

10 Data and Probability

Dear Student,

If you toss a coin, how likely is it that the coin will come up heads? If you toss a coin 10 times in a row, about how many times would you expect to get heads? Could you get 10 heads in a row? Would it surprise you if that happened?

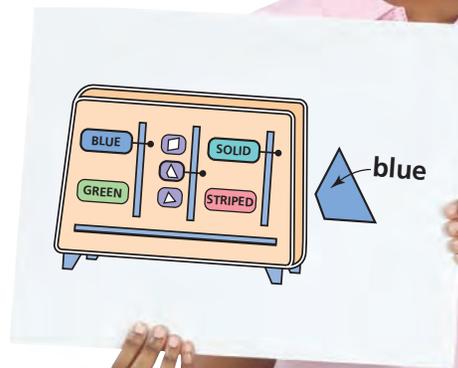
These are all questions about probability: how likely it is that some particular thing will happen.

Imagine a machine that prints out cards with figures on them. There are three possible figures: a parallelogram, a trapezoid, and a triangle. The figures can be either blue or green, and either striped or solid-colored. You can set each of the levers separately to pick the color, shape, and pattern that the machine will print on a card. In this picture, the machine has been set to print a solid blue trapezoid.

How many different combinations of color, shape, and pattern do you think the machine can make? How many of those combinations would be blue figures?

If you set the switches without looking, how likely is it that the machine will print a blue figure? You'll be talking about questions like this as you learn about probability.

Mathematically yours,
The authors of *Think Math!*



You Quack Me Up!

Whether it is a state fair, a county fair, or a school fair, there is something for everyone to smile about at a fair.



There is a children's duck pond game at Center Elementary School Fair. Twelve plastic ducks are in the pond and each duck has a star, circle, or triangle hidden on its bottom. You pick a duck at random from the pond. You will win a pencil top eraser prize depending on which symbol is on the bottom of the duck you pick. The table shows how many ducks have each symbol, and which pencil top eraser you will receive.

Duck Pond Game		
Symbol	Pencil Top Eraser	Number of Ducks
★	dinosaur	2
●	train	4
▲	smile face	6

FACT • ACTIVITY 1

- 1 What portion of the plastic ducks have a star? a circle? a triangle? Write each portion as a fraction.
- 2 If you pick a plastic duck at random, which pencil top eraser are you most likely to receive?
- 3 How many ducks with stars would there have to be to make the likelihood of receiving a dinosaur pencil top eraser $\frac{1}{12}$?

Another game at the school fair has a grid of squares with different colors. You toss a bean bag onto the grid. You then receive a pencil with a special message depending on the color of the square your bag lands on.

Bean Bag Toss Game	
Color	Message
White	Have a great day!
Yellow	You are so cool!
Red	Kids rule!

Yellow	White	Red	White	White
White	White	White	White	Yellow
White	Yellow	White	White	White
White	White	White	Red	White
White	White	Yellow	White	White

FACT • ACTIVITY 2

Use the chart and grid to answer the questions.

- 1 If your bag is equally likely to land on each square, what fraction of the game board wins the pencils that say, *Have a great day!*; *You are so cool!*; *Kids rule!*?

Olivia played the game 10 times and landed on: white, white, yellow, white, yellow, white, white, white, red, white.

- 2 Draw a bar graph to show the results of Olivia's 10 throws.
- 3 Based on Olivia's results, what fraction of the pencils she won say, *Have a great day!* or *Kids rule!*?

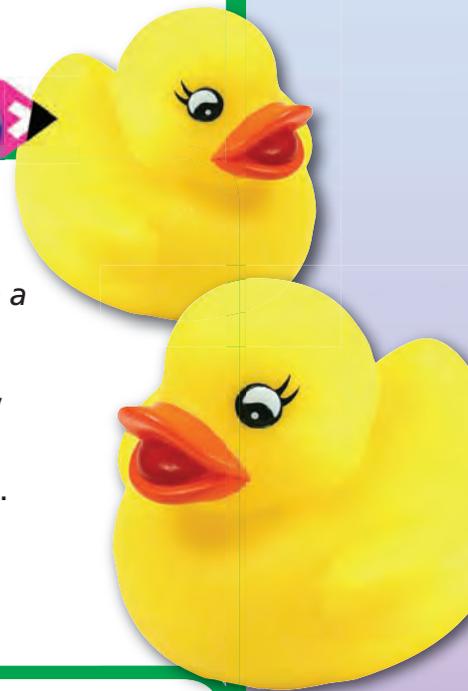
CHAPTER PROJECT

Sometimes spinners are used in games of chance. Design your own *Spin the Wheel* game. Draw a circle on cardboard. Divide the circle into 6 or 12 equal sections. Fill the sections using 3 different colors. Cut out the circle. Put the tip of a pencil through the center of the circle's top side. Place a paper clip around the pencil tip. Flick the paper clip to make it spin. Describe the rules of your game. Which color is the spinner most likely to land on? least likely?

- Play the game 20 times and collect the data. Show the data in a table and a bar graph.
- Using your table, determine the probability of each outcome as a fraction. Make a prediction of the next spin.

ALMANAC Fact

The first Texas State Fair was held in Fair Park, Dallas in 1886. Today, the 277-acre Fair Park is an education, entertainment, and recreation center where you can find museums, a music hall, and the famous Cotton Bowl Stadium.



EXPLORE

How Likely is It?

Becky and Sammi played “Fish” with the deck of attribute cards. Becky said the game wasn’t fair because some kinds of cards came up more often than others. You decide to explore this idea.

- 1 If you draw one card from your deck of attribute cards, what might it be? List all possibilities.
-
- 2 If you draw one card from your deck, is it *certain*, *likely*, *unlikely*, or *impossible* that the card will have a figure that is:
 - either striped or solid?
 - either a parallelogram or a triangle?
 - a trapezoid?
 - yellow?
 - a blue trapezoid?
 - green or striped or both?Be prepared to explain and discuss why you chose your answer.
-
- 3 Think of some other possibilities that are *certain*, *likely*, *unlikely*, or *impossible* if you draw one attribute card.

REVIEW MODEL

Writing Probabilities

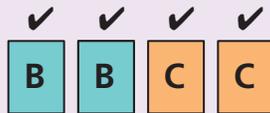
What is the probability of choosing a shaded card?



You can use fractions to write probabilities.

Step 1

Count to find the number of shaded cards.



There are 4 shaded cards.

Step 2

Count to find the total number of cards.

There are 8 cards altogether.

Step 3

Write the probability.

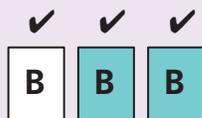
$$\text{probability} = \frac{\text{shaded cards}}{\text{total cards}}$$

$$\text{probability} = \frac{4}{8} \text{ or } \frac{1}{2}$$

What is the probability of choosing "B"?

Step 1

Count to find the number of "B" cards.



There are 3 "B" cards.

Step 2

Count to find the total number of cards.

There are 8 cards altogether.

Step 3

Write the probability.

$$\text{probability} = \frac{\text{"B" cards}}{\text{total cards}}$$

$$\text{probability} = \frac{3}{8}$$

Check for Understanding

1 What is the probability of choosing a striped card?

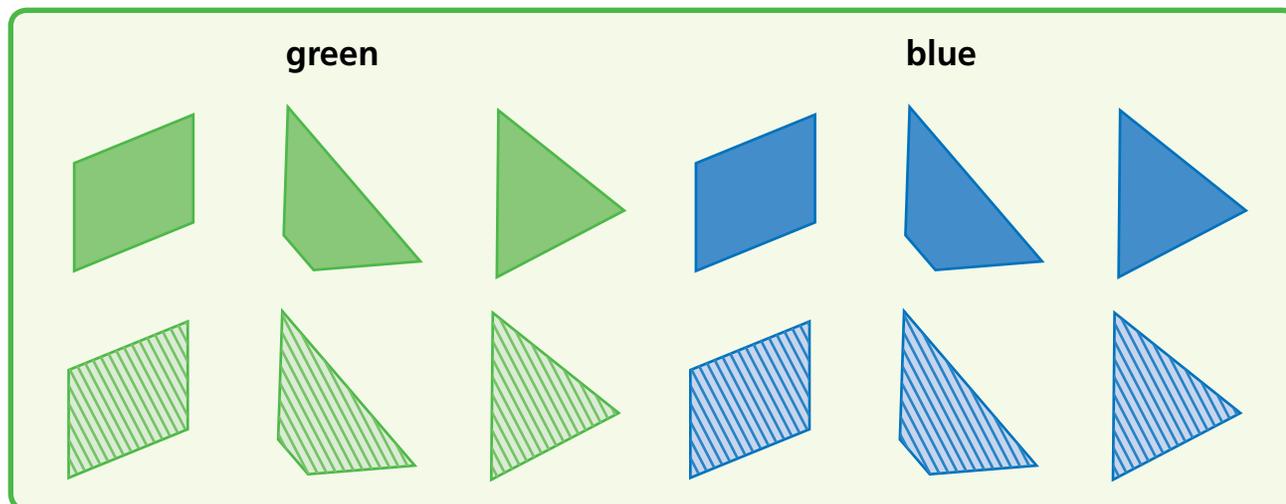


2 What is the probability of choosing a "Y"?

3 What is the probability of choosing an unshaded, unstriped "X"?

EXPLORE

How Likely is Drawing a Trapezoid?



Imagine that you:

- draw one attribute card randomly from the deck
- write down what is on the card
- return the card to the deck
- shuffle the deck

1 If you repeat these steps 30 times, about how many times do you think you will pick a card with a trapezoid on it?

2 About what fraction of the cards you drew do you predict will have trapezoids?

3 Write at least 3 fractions equivalent to the one you wrote for Problem 2.

REVIEW MODEL

Finding Equivalent Fractions Using Patterns

You can use patterns to write a fraction that is equivalent to another fraction. Look for a relationship between the top and bottom numbers in the first fraction. The relationship should involve multiplication or division. Use the same relationship to write an equivalent fraction.

Find a fraction equivalent to $\frac{2}{6}$.

Step 1

How are the top and bottom numbers related?

$\frac{2}{6}$ The bottom number is 3 times the top number.

$$6 = 3 \times 2$$

Step 2

Use the same relationship to write an equivalent fraction.

One Way

$\frac{1}{3}$ The top number is 1.
The bottom number is 3×1 .

Another Way

$\frac{5}{15}$ The top number is 5.
The bottom number is 3×5 .

Find a fraction equivalent to $\frac{8}{10}$.

Step 1

How are the top and bottom numbers related?

$\frac{8}{10}$ Multiply (or divide) both top and bottom by the same number.

$$\frac{8 \div 2}{10 \div 2} = \frac{4}{5}$$

Step 2

Use the new fraction to write an equivalent fraction.

One Way

$$\frac{4 \times 3}{5 \times 3} = \frac{12}{15}$$

Another Way

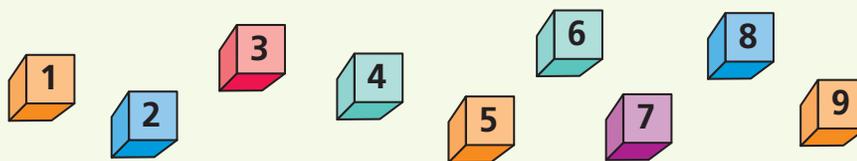
$$\frac{4 \times 7}{5 \times 7} = \frac{28}{35}$$

Check for Understanding

1 Find two fractions equivalent to $\frac{2}{8}$.

2 Find two fractions equivalent to $\frac{6}{10}$.

9-Block Experiment



1 If you put these blocks into a bag and drew one without looking, what is the probability that the number on your block would be:

- even?
- a multiple of 3?
- a square number?
- at least 5?

2 If you draw a block as in Problem 1 and do this 27 times, putting the block back each time, about how many blocks would you expect to draw whose number is:

- even?
- a multiple of 3?
- a square number?
- at least 5?

3 Think of at least 2 more predictions you can make about the experiment described in Problem 2.

REVIEW MODEL

Making a Bar Graph

Making a bar graph is like building towers out of blocks. You can compare sets of data by comparing the heights of the towers.

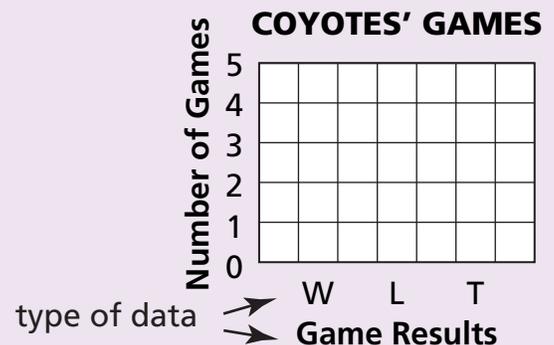
At the right are the results of the Coyotes' first 8 soccer games (W = win, L = loss, T = tie). Draw a bar graph of the results.

RESULTS OF COYOTES' GAMES

W W L T T W T W

Step 1

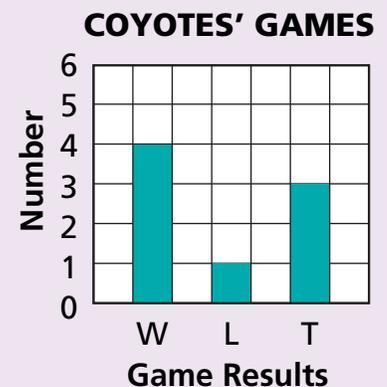
Draw and label a grid. Let the horizontal axis represent the type of game result. Let the vertical axis represent the number of games for each type of data.



Step 2

Graph the data. Start at the bottoms of columns. Shade one square for each win, one square for each loss, and one square for each tie.

The completed graph allows you to compare numbers of wins, losses, and ties visually as well as numerically.



✓ Check for Understanding

Below are the ways 15 students get to school (W = walk, C = car, B = bus, S = subway). Draw a bar graph of these means of transportation.

SCHOOL TRANSPORTATION

S C B W B C B C
S S W B S C B

REVIEW MODEL

Problem Solving Strategy

Make a Graph

Listed below are the types of instrument played by the members of the school band (B = brass, P = percussion, S = string, W = woodwind). One student was absent from rehearsal yesterday. What is the probability that the student plays a brass instrument?

S B W B B P S B W P B W W B S S B S W S B W W P

Strategy: Make a Graph

Read to Understand

What do you know from reading the problem?

the types of instruments played by the band members

What do you need to find out?

the probability that a student plays a brass instrument

Plan

How can you solve this problem?

Display the data in a bar graph. Then count squares to find the probability.

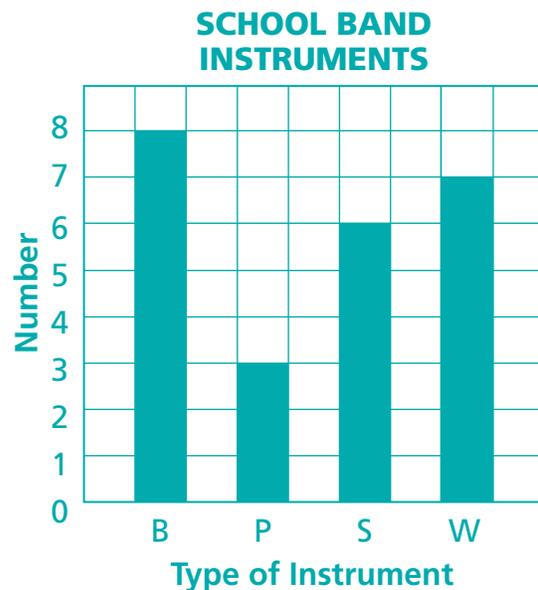
Solve

How can you find the probability?

First, make and label a grid. For each item of data, shade a square in the correct column. Eight of the 24 shaded squares represent brass instruments. The probability is $\frac{8}{24}$ or $\frac{1}{3}$.

Check

Look back at the problem. Did you answer the question that was asked? Does the answer make sense?



Problem Solving Practice

Use the strategy *make a graph* to solve.

- 1 Ashton tossed a number cube 18 times. She tossed three 1s, two 2s, four 3s, five 4s, zero 5s, and four 6s. Show how she can display the data to allow easy analysis of her results.
- 2 Look at the graph you made in Problem 1. Which number did Ashton toss the expected number of times? How does the graph show this? How does the graph show this?

Problem Solving Strategies

- ✓ Act it Out
- ✓ Draw a Picture
- ✓ Guess and Check
- ✓ Look for a Pattern
- ✓ **Make a Graph**
- ✓ Make a Model
- ✓ Make an Organized List
- ✓ Make a Table
- ✓ Solve a Simpler Problem
- ✓ Use Logical Reasoning
- ✓ Work Backward
- ✓ Write an Equation

Mixed Strategy Practice

Use any strategy to solve.

- 3 Jake scored 100 points total on three math quizzes. He scored 29 and 42 on the first two quizzes. What did he score on the third quiz?
- 4 A square has an area of 100 square inches. What is the perimeter of the square?
- 5 Baseball cards sell for \$12 each. Football cards sell for \$9 each. Mason bought 3 baseball cards and 5 football cards. He paid for his purchase with a \$100 bill. How much change did he receive?
- 6 Movie tickets cost \$8. The movie theater has 18 rows of seats with 14 seats in each row. At the last show there were 96 empty seats. How much was spent on the purchase of tickets for that show?
- 7 Eggs sell for \$1.45 per dozen. Becky bought 72 eggs. How much did the eggs cost?
- 8 Mrs. Fritz is 3 times as old as her son Marco. Her daughter Hallie, who is 7 years old, is half as old as Marco. How old is Mrs. Fritz?
- 9 Apples sell for \$1.56 per pound. Peaches sell for \$1.66 per pound. If the price of apples increases 4¢ per week and the price of peaches increases 2¢ per week, what will the price be when both items sell for the same price?
- 10 Blake scored 28 points in a basketball game. Some of his points came from 2-pointers and the rest came from 3-pointers. He scored twice as many 2-pointers as 3-pointers. How many 3-pointers did he score?

Choose the best vocabulary term from Word List A for each sentence.

- 1 A(n) ___?___ is often stated as some number from 0 to 1.
- 2 Information can also be called ___?___.
- 3 An event with a probability of less than $\frac{1}{2}$ is a(n) ___?___ event.
- 4 To find the ___?___ of a set of data, subtract the smallest number from the largest number.
- 5 An event with a probability of 1 is a(n) ___?___ event.
- 6 An event with a probability of 0 is a(n) ___?___ event.
- 7 A measurement of 1 hour has less ___?___ than a measurement of 54 minutes.
- 8 A(n) ___?___ is a possible result of an action.
- 9 The ___?___ of a set of data is the item that appears more often than any of the other items.

Word List A

attribute
certain
data
impossible
likely
median
mode
outcome
precision
probability
range
unlikely

Complete each analogy. Use the best term from Word List B.

- 10 Usually is to ___?___ as always is to certain.
- 11 B is to ABC as ___?___ is to a set of data.

Word List B

likely
median
probability

Talk Math

Discuss with a partner what you have just learned about data and probability. Use the vocabulary terms *certain*, *impossible*, *likely*, and *unlikely*.

- 12 Suppose temperatures increased 2 degrees each day last week. How can you describe temperatures for the next day?
- 13 A coin is flipped 100 times. How can you describe the outcomes?

Word Line

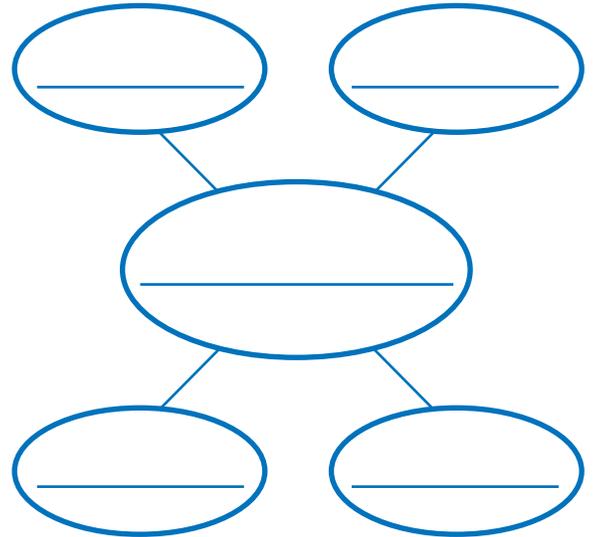
- 14 Create a word line for the terms *certain*, *impossible*, *likely*, and *unlikely*. Arrange the words from 0 to 1.

Words:

Sequence:

Concept Map

- 15 Create a concept map for *Describe Data*. Use what you have learned about ways to describe a set of data.



What's in a Word?



DATA Ancient Romans did not have e-mail, so they wrote messages by hand. At the end of a message, they wrote “*datum*,” meaning “given” and the month and day. More than one *datum* is *data*. The Romans used *data* to mean “the time and place stated.”

Today, we use the word *data* to mean information collected about people or things. Weights, heights, lengths, dates, and populations are all *data*.



Technology

Multimedia Math Glossary

www.harcourtschool.com/thinkmath

GAME

Attribute Memory

Game Purpose

To practice identifying common attributes

Materials

- Activity Master 90: Machine Cards
- blue and green pencils
- scissors

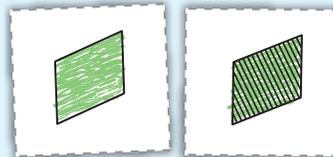


How To Play The Game

- 1** Play this game with 3 players. On, Activity Master 90, shade the top 6 figures blue. Shade the bottom 6 figures green. Cut out the cards.
- 2** Mix up the cards, and place all 12 face down in a 4-by-3 array.
- 3** The first player turns over two cards.
 - If the figures on the cards have two attributes in common and one that is different, the player keeps the cards.
 - If the figures have no attributes in common, the player puts the cards back face down where they were in the array.
- 4** Players take turns repeating Step 3 until no more cards can be taken. There could be up to 4 cards left on the table when no more can be taken.
- 5** The player with the greatest number of cards wins.

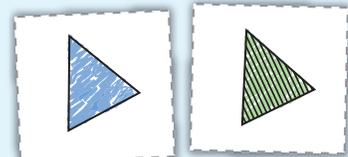
Example:

You could keep this pair.



The shape and color are the same, but the shading is different.

You could not keep this pair.



The shape is the same, but the shading and color are different.

GAME

Attribute Card Forecast

Game Purpose

To practice estimating probabilities

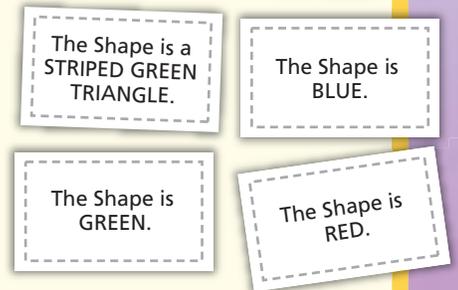
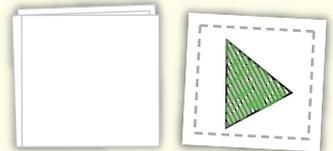
Materials

- Set of 12 Attribute Cards
- Activity Masters 92, 93, and 94 (Event Cards)

How To Play The Game

- 1** Play this game with 3 or 4 players.
 - Mix up the Attribute Cards. Place the pile of Attribute Cards face down on the table.
 - Mix up the Event Cards. Pass out the Event Cards equally among the players. Set aside any leftover cards.
 - Decide who will go first.
- 2** Player 1 chooses one Event Card from his or her cards and puts it face up on the table. The other players take turns doing the same, moving clockwise from Player 1.
- 3** Player 1 turns the top Attribute Card face up.
- 4** Any player whose Event Card describes the Attribute Card scores 1 point. The description must be correct. It does not have to be complete.

Example: For this round, two Attribute cards are correct. Can you find them?
- 5** Put the Attribute Card back in the pile, and mix up the cards.
- 6** Repeat Steps 2–4. This time, the player to the left of Player 1 goes first.
- 7** Play the game until all the Event Cards have been used. The player with the greatest number of points is the winner.



CHALLENGE

Play these games with a partner. You'll need two number cubes labeled 1 to 6. Decide who will be Player 1 and Player 2 for each game. Play both games several times. Then use what you know about probability to decide whether each is a fair game. Any game is fair if all players have an equal chance of winning.

Play a Subtraction Game

- Take turns tossing both number cubes and subtracting the smaller number from the larger number.
 - Player 1 gets 1 point if the difference is an odd number.
 - Player 2 gets 1 point if the difference is an even number. Remember, 0 is an even number.
- The first player with 10 points wins the game. Play again.
- After you have played several times, copy the above table. Complete the table, and use it to help you decide if this a fair game.

-	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

Play a Multiplication Game

- Take turns tossing both cubes and multiplying the numbers.
 - Player 1 gets 1 point if the product is an odd number.
 - Player 2 gets 1 point if the product is an even number.
- The first player with 10 points wins the game. Play again.
- After you have played several times, copy this table. Complete the table, and use it to help you decide if this a fair game.

x	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						