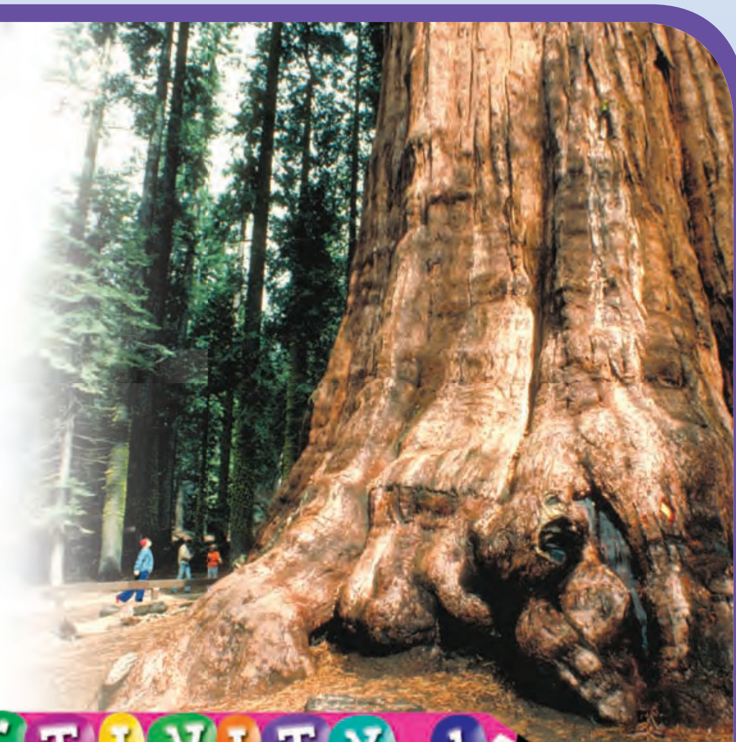
In this chapter, you'll use what you already know about addition, subtraction, multiplication, and division to solve puzzles and discover some interesting things about magic squares.

Mathematically yours,
The authors of
Think Math!



THE WORLD ALMANAC FOR KIDS

Tree Tales

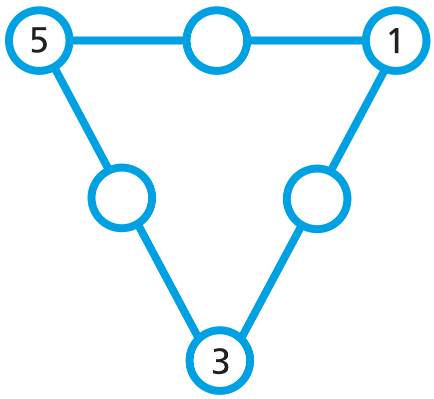


There are an enormous number of trees in the world. The tallest and most massive trees are California sequoia. Some are more than 300 feet tall. The largest is so wide that it might take 25 children holding hands to circle it completely! Most trees are much smaller. Many people plant small flowering trees around their homes.

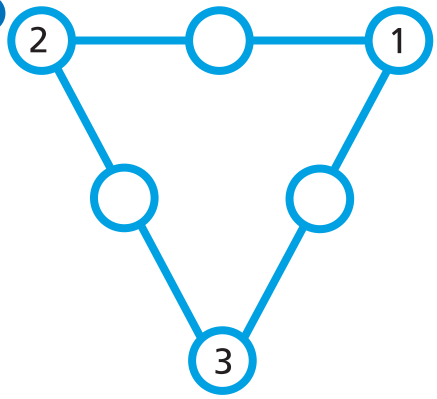
FACT·ACTIVITY 1

Larry the landscaper wants to plant groups of small flowering trees in a triangular pattern. The number of trees at the corners are shown. How many trees should he plant along each side so there are 10 trees along each line of the triangle?

1



2



FACT · ACTIVITY 2

In an effort to improve the environment, a fourth grade class helps a park ranger plant a total of 136 seedlings. The map shows the number of trees already planted in each of 16 regions of the park.

A student notices that the arrangement of trees planted so far resembles a magic square.

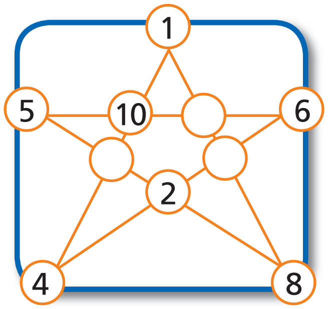
1. Copy and complete the square. How many seedlings need to be planted in each space to make the arrangement a magic square? You can work backward.
2. What will every sum be?

12		14	3
		2	15
1	16	7	10
13	4	11	

CHAPTER PROJECT

The magic star works similar to a magic square. The sum along any line must be 24.

- Work in groups to find the solution to this magic star.
- Now make your own magic square or magic star. You can use the square or star from this activity to help you get started.



ALMANAC Fact

Trees help keep the environment clean. An average mature tree will remove about 20 tons of pollution from the air each year.

Chapter 1
Lesson 3

EXPLORE
Subtracting Magic Squares

The picture shows the addition of magic squares A and B.

A

5	0	7
6	4	2
1	8	3

+

B

11	2	5
0	6	12
7	10	1

=

C

■	■	■
■	■	■
■	■	■

1. Find **C**. Is **C** a magic square?

What happens when B is subtracted from C?

Subtract the number in the upper left box of B from the number in the upper left box of C to find a number in the new grid.

C

■	■	■
■	■	■
■	■	■

−

B

11	2	5
0	6	12
7	10	1

=

C − B

5	■	■
■	■	■
■	■	■

2. Find **C − B**. Is **C − B** a magic square?

---

3. Can you predict what **C − A** will be without doing any additions or subtractions?

---

4. Write a subtraction sentence to show how you get one of the numbers in **C − B**.

---

5. Complete the fact family for the answer to **Problem 4**.

Chapter 1

Lesson 3

REVIEW MODEL
Subtracting with Magic Squares

The difference of two magic squares is a magic square.

A

19	10	16
12	15	18
14	20	11

−

B

1	8	6
10	5	0
4	2	9

=

A − B

18	2	10
2	10	18
10	18	2

Step 1 Verify the sum of each row, column, and diagonal in A is the same. The sum here is 45. A is a magic square.

Step 2 Verify the sum of each row, column and diagonal in B is the same. The sum here is 15. B is a magic square.

Step 3 Find the difference of the numbers in the corresponding boxes of magic squares A and B. Verify the sum of each row, column, and diagonal in A − B is the same. The sum here is 30. A − B is a magic square. Since the sums in A are 45 and the sums in B are 15, the sum of each row, column, and diagonal in A − B is 45 − 15 = 30.

19 − 1 = 18	10 − 8 = 2	16 − 6 = 10
12 − 10 = 2	15 − 5 = 10	18 − 0 = 18
14 − 4 = 10	20 − 2 = 18	11 − 9 = 2

Check for Understanding

1 Find the difference of magic squares D and E and verify the new grid is a magic square.

D

14	5	11
7	10	13
9	15	6

−

E

6	2	4
2	4	6
4	6	2

=

D − E

■	■	■
■	■	■
■	■	■

Chapter 1
Lesson 4

EXPLORE
Multiplying Magic Squares by Numbers

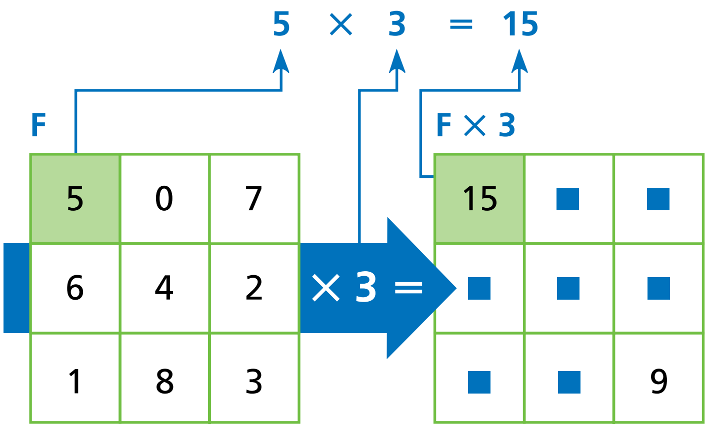
Let's see what happens when you multiply a magic square by a number.

1. Check that **F** is a magic square.

F

			■
5	0	7	■
6	4	2	■
1	8	3	■
■	■	■	■

Let's multiply **F** by 3. To find the number in the upper left box of the new grid, multiply the number in the same box of **F** by 3. Do the same for each box in the new grid.



2. Multiply **F** by 3. Is the result a magic square?

3. Do you think the product of a magic square and a number is always a magic square? Why or why not?

Chapter 1

Lesson 4

REVIEW MODEL
Multiplying a Magic Square by a Number

A product of a magic square and a number is a magic square.

Step 1

Check that C is a magic square.

The rows, columns, and diagonals all add to 27, so C is a magic square.

C

13	4	10
6	9	12
8	14	5

			27
13	4	10	27
6	9	12	27
8	14	5	27
27	27	27	27

Step 2

Multiply C by 4. To find the number in each box in the new grid, multiply the number in the corresponding box by 4. The sum of the rows, columns, and diagonals in C × 4 is 108 which is 4 × 27, the sum in magic square C.

C × 4

52	16	40
24	36	48
32	56	20

Check for Understanding

1. **Find the product of magic square T and 6. Verify it is a magic square.**

T

21	12	18
14	17	20
16	22	13

× 6 =

T × 6

			■
■	■	■	■
■	■	■	■
■	■	■	■
■	■	■	■

Chapter 1 Lesson 5

EXPLORE
Dividing Magic Squares

1 What happens when you divide a magic square by a number?

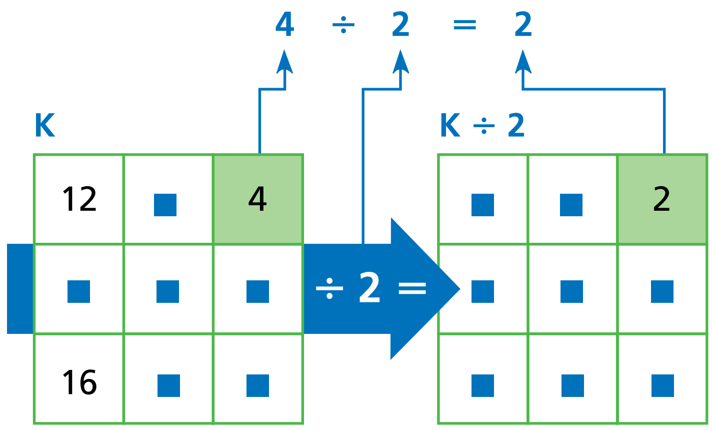
Complete magic square **K**.

K

			30
12	■	4	■
■	■	■	■
16	■	■	■
■	■	■	30

---

2 To find the number in the upper right box of **K** ÷ **2**, divide the number in the same box of **K** by **2**.



A Find **K** ÷ **2**.

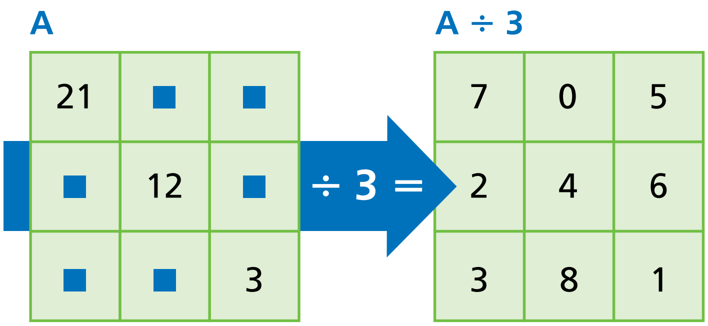
B Is the result a magic square? Why or why not?

---

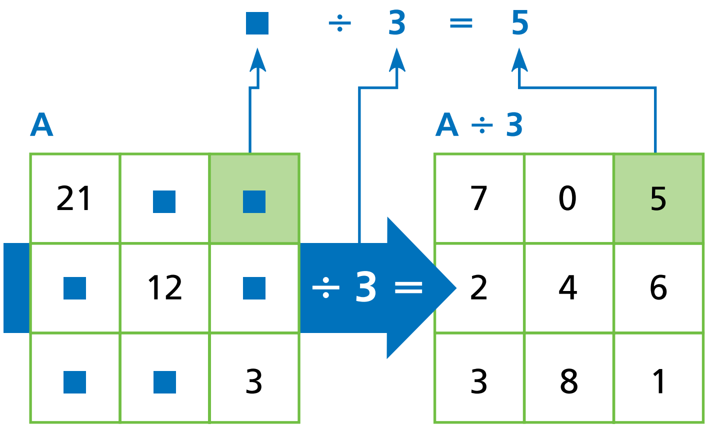
3 Do you think dividing a magic square by a number will always result in a magic square? Why or why not?

EXPLORE
Working Backward to Solve Division Puzzles

Here's a puzzle with magic squares.



Most of the numbers in the first magic square are missing, but you can use the numbers in the second magic square to help you fill them in.



This division sentence shows how to find the number in the upper right box of the magic square.

You can also rewrite it as a multiplication sentence: **3** × **5** = ■

1. Write a division sentence and a multiplication sentence about the lower left boxes of this puzzle. Does either of these sentences help you figure out what number to fill in the first magic square?

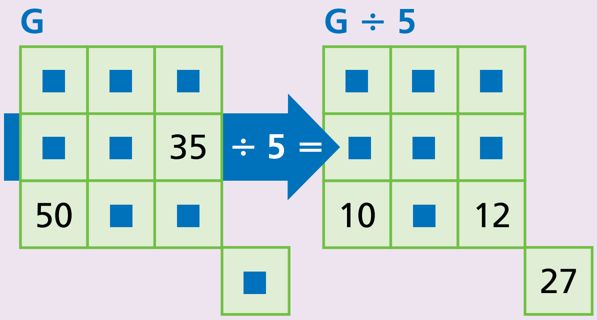
2. Use the numbers in the second magic square to help you complete the first magic square.

Chapter 1

Lesson 7

REVIEW MODEL
Problem Solving Strategy
Work Backward

Copy the magic squares on paper. Fill in the missing numbers to complete the magic squares.



Strategy: Work Backward

Read to Understand

What do you need to find?

I need to fill in the missing numbers so that each is a magic square and the division sentence is correct.

Plan

How can you solve this problem?

I can use the problem solving strategy work backward to fill in some of the missing numbers.

Solve

How can working backward help you find the missing numbers?

I can find the number in the lower right corner by working backward: $12 \times 5 = 60$. I can also work backward to find the sum of magic square G: $27 \times 5 = 135$.

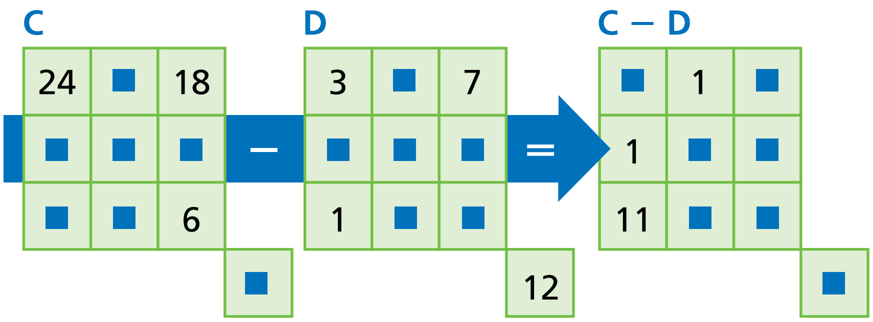
Check

Look back at the original problem. Does the answer make sense?

Yes. Each grid is a magic square and the division sentence is correct.

Problem Solving Practice

Use the strategy *work backward* to solve.

1. 

Problem Solving Strategies

- Act It Out
- Draw a Picture
- Guess and Check
- Look for a Pattern
- Make a Graph
- Make a Model
- Make an Organized List
- Make a Table
- Solve a Simpler Problem
- Use Logical Reasoning
- **Work Backward**
- Write an Equation

Mixed Strategy Practice

Use any strategy to solve. Explain.

2. Henry has 45 action figure cards. He starts adding 9 more to his collection each week. How many weeks until he has 81 cards?

3. Leonardo is buying 5 pounds of ground meat for $3 a pound and 5 packages of buns for $2 each. If he pays with a $50 bill, how much change should he receive?

For 4–5, use the table.

FAVORITE ICE CREAM FLAVOR	
Flavor	**Number of Students**
Chocolate	■
Mint Chip	54
Strawberry	21
Vanilla	82

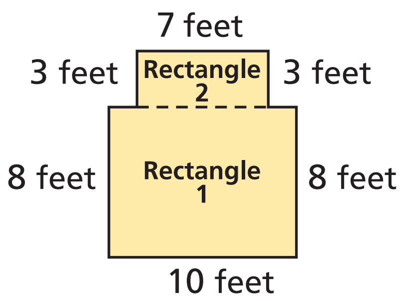
Andre surveyed 267 students about their favorite ice cream flavor.

4. How many students picked chocolate as their favorite flavor?

5. Put the ice cream flavors in order from most liked to least liked.

6. The band director had a special stage built for school performances.

   What is the area of this stage? Explain what strategy you used and how you solved the problem.



Chapter 1 Vocabulary

Choose the best vocabulary term from Word List A for each sentence.

1. Multiplication __?__ have at least one multiplication problem and at least one division problem.
2. Operations that undo each other, such as multiplication and division, are __?__.
3. When you multiply, the answer is the __?__.
4. In a magic square, two squares of a __?__ are the lower right square and the upper right square.
5. In a magic square, two squares of a __?__ are the lower right square and the lower left square.
6. A(n) __?__ is one of the numbers being added to make a sum.
7. When you divide, the answer is the __?__.
8. In a magic square, each number in a __?__ is in a different row and column.

Word List A

- addend
- column
- diagonal
- fact families
- inverse operations
- lower
- product
- quotient
- right
- row
- sum

Complete each analogy using the best term from Word List B.

9. Sum is to addition as __?__ is to multiplication.
10. Difference is to sum as __?__ is to product.

Word List B

- addend
- sum
- product
- quotient

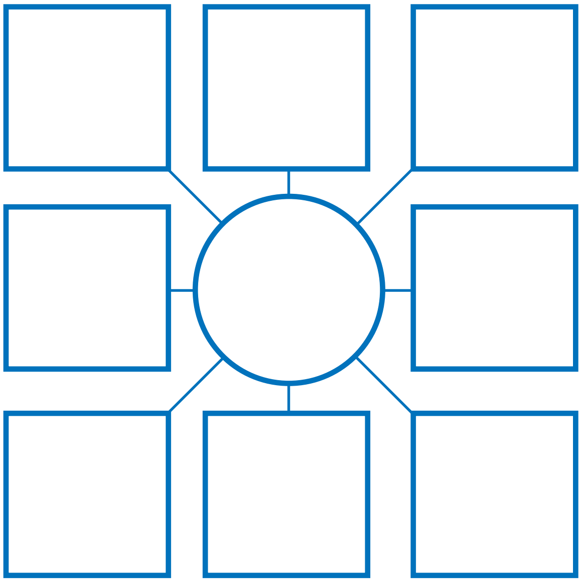
Talk Math

Describe what you have just learned about magic squares with a partner using the vocabulary terms in Word List A.

11. How can you use subtraction to create a new magic square?
12. How can you find the original magic square if a related magic square was made by dividing each number by 3?

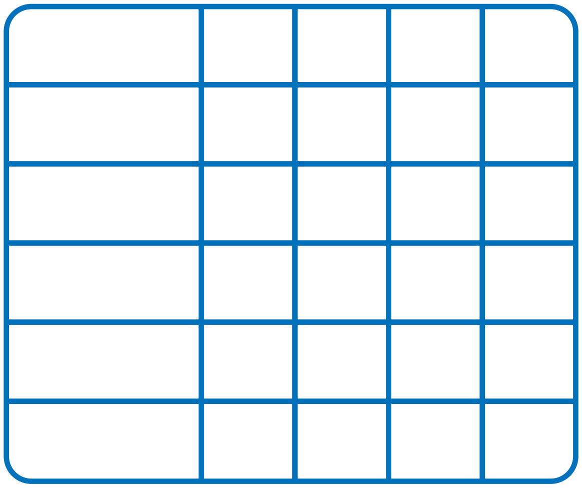
Concept Map

13 **Create a concept map for the words describing the positions of the squares of a magic square. Imagine the diagram as 3 rows and 3 columns of a magic square. Use the words *upper, lower, middle, right,* and *left.***



Analysis Chart

14 **Create an analysis chart for the terms *addend, sum, product,* and *quotient.***



What's in a Word?

COMMUTATIVE The term *commute* means "to change" or "to exchange one thing for another." Another meaning of *commute* is "to travel back and forth regularly." People generally commute between their homes and work. In mathematics, the term *commutative* means that when you add or multiply, changing the order of the numbers does not change the result.

GO ONLINE **Technology**
Multimedia Math Glossary
www.harcourtschool.com/thinkmath

GAME

Hit the Target!

Game Purpose
To practice addition and subtraction facts

Materials
- Activity Master 5 (*Number Cards*)
- index cards
- stopwatch or clock with second hand

How To Play The Game

1. Play this game with a partner. Cut out the number cards from Activity Master 5. Use index cards to make at least two sets of operation cards for $+$, $-$, and $=$.

2. Mix up the number cards and put them face down in a pile. Player 1 turns over the top card. This is the target number.

3. Player 2 turns over 4 more number cards. Player 2 has 1 minute to use all the number cards and any of the $+$, $-$, and $=$ cards to make the target number. Player 1 keeps track of the time.

   Example: The target number is 8. Player 2 has 2, 1, 6, and 3. Player 2 makes this number sentence and scores 1 point.

   $1 + 3 - 2 + 6 = 8$

   - If Player 2 cannot make a number sentence, Player 1 has 1 minute to try. If successful, Player 1 scores 1 point.
   - If neither player can make a number sentence, no point is scored.

4. Put all the cards back together. Mix them up, and switch roles.

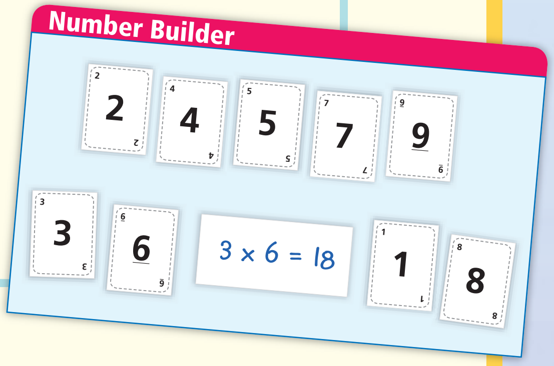
5. When time is called, the player with the most points wins.

GAME

Number Builder

Game Purpose
To practice facts

Materials
- Activity Master 5 (*Number Cards*)
- index cards
- stopwatch or clock with second hand



How To Play The Game

1. Play this game with a partner. Cut out the number cards from Activity Master 5. Use the index cards to make operation cards for $+$, $-$, $\times$, $\div$, (,), and $=$.

2. Mix up the number cards and put them face down in a pile. Player 1 turns over the top two cards to make a 2-digit number. This is the target number.

3. Turn the rest of the cards face up. Player 2 has 2 minutes to make the target number. The numbers on the cards can be used only as 1-digit numbers. Player 1 keeps track of the time.

   Example: The first 2 cards are 1 and 8, so the target number is 18.

   $(3 + 7) \times 2 - 6 + 4 = 1\ 8$

   - If Player 2 cannot make a number sentence, Player 1 has 2 minutes to try. If successful, Player 1 scores 1 point.
   - If neither player can make a number sentence, no point is scored.

4. Put all the cards back on the table. Mix them up, and trade roles.

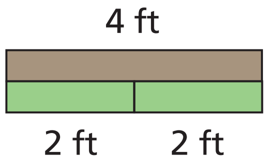
5. When time is called, the player with the most points wins.

CHALLENGE

Frank builds fences. He uses different lengths of logs to build different styles of fences. Below are plans for some of his fences.

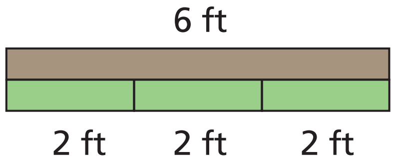
Frank has written out one way of finding the total number of logs and the total number of feet he needs for each fence.

Look at the shorter way. Then write the total number of feet.



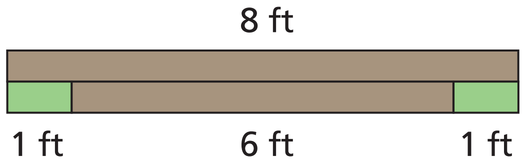
1. This fence will have 20 sections like this one.

   $(20 \times 4) + 20 \times (2 + 2) =$
   $20 \times 4 + 20 \times 4 = 20 \times 8 = \blacksquare$ feet



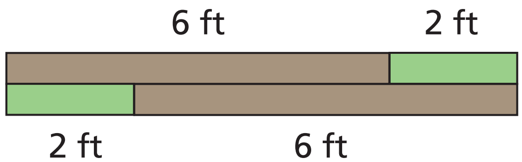
2. This fence will have 18 sections like this one.

   $(18 \times 6) + 18 \times (2 + 2 + 2) = \blacksquare$ feet



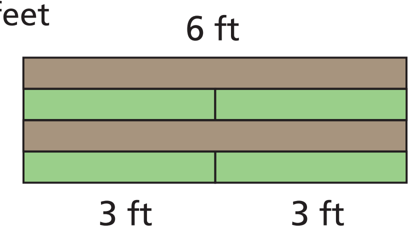
3. This fence will have 22 sections like this one.

   $(22 \times 8) + 22\ (1 + 1) + (22 \times 6) = \blacksquare$ feet



4. This fence will have 15 sections like this one.

   $15 \times (6 + 2) + 15 \times (6 + 2) = \blacksquare$ feet



5. This fence will have 19 sections like this one.

   $19 \times (2 \times 6) + 19 \times (4 \times 3) = \blacksquare$ feet