

# 11 Three-Dimensional Geometry

## Dear Student,

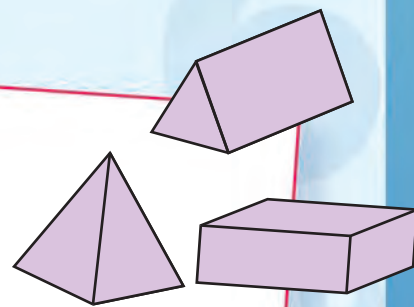
Three-dimensional objects have height, width, and depth. Most such objects, especially those that occur in nature, have complicated shapes (think of trees, people, and clouds). But many things that people make have simple three-dimensional shapes (think of milk cartons, tin cans, the room you're in, and spaghetti). Even some natural objects such as crystals and water drops have simple shapes.

In this chapter, you'll explore many such simple shapes. You'll make some of them by folding paper.

Some three-dimensional figures have curved surfaces. If all the surfaces of a figure are flat, we call that figure a *polyhedron*. In this chapter you'll learn why we refer to the surfaces of a polyhedron as "faces," rather than the more familiar word "sides." You'll find the total area of a polyhedron's faces, which tells how much paper you would need to wrap the polyhedron if you were giving it to someone as a gift. And you'll learn how to measure the volume *inside* a polyhedron, which tells the amount of air, water, wood, or metal it could contain.

Now, it's time to start building! Have fun!

Mathematically yours,  
The authors of *Think Math!*



# Wrapping It Up!



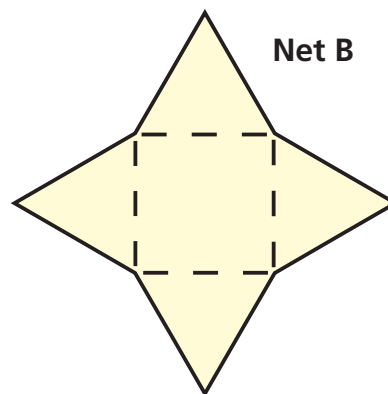
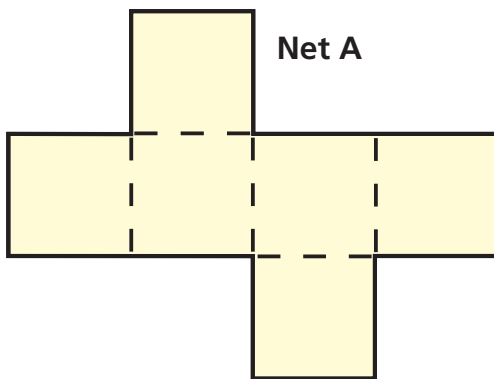
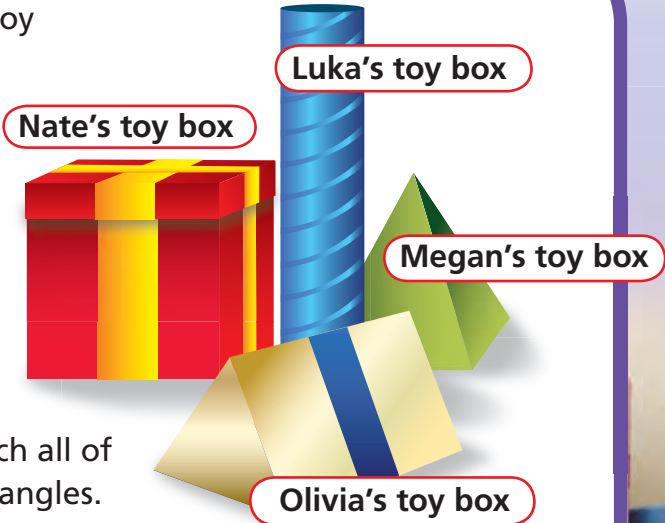
Cardboard was invented in China in the 1600s. About 200 years later the English used cardboard to make cardboard boxes.

## FACT • ACTIVITY 1

Luka, Megan, Nate, and Olivia each buy a toy from a toy shop. The toys are packaged in cardboard boxes of various shapes.

Use the toy boxes to answer 1–4.

- 1 Name the three-dimensional figure represented by each toy box.
- 2 Describe the faces of Megan's toy box. How many faces are there? Are the faces congruent to each other?
- 3 Which person's toy box has faces in which all of the angles are congruent? Describe the angles.
- 4 Which of the nets below is a net for Megan's toy box?

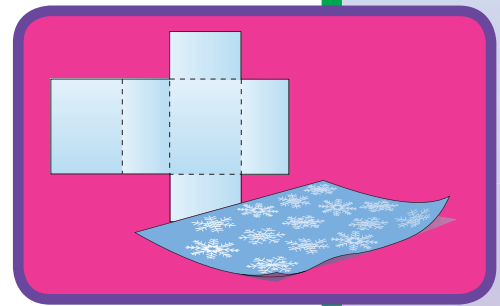
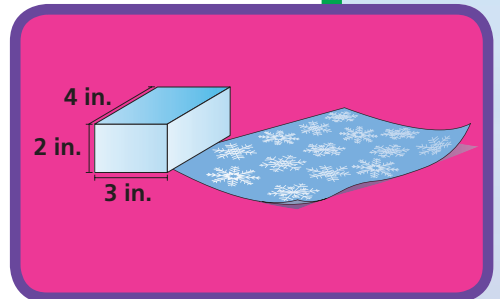


## FACT • ACTIVITY 2

People have been wrapping gifts for almost 2,000 years when paper was invented in China. Today, you can buy all sorts of fancy gift wrap.

### Max needs to wrap this gift box.

- 1 Trace the net below on a piece of paper. Label the net with the measurements of each edge using the box drawing at the right.
- 2 Find the total area of the faces of the box.
- 3 Suppose Max's gift wrap measures 5 in.  $\times$  10 in. Does he have enough to wrap the box? Explain.
- 4 Max wants to fill the box with candy that originally filled a box that was 6 in. long, 2 in. wide, and 2 in. high. Will the candy fit in the box? Explain.



## CHAPTER PROJECT

### Materials: empty boxes, one-inch cubes, inch ruler

How good are you at estimating volume? Work in groups of 3 or more. Gather a collection of empty boxes shaped like rectangular prisms, such as cereal boxes, shoe boxes, or tissue boxes. Use various sizes.

- Write down the number of one-inch cubes you think will fit in each box. Carefully place as many cubes as you can in each box. Record your results. Compare your estimates to the number of cubes that actually fit in the boxes.
- Then, measure to the nearest inch the length, width, and height of each box and find the volume for each box. Record your results.
- How do you explain the difference between the volume found by placing the cubes in the box and the volume found using the formula?

## ALMANAC Fact

Edwin Binney and C. Harold Smith made their first box of crayons in 1903. There were only 8 colors back then. They now make bigger boxes with as many as 120 colors.

# REVIEW MODEL

## Recognizing Three-Dimensional Figures

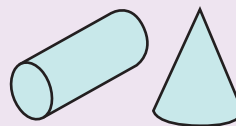
You can use the faces of a three-dimensional figure to find the name of the figure.

**Step 1** Decide: Are the faces flat and polygon-shaped?

**YES.** The faces are flat and polygon-shaped. The figure is a **polyhedron**.



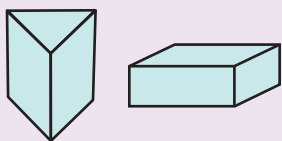
**NO.** At least one face is curved. The figure is *not* a **polyhedron**.



**Step 2** If the shape is a polyhedron, decide:

Can it be placed on a table so that the top and bottom faces are parallel and congruent?

**YES.** The figure is a **prism**.



**NO.** The figure is *not* a **prism**. But if it can sit flat on one face on a table, and if its other faces are triangles that meet at a point, it is a **pyramid**.



**Step 3** If at least one face is curved, decide:

Does it have a sharp point?

**YES.** The figure is a **cone**.



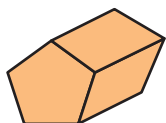
**NO.** The figure is *not* a **cone**. But if it can be placed on a table so that the top and bottom faces are parallel and congruent, it is a **cylinder**.



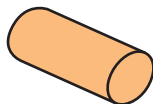
### ✓ Check for Understanding

Name the figure.

1



2



3



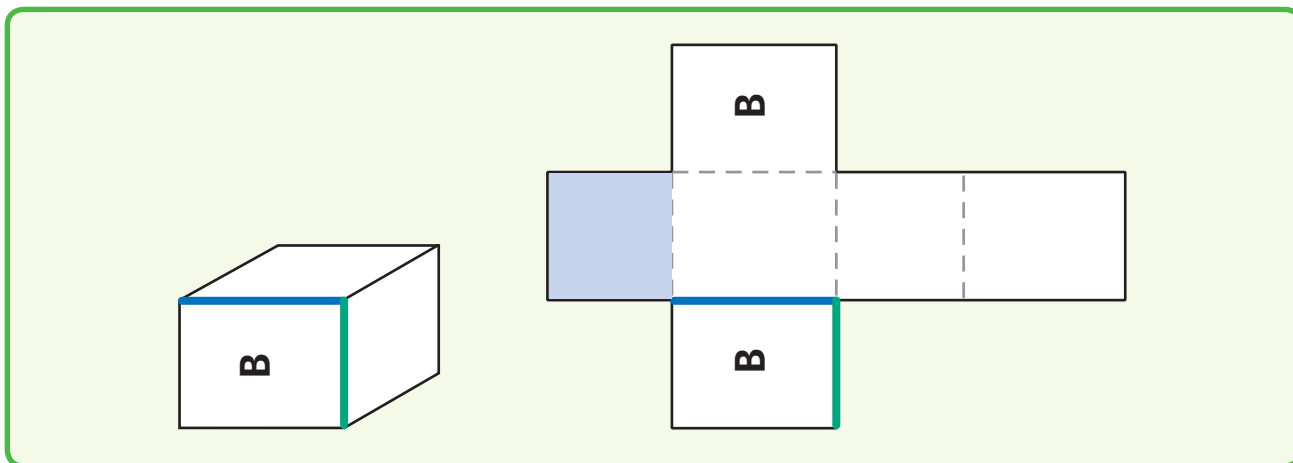
4



# EXPLORE

## Finding Areas

The faces of Figure B are all rectangles.



Use the net of Figure B page to answer these questions.

- 1 Estimate the length of the green edge of Face B in inches.
- 2 Estimate the length of the blue edge of Face B in inches.
- 3 Estimate the area of Face B in square inches.
- 4 Estimate the perimeter of the net of Figure B.

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Use a ruler to measure the edges of Figure B.

- 5 Using your measurements, find the perimeter of the net of Figure B.
- 6 Using your measurements, find the area of Face B in square inches.
- 7 Using your measurements, find the area of the shaded face in square inches.
- 8 Find the total area of all of the faces of this polyhedron in square inches.

# REVIEW MODEL

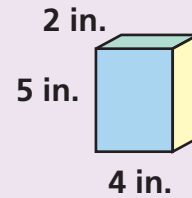
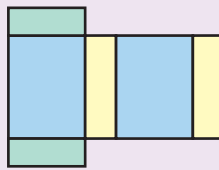
## Finding Areas of Faces

You can use the net of a prism to find the total area of the faces of the prism.

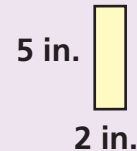
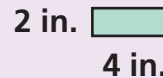
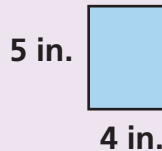
Find the total area of the faces of the prism at the right.

**Step 1** Look at a net of the prism. Decide which faces are congruent.

- The blue faces are congruent.
- The yellow faces are congruent.
- The green faces are congruent.



**Step 2** Use the prism to find the length and width of each different face.



**Step 3** Multiply the length by the width to find the area of each face.

$$5 \times 4 = 20$$

$$2 \times 4 = 8$$

$$5 \times 2 = 10$$

**Step 4** Add the areas of the faces to find the total area. Remember to include the areas of the congruent faces.

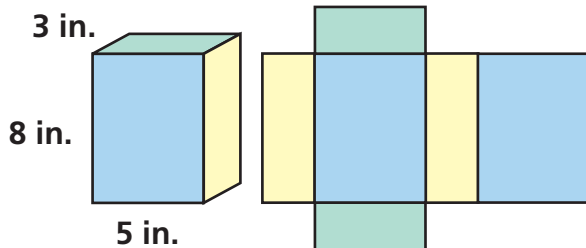
$$20 + 20 + 8 + 8 + 10 + 10 = 76$$

The total area of the faces is 76 square inches.

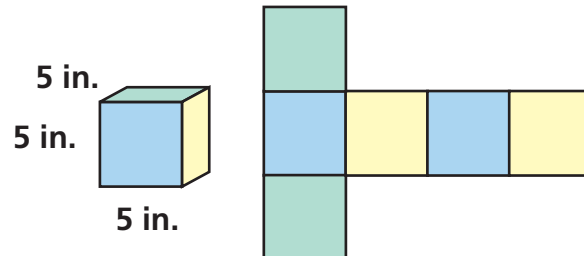
### Check for Understanding

Find the total area of the faces of the prism.

1

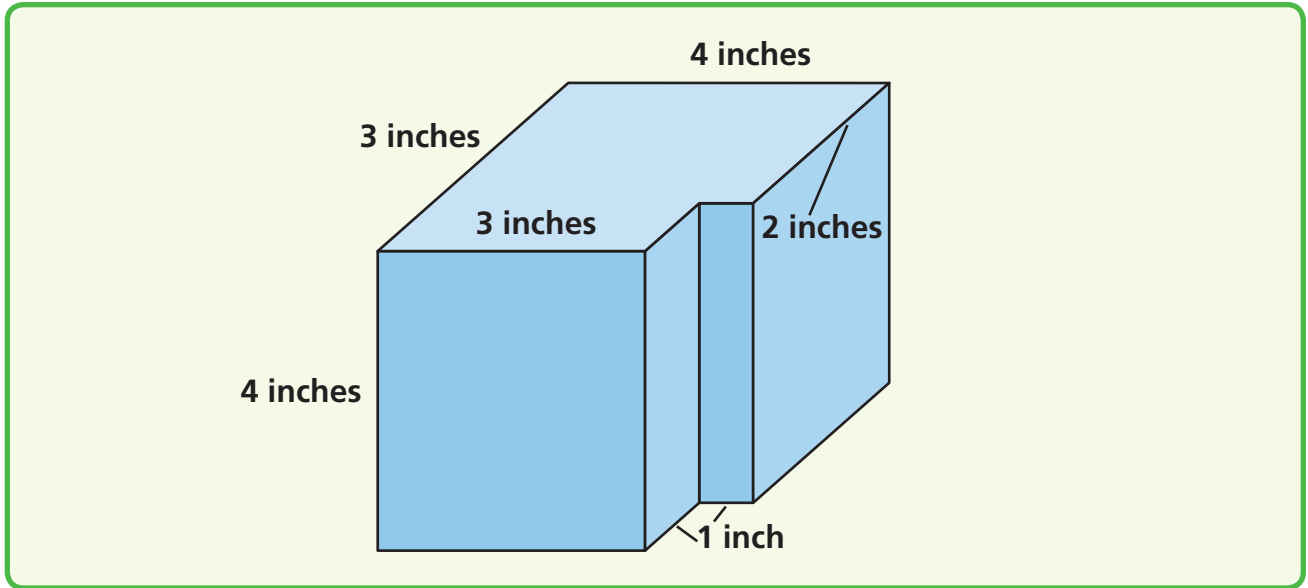


2



# EXPLORE

## Exploring Volume



**In a minute, you're going to build this three-dimensional figure out of inch cubes.**

- 1 How many cubes do you think you will need?

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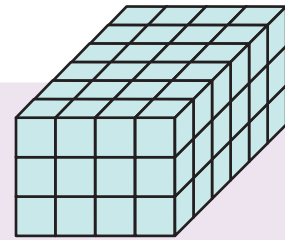
**Now build the shapes with cubes.**

- 2 How many cubes did you use?

## REVIEW MODEL

## Finding the Volume of a Three-Dimensional Figure

The *volume* of an object is a measure of the amount of space it takes up. You can count inch cubes and use multiplication to find the volume of a rectangular prism.

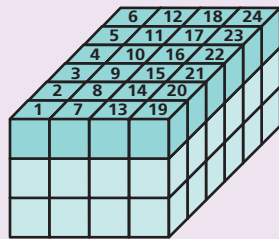


Find the volume of the prism at the right.

**Step 1** Find the number of cubes in **one layer** of the prism.

**One Way**

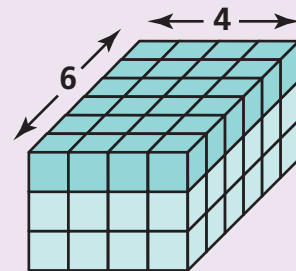
Count cubes.



There are 24 cubes in the top layer.

**Another Way**

Multiply the lengths of the two sides of the layer.



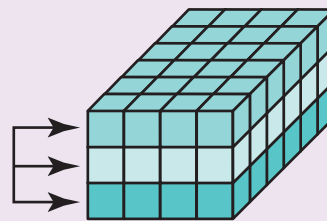
There are 24 cubes in the top layer.

**Step 2** Multiply your answer by the number of layers in the prism.

$$3 \times 24 = 72$$

The volume of the prism is 72 cubic units.

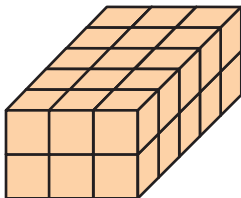
3 layers



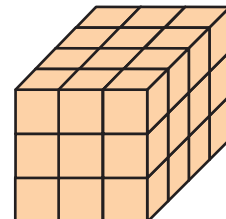
### Check for Understanding

Find the volume of the rectangular prism.

1



2

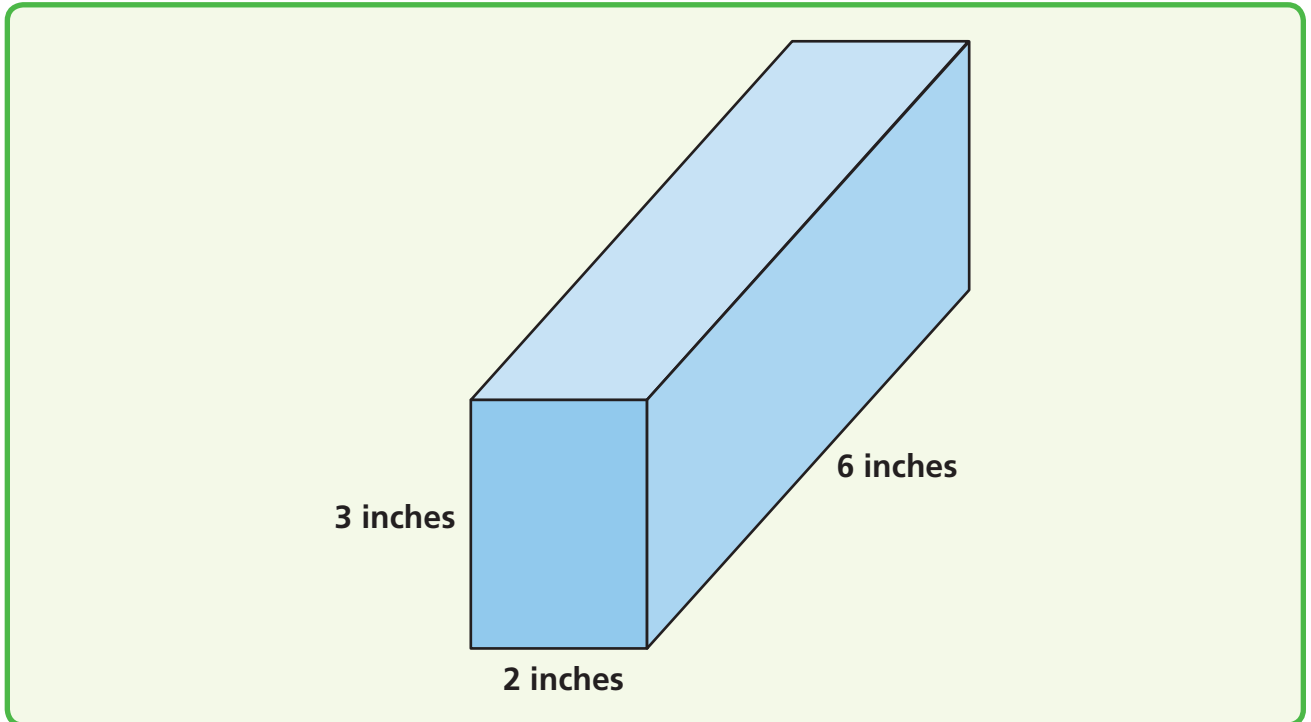




## EXPLORE

## Prisms with the Same Volume

Shelby built this 3 inch  $\times$  2 inch  $\times$  6 inch rectangular prism.



- 1 Build and then draw a sketch of a different rectangular prism with the same volume as Shelby's prism. Write an expression like the following to describe your prism.

$$3 \text{ in.} \times 2 \text{ in.} \times 6 \text{ in.}$$

- 2 Try to build another rectangular prism with this same volume. Write an expression to describe your prism.

## REVIEW MODEL

# Problem Solving Strategy

## Act It Out

Gina used 12 inch-cubes to build a rectangular prism. She figured out the total area of the faces of the prism. What is the largest total area the prism could have had?

### Strategy: Act It Out

#### Read to Understand

What do you know from reading the problem?

The prism was a rectangular prism built from 12 inch-cubes.

What do you need to find out?

the largest possible total area of the faces of the prism

#### Plan

How can you solve this problem?

You could act out the situation described in the problem.

#### Solve

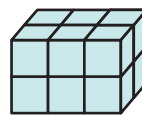
How can you act out the problem?

You can build all possible rectangular prisms using 12 inch-cubes. Then you can find the total areas of their faces by counting the faces of the cubes. Each face has an area of 1 square inch.

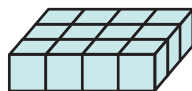
Four prisms are possible:



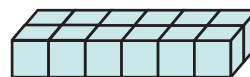
$$1 \times 1 \times 12: 50 \text{ sq in.}$$



$$3 \times 2 \times 2: 32 \text{ sq in.}$$



$$4 \times 3 \times 1: 40 \text{ sq in.}$$



$$6 \times 2 \times 1: 40 \text{ sq in.}$$

The largest possible total area is 50 square inches.

#### Check

Look back at the problem. Did you answer the question that was asked? Does the answer make sense?

## Problem Solving Strategies

### ✓ Act It Out

- ✓ Draw a Picture
- ✓ Guess and Check
- ✓ Look for a Pattern
- ✓ Make a Graph
- ✓ Make a Model
- ✓ Make an Organized List
- ✓ Make a Table
- ✓ Solve a Simpler Problem
- ✓ Use Logical Reasoning
- ✓ Work Backward
- ✓ Write an Equation

## Problem Solving Practice

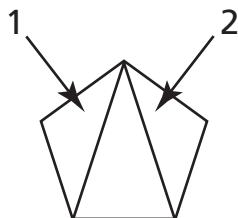
Use the strategy *act it out* to solve.

- 1 A man and his wife each weigh 160 pounds. Each of their twin sons weighs 80 pounds. The four must cross a stream in a rowboat that holds only 160 pounds. How can they cross the stream?
- 2 A rectangular piece of wood measures 3 feet by 6 feet. A carpenter wants to cut the board into three pieces that can be joined together to make a board measuring 2 feet by 9 feet. How can the carpenter do this?

## Mixed Strategy Practice

Use any strategy to solve.

- 3 The drawing shows that exactly two lines can be drawn from a corner of a 5-sided polygon to other corners. How many lines can be drawn from a corner of a 25-sided polygon to other corners?



- 4 An empty room is in the shape of a rectangular prism measuring 15 feet by 12 feet by 8 feet. Suppose you painted all four walls, the ceiling, and the floor. How many gallons of paint would you need if each gallon covered an area of 300 square feet?
- 5 A rectangle with a perimeter of 30 inches is twice as long as it is wide. What is the area of the rectangle?
- 6 Jon bought twelve 39-cent stamps and paid for them with a \$10 bill. How many 4-cent stamps can he buy with the change he received?
- 7 Christy ran for an hour around a track that was 500 yards long. Her average speed was 8 miles per hour. How far did she run?
- 8 Lee, Kara, and Jared are shelving books in the library. Lee took half the books. Kara took two-thirds of the books that remained. Jared took the last 6 books. How many books were there to begin with?

Choose the best vocabulary term from Word List A for each sentence.

- 1 A polygon that is one side of a polyhedron is called a(n) \_\_\_\_?
- 2 A polyhedron with a polygon base and other faces that are triangles is a(n) \_\_\_\_?
- 3 The place where three or more edges of a polyhedron intersect is called a(n) \_\_\_\_?
- 4 A three-dimensional figure with polygonal faces is called a(n) \_\_\_\_?
- 5 A polyhedron with two congruent polygonal bases and other faces that are rectangles is a(n) \_\_\_\_?
- 6 A line segment that forms the boundary of a face of a polyhedron is called a(n) \_\_\_\_?
- 7 A(n) \_\_\_\_? is a two-dimensional pattern of a three-dimensional figure.

Complete each analogy using the best term from Word List B.

- 8 Square is to area as \_\_\_\_? is to volume.
- 9 \_\_\_\_? is to vertices as polyhedron is to polyhedra.

### Word List A

area  
edge  
face  
height  
net  
polyhedron  
prism  
pyramid  
side  
vertex  
volume  
width

### Word List B

cube  
length  
total area  
vertex

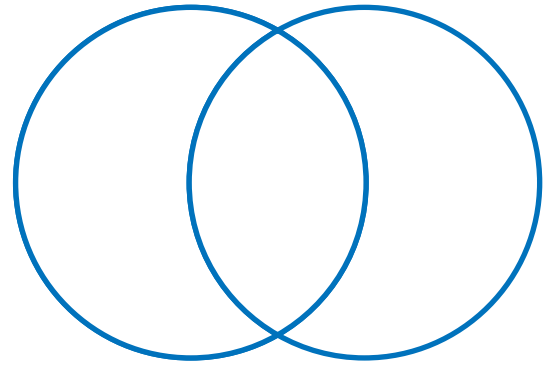
## Talk Math

Discuss with a partner what you have learned about polyhedra. Use the vocabulary terms *face*, *net*, and *three-dimensional figure*.

- 10 How can you recognize a polyhedron?
- 11 How are prisms and pyramids similar? How are they different?
- 12 How can you find the surface area of a polyhedron?

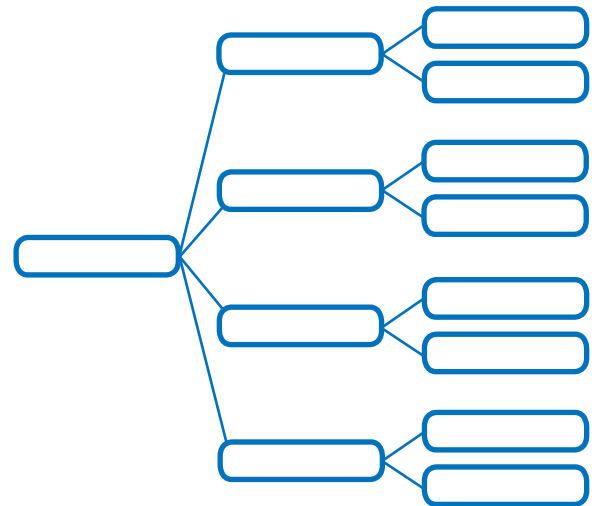
## Venn Diagram

- 13 Create a Venn diagram for the words *area*, *cubic*, *face*, *length*, *height*, *nets*, *polyhedron*, *total area*, *volume*, and *width*.



## Tree Diagram

- 14 Create a tree diagram using the word *polyhedra*. Use what you know and what you have learned about three-dimensional figures.



### What's in a Word?



**FACE** *Face* usually means the front of something. The front of your head from your chin to your forehead is your *face*. Other fronts are the *face* of a building, the *face* of a clock, and the *face* of the moon. *Face* is also an action, such as “*face* the board,” which means “turn toward the board.” In math, *face* has a meaning similar to “front.”

A *face* is any of the plane surfaces of a polyhedron.



#### Technology

Multimedia Math Glossary

[www.harcourtschool.com/thinkmath](http://www.harcourtschool.com/thinkmath)

# GAME

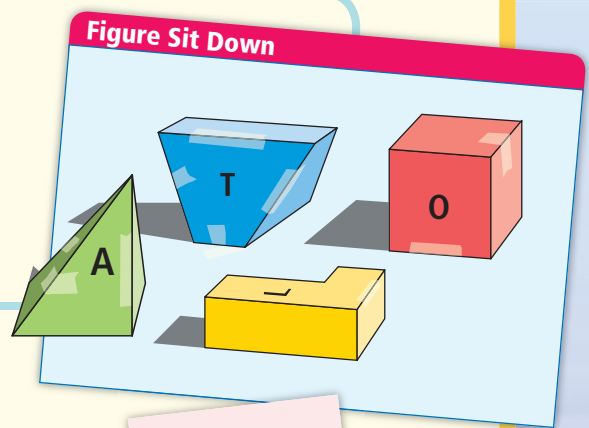
## Figure Sit Down

### Game Purpose

To practice identifying attributes of three-dimensional figures

### Materials

- Figure Zoo figures from Lesson 11.1
- Index cards



### How To Play The Game

- 1** This is a game for a group of 6 to 10 players. Together, make a set of Attribute Cards. Write a different attribute of a three-dimensional figure on an index card. Write as many as you can think of. Try not to write attributes that belong to all prisms or all pyramids. Here are some suggestions:
- 2** Decide who will be the Zookeeper. The Zookeeper mixes up all the Attribute Cards and puts them face-down in a pile. The Zookeeper gives one Figure Zoo figure to each player. All the players stand up holding their Figure Zoo figures.
- 3** The Zookeeper picks the top card and reads it aloud. Each player decides whether his or her figure matches the attribute. If it does not, the player sits down. The Zookeeper picks another card. Play until there is only one figure left. The last player standing is the winner.
- 4** Choose a different Zookeeper. Trade figures with another group of players. Mix up the cards, and play again. Play as many games as you can. Try to use all the figures in the Figure Zoo.

It has 6 faces.

It has more than 5 vertices.

It has fewer than 8 right angles.

Some faces are triangles.

# GAME

## Volume Builder

### Game Purpose

To practice estimating and finding volume

### Materials

- Inch cubes
- Coin
- Scratch paper



### How To Play The Game

- 1** This game is for two players. The object of the game is to score points by building prisms with the greatest possible volumes.
- 2** Start by placing a 1-inch cube on a flat surface between you and your partner. The volume of the cube is 1 cubic inch. Decide who will go first. Then take turns.
- 3** Toss the coin. Heads means 1, and tails means 2.
  - If the coin lands on heads, you may add 1 layer horizontally or vertically to the prism.
  - If the coin lands on tails, you may add 2 layers horizontally or vertically to the prism.
- 4** Think about the prism you want to build, and estimate its volume. That will help you decide which direction—horizontal or vertical—will give the prism with the greater volume.
  - Add your layer or layers according to the coin toss.
  - Compute the volume, and record it on scratch paper. The prism's volume is your score for the round.
- 5** Take turns. Total your score after each round. The first player to score 200 points wins!

# CHALLENGE

Many different nets can be used to make a cube. Only two of the three nets shown below can be folded into a cube. Can you tell which net will not form a cube?

**For each net:**

- Decide whether it can be folded to make a cube. If it can be folded into a cube, predict which face will be opposite the purple face.
- Test your prediction. Copy the net. Cut it out and fold it to make a cube.

