14 Algebraic Thinking

Dear Student,

Try this number puzzle:

Did other students also get 1? If not, tell them that they must have made a mistake. That will surprise them!

In this chapter, you will learn how such puzzles work and have a chance to make up your own "think of a number" puzzles. When you do, try them out on your friends and family. See if they can figure out the "tricks" of these puzzles.

Have fun puzzling through this chapter!

Mathematically yours,
The authors of *Think Math!*

☑ Think of a number.
☑ Add 3 to it.
☑ Double your result.
☑ Subtract 4.
☑ Divide your result by 2.
☑ Subtract the number you thought of first.

Aha! Your result is 1!

Model Trains: More Than Just Toys

The Great Train Story is a famous 3,500 square foot model railroad exhibit at the Museum of Science and Industry in Chicago, Illinois. It has **34** trains running along **1,425** feet of track between the miniature cities of Chicago and Seattle. At night, **80,000** windows and **1,291** streetlights light up the scene.

Copy the puzzle below on a piece of paper. If you follow the steps, the last line of the puzzle reveals some interesting facts about the Great Train Story.

Operations \ Facts	A — Gallons of glue used	B — Height of Sears Tower (feet)	C — Number of people worked on this model	D — Pounds of dirt used on layout
Start with a number.	47			
Add 5.		16		
Multiply by 2.			84	
Subtract 4.				1,200
Divide by 2.				

FACT·ACTIVITY 1

Complete the puzzle and the sentences below.

A ▪ gallons of glue were used.

B ▪ feet is the height of the Sears Tower in the model.

C ▪ people worked on the project.

D ▪ pounds of dirt were used on the layout.

Ernesto and Sally are having a discussion about the *mysterious x*. They have come up with several ways to use *x* in the puzzle in Fact Activity 1.

For 1–4, replace the starting number in each column with *x*. Write the shorthand notation.

1. Use *x* to write an expression for a number in the blue row.

2. What is the shorthand notation for a number in the yellow row?

3. Use *x* to write an expression for a number in the pink row.

4. What is the shorthand notation for a number in the purple row?

CHAPTER PROJECT

Materials: small empty boxes (tissue or shoe box), paint, construction paper, yarn, glue

Collect empty cardboard boxes to build a link of trains. To link the trains together, pierce 2 holes on one end of each box. Feed a strand of yarn through the holes and secure by tying knots. For the wheels, cut out round pieces of cardboard and glue them on the sides of the train. Paint or glue construction paper to your train to create features such as windows, door handles, and ladders.

- Keep track of the materials you used. How many boxes, wheels, windows, etc. are there?

- Write down clues for your partner to solve the mystery numbers of your train. Your clues must include 3 operations. You must also provide a final number so that your partner will work the clues backward to find the mystery number.

- Write the algebraic expression for each step.

- Finally, have your partner count the pieces from your model to verify the answers.

ALMANAC
Fact

Model railroads began as a hobby in the 1840s. One of the largest model railroads is Northlandz in Flemington, New Jersey. It has more than 100 trains that run on 8 miles of track.

EXPLORE
Number Puzzle Mystery

Ryan discovered a number puzzle where the directions for each step are given as a picture.

		A	B	C	D	E
Step 1	🛍	9	▪	▪	▪	▪
Step 2	🛍🛍	?	?	?	?	?
Step 3	🛍🛍 •••	?	?	?	?	?
Step 4	🛍 •••	12	5	27	3	16

1 What are the starting numbers for each round of this puzzle?

2 Describe a single step for getting from the starting number to the final number.

3 Describe a single step for getting from the final number to the starting number.

REVIEW MODEL
Using Bags and Counters

You can use bags ⏳ and counters •• to create number puzzles, and to see how number puzzles work. When you work a number puzzle, you can add, subtract, multiply, and divide bags and counters as though they were whole numbers.

Example

What the puzzle says:	Joe's number	Taylor's Number	Bags and counters
Think of a number.	7	15	⏳
Double it.	$7 \times 2 = 14$	$15 \times 2 = 30$	⏳ ⏳
Add 10.	$14 + 10 = 24$	$30 + 10 = 40$	⏳ ⏳ •••• •
Divide by 2.	$24 \div 2 = 12$	$40 \div 2 = 20$	⏳ •••• •
Subtract 3.	$12 - 3 = 9$	$20 - 3 = 17$	⏳ ••

The last picture ⏳•• shows that no matter what number you start with ⏳, you will end with the same number ⏳ plus 2 more ••. Joe started with 7 and ended with $7 + 2 = 9$. Taylor started with 15 and ended with $15 + 2 = 17$.

✔Check for Understanding

On a separate sheet of paper, draw bags and counters to represent the following four steps in a number puzzle.

1 Think of a number.

2 Add 3.

3 Double it.

4 Subtract 5.

5 Think of a number.

6 Double it.

7 Add 6.

8 Subtract 2.

REVIEW MODEL
Using Shorthand Notation

When a number puzzle says, "Think of a number and double it," it's easy to work the puzzle with bags: 🎒🎒. But suppose the puzzle says, "Think of a number and multiply it by 50." Would you like to draw 50 bags? There's an easier way to work number puzzles.

- Use x or another variable instead of 🎒.
- Use whole numbers to represent the numbers of bags and counters.

	Bags and Counters	Shorthand Notation
Think of a number.	🎒	x
Multiply it by 8.	🎒🎒🎒🎒🎒🎒🎒🎒	$8x$
Add 14.	🎒🎒🎒🎒🎒🎒🎒🎒 •••••• ••••	$8x + 14$
Divide by 2.	🎒🎒🎒🎒 •••••••	$4x + 7$
Subtract 3.	🎒🎒🎒🎒 ••••	$4x + 4$

✔Check for Understanding

Use shorthand notation to write the four steps of a number puzzle.

1 **A** Think of a number. **B** Multiply it by 20. **C** Add 48. **D** Divide by 2.

2 **A** Think of a number. **B** Multiply it by 100. **C** Subtract 20. **D** Divide by 4.

EXPLORE
Finding Your Number

Words	Shorthand	Betty	Ted	Jun	Karina
Think of a number.	x	?	?	?	?
Step ❶	$x + 6$	10		21	
Step ❷	$x + 2$	6			2
Step ❸	$2x + 4$	12	26		

❶ What number did each student think of? Use base-ten blocks or counters to help you.

❷ Use words to describe **Step 1** of this puzzle.

❸ Use words to describe **Step 2** of this puzzle.

❹ Use words to describe **Step 3** of this puzzle.

EXPLORE
Product Near Square Numbers

The 4th graders are making a class flag.
They want its area to be as big as possible.
Their teacher offers them some choices for
the sizes of their flags.

Which should they choose? Why?

1

Flag A

6 feet

8 feet

or

Flag B

7 feet

7 feet

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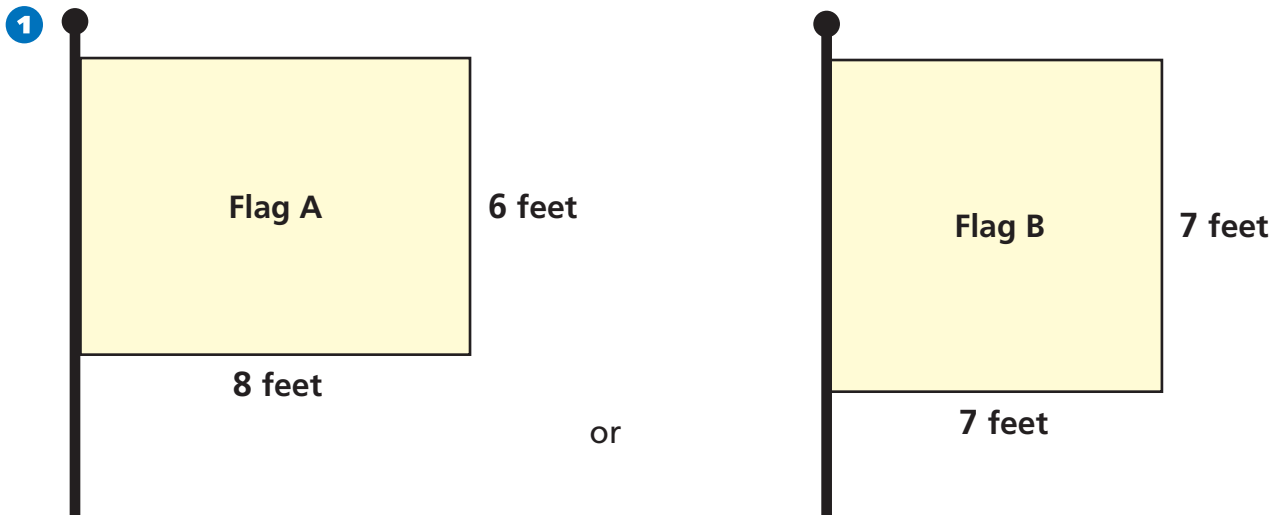
2 Flag A is **15-by-17 feet**   or   Flag B is **16-by-16 feet**

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3 Flag A is **28-by-30 feet**   or   Flag B is **29-by-29 feet**

What do you notice?

REVIEW MODEL
Applying a Squaring Pattern

You can use an amazing number pattern to help you multiply large numbers. The pattern uses the *square of a number* (the number multiplied by itself) and the *nearest neighbors* of the number.

Example 1

number: 7 → square of number: $7 \times 7 = \boxed{49}$

nearest neighbors: 6 and 8 → product of nearest neighbors: $6 \times 8 = \boxed{48}$

Example 2

number: 12 → square of number: $12 \times 12 = \boxed{144}$

nearest neighbors: 11 and 13 → product of nearest neighbors: $11 \times 13 = \boxed{143}$

Notice that in both examples, the square of the number is 1 more than the product of its nearest neighbors.

Example 3

Use mental math to find the product 49×51.

$50 \times 50 = 2,500$.

So, $49 \times 51 = 2,500 - 1 = 2,499$.

Think: The square of 50 is 1 more than the product of 49 and 51.

Example 4

If $73 \times 75 = 5,475$, what is 74×74?

$73 \times 75 = 5,475$.

So, $74 \times 74 = 5,475 + 1 = 5,476$.

Think: The square of 74 is 1 more than the product of 73 and 75.

✔Check for Understanding

Use mental math to find the product.

1 19×21 **2** 39×41 **3** 79×81 **4** 99×101

Solve.

5 If $36 \times 38 = 1,368$, what is 37×37? **6** If $47 \times 49 = 2,303$, what is 48×48?

REVIEW MODEL
Problem Solving Strategy
Work Backward

Kyle bought two tickets to the Spring Concert. The total cost of the tickets, including a $3 service charge, was $39. The equation $2x + 3 = 39$ represents the total cost of the tickets. How can you solve the equation to find the cost of one ticket?

Strategy: Work Backward

Read to Understand

What do you know from reading the problem?

Kyle bought 2 tickets. The service charge was $3. The total cost was $39. The equation $2x + 3 = 39$ represents the total cost.

What do you need to find out?

the price of a ticket

Plan

How can you solve this problem?

You could work backward to solve the equation $2x + 3 = 39$. The value of x will be the cost of one ticket.

Solve

How can you find the value of x in the equation?

If $2x + 3 = 39$, then 39 must be 3 *more than* $2x$. That makes sense, because $39 is $3 more than the cost of two tickets, due to the $3 service charge. So, working backward, $2x$ must be 3 *less than* 39, or 36. If $2x = 36$, then 36 is 2 *times x.* That means that x must be 36 *divided by* 2, or 18. So, the cost of one ticket is $18.

Check

Look back at the problem. Did you answer the questions that were asked? Does the answer make sense?

The answer makes sense because the total cost of 2 tickets ($2 \times $18 = 36) plus a $3 service charge ($36 + $3 = 39) is $39.

Problem Solving Practice

Use the strategy *work backward* to solve.

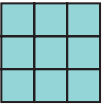**1** Damon bought 6 apples and 5 peaches. He spent a total of $4.15. Each apple cost $0.40. How much did each peach cost if all the peaches were the same price?

2 Hallie walks to school from her house by walking 3 blocks north, 4 blocks west, and 1 block south. She walks home by the same route. Describe the route that she follows home.

Mixed Strategy Practice

Use any strategy to solve. Explain.

3 A photograph has a width of 8 inches and a perimeter of 36 inches. What is its area?

4 How many squares are in the figure at the right?

5 The Sharks soccer team played 28 games. Five games ended in ties. The team won 5 more games than it lost. How many games did it win?

6 Mark mows his lawn every 6 days and waters his lawn every 4 days. He watered and mowed on July 1. When was the next day he watered and mowed on the same day?

7 Amber, Michael, and Josh are students. One is in 3rd grade, one is in 4th, and one is in 5th. Michael is not in 5th grade. The 3rd grader is on the track team with Amber and in the chorus with Michael. Which student is in 4th grade?

8 Chelsea's flight to Chicago leaves at 8:20 A.M. She wants to be at the airport 1 hour 45 minutes early. It will take her 45 minutes to drive to the airport. What time should she leave her house?

9 A number cube measures 2 inches on a side. An empty box is cube-shaped and measures 4 inches on a side. How many number cubes can you pack in the box?

10 A donut-frosting machine can frost 6 donuts every 5 seconds. How many donuts can it frost in 1 minute?

Vocabulary

Choose the best vocabulary term from Word List A for each sentence.

1 A study of number patterns with symbols is called ___?___.

2 The most commonly used variable is ___?___.

3 A letter or symbol that stands for one or more numbers is called a(n) ___?___.

4 A(n) ___?___ is a number sentence that shows that two quantities are equal.

5 In algebra, a raised ___?___ between two numbers means to multiply those two numbers.

6 Symbols used to show which operation or operations in a expression should be done first are called ___?___.

7 The product of a number and itself is called the ___?___ of the number.

Word List A

algebra
dot
equation
parentheses
square
variable
x
y
z

Complete each analogy using the best term from Word List B.

8 A plus sign is to addition as a(n) ___?___ is to multiplication.

9 Sentence is to language as ___?___ is to algebra.

10 Number is to 7 as ___?___ is to *x*.

Word List B

algebra
dot
equation
variable
y

Talk Math

Discuss with a partner what you have learned about algebra. Use the vocabulary terms *equation* and *variable*.

11 How can you record a number puzzle that works for all numbers?

12 How can you use symbols to represent a square number?

Analysis Chart

13 Create an analysis chart for the terms *algebra, equation, variable,* and *square.* Use what you know and what you have learned about algebra and rules.

Word Web

14 Create a word web using the word *square.*

?

ALGEBRA In the ninth century, a Persian mathematician named Abu Ja'far Muhammad ibn Musa al-Khwarizmi wrote a book about math that described algebra. The book was called *The Compendious Book on Calculation by Completing and Balancing.* He wrote the book in Arabic. The word *completing* in the title is *al-jabr* in Arabic. *Al-jabr* became *algebra* in English.

GO ONLINE
Technology
Multimedia Math Glossary
www.harcourtschool.com/thinkmath

GAME

Make a Puzzle

Game Purpose
To gain further experience with the math behind number puzzles and the strategy *work backward*

Materials
- Activity Masters 147, 148, 149
- counters
- scissors

Make a Puzzle

How To Play The Game

1 This is a game for two players. The object is to make and solve puzzles. Each player will need a *Make a Puzzle* game board. Cut out each set of *Make a Puzzle* cards and place the cards face down.

2 Take turns picking an operation card and a number card. For each pair of cards, record the step in a blank under "Draw a number" on the game board. Fill in the first five blanks above the heavy line. You may mix up each set of cards again at any time.

3 Next, take turns picking a number card. Write the number in the first blank beside "Draw a number." Use that number as the starting number in a puzzle. Work downward through the five steps. You may use counters to help. Complete all your puzzles.

4 Now, write 1, 2, or 3 rules to try to get back your original numbers. If you do not need a row on the game sheet, leave it blank.

5 Complete your puzzles. If you cannot complete any arithmetic because it would mean an uneven division or lead to a negative number, stop working in that column.

6 Find the difference between the original and last number recorded in each column. Add all the differences. The sum is your score. Whoever has fewer points wins.
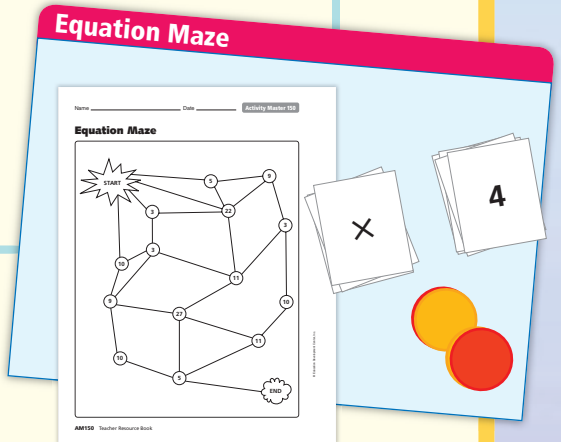
GAME

Equation Maze

Game Purpose
To practice finding the value of *x* in number sentences written in shorthand notation

Materials
- Activity Masters 150 and 151
- game token
- scissors

How To Play The Game

1 This is a game for two players. The goal is to be the first player through the *Equation Maze.* Cut out all the cards. Place them face down in a stack.

2 Put your game tokens at the start of the maze. Decide who will go first, and then take turns.

3 Pick a card. Find the value of *x.*
- If the value of *x* matches a number in a circle that is connected to the circle where you are, move your token to the new circle.
- If the value of *x* matches more than one circle connected to yours, move your token to either circle.
- If the value of *x* does not match a circle connected to yours, do not move your token.

Examples: Mitzi is at Start. The value of *x* is 10.
Mitzi can move forward.

Jin is at 22. The value of *x* is 3.
Jin has two choices.

Mitzi is at 27. The value of *x* is 22.
Mitzi must stay where she is.

4 Play until one player reaches the end of the maze. That player is the winner.

CHALLENGE

Here are two algebra tricks you can try on your family or friends. Before you try them on someone else, test them yourself so you see how they work. Look for a pattern in each trick.

Hint: Try using a variable. That will help you understand how the tricks work.

Algebra Trick #1

✔ Choose any number from 1 to 10.

✔ Add 5 to the number.

✔ Multiply the result by 2.

✔ Subtract 10.

✔ Divide the result by 2.

What number are you left with?

Algebra Trick #2

✔ Choose any number from 1 to 10.

✔ Multiply the number by 2.

✔ Add 2 to the result.

✔ Multiply the result by 2.

✔ Divide the result by 4.

✔ Subtract 1 from the result.

What number are you left with?

❶ Now that you have seen how these two tricks work, do you think they will work with any starting number? Explain.

❷ Make up an algebra trick of your own. Test it to be sure it works. Then try it on someone else.