

15 Estimation

Dear Student,

Congratulations on making it to the final chapter!

This chapter is all about estimating. You will be estimating lengths, areas, capacities, and money. In what kinds of situations might you estimate? What units might you estimate these things with? What things might you estimate the length, area, or capacity of? What tools could you use to check your estimates?

You will find answers to all of these questions. As you do, we hope you will learn to value estimating and use it to simplify computations in everyday life.

Mathematically yours,
The authors of *Think Math!*



Bee-havior

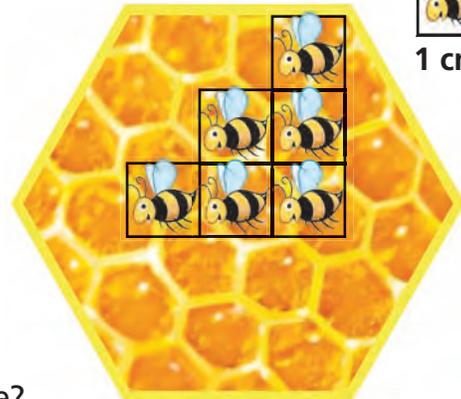


In a natural beehive, the working bees build honeycombs attached to each other from top to bottom. These honeycombs are made of beeswax and they form hexagonal cells. It takes about 15 pounds of beeswax to form the entire structure of the honeycomb. The cells of the honeycomb are used for storing honey and raising the young.

FACT • ACTIVITY 1

Look at the honeycomb.

- 1 Estimate the perimeter and area of the honeycomb. Use the fact that the picture of the bee to the right of the honeycomb is 1 square centimeter.



For 2–3, use the drawing.

- 2 How far does a bee have to fly from the beehive to reach the flowerbed?
- 3 About how far is the boy from the beehive?



beehive

flowerbed

13 meters

20 meters

A single bee hive can have more than 30,000 bees and produce about 300 pounds of honey in a year. During their lives, 12 worker bees will gather only 1 teaspoon of honey.

FACT • ACTIVITY 2

- 1 Suppose the capacity of a jar is 8 ounces. How many jars will you need to hold 1 gallon of honey?
- 2 An American consumes about 594 grams (1.31 pounds) of honey per year. Estimate the amount of honey one person will consume in 5 years. Explain how you found your answer.
- 3 About how much honey is produced by a colony of bees in a year, in kilograms? Explain.
- 4 One gallon of honey weighs about 12 pounds. Estimate how many gallons of honey a single hive can produce in a year.

CHAPTER PROJECT

- Use your new knowledge of bees and honey to estimate how many bees it takes to produce the honey for this recipe.
- Suppose you were to make enough servings of this snack for everyone in your class. Estimate how much more honey you will need for a recipe large enough for everyone in your class.
- Does $\frac{1}{2}$ cup honey weigh the same as $\frac{1}{2}$ cup water? Measure $\frac{1}{2}$ cup of each into identical paper cups and weigh each one. Record your results. Then try other liquids, such as olive oil or juice. Weigh $\frac{1}{2}$ cup of each. Make a chart to show the results. Does the same volume of different liquids weigh the same? Explain.

Honey Snacks

Makes 8 servings:

- $1\frac{1}{3}$ cups toppings: ground toasted almonds, ground coconut, candy sprinkles, or graham cracker crumbs
- 4 just-ripe bananas, peeled
- $\frac{1}{2}$ cup honey
- 8 popsicle sticks

Combine one or more toppings in a mixing bowl to make $1\frac{1}{3}$ cups. Slice each banana in half crosswise. Insert a popsicle stick into each half banana. Spread honey on each banana to coat evenly. Roll each banana half in the toppings to coat.

ALMANAC Fact

Honey bees fly up to 24 km/hr (15 mph) and their wings beat 200 times/sec (12,000 beats/min).



The Lemonade Stand

Five friends set up a lemonade stand by the side of the road. They sold cups of lemonade for 10¢ each and cookies for 25¢ each.



They earned \$8.15 on Saturday and \$9.65 on Sunday. They decided that they should each get \$4.00.

1 Why won't this work?

2 How much should each friend get?

EXPLORE

Estimating Perimeter

Paul Perimeter is $1\frac{1}{4}$ meters tall. His pencil is a little more than 10 centimeters long.

Paul is going to glue a border around the sides of his door. He is trying to find the door's perimeter to figure out how much border he needs.

- 1 How can Paul use his height to estimate the perimeter in meters?

-
- 2 How can Paul use his pencil to estimate the perimeter of the door in centimeters?

-
- 3 Would these two estimates of the door's perimeter make sense? Why or why not?

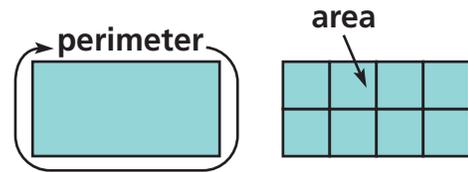
4 times Paul's height

100 pencils

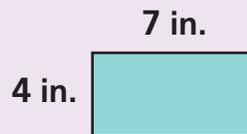
REVIEW MODEL

Finding Perimeter and Area

The *perimeter* of a figure is the distance around the outside. The *area* of a figure is the number of square units on the inside.

**Example 1**

Find the perimeter of the rectangle.

**One Way**

Add the lengths of the four sides.
 $4 \text{ in.} + 7 \text{ in.} + 4 \text{ in.} + 7 \text{ in.} = 22 \text{ in.}$

The perimeter is 22 in.

Another Way

Add two adjacent sides:
 $4 \text{ in.} + 7 \text{ in.} = 11 \text{ in.}$

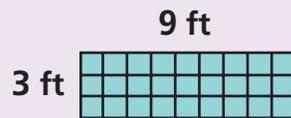
Multiply the sum by 2:
 $11 \text{ in.} \times 2 = 22 \text{ in.}$

Example 2

Find the area of the rectangle.

**One Way**

Count the number of square units inside. There are 27 squares.



The area is 27 sq ft.

Another Way

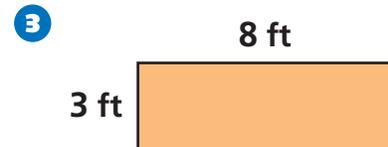
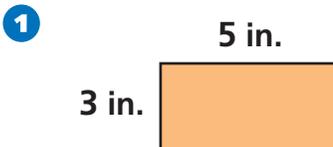
Multiply the length by the width.

$$9 \text{ ft} \times 3 \text{ ft} = 27 \text{ sq ft.}$$

The area is 27 sq ft.

✓ Check for Understanding

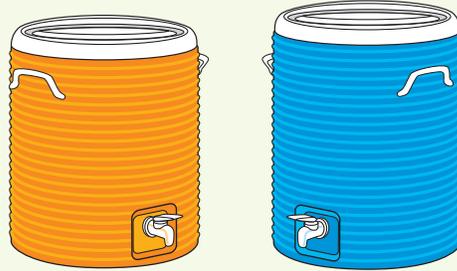
Find the perimeter and area of each rectangle.



EXPLORE

Comparing Liters and Gallons

Felisha is going to the beach and wants to bring lots of water with her. She has two water coolers. One of the coolers holds 1 gallon and the other holds 4 liters.



- 1 What can Felisha do to figure out which cooler holds more water?

- 2 Felisha just remembered that 1 liter is a little bit more than 1 quart. How can this help her decide which cooler is bigger?

- 3 Felisha changed her mind. She wants to bring lemonade instead of water to the beach. To make lemonade, she mixes one lemonade packet with 8 cups of water. About how many packets should she use to make enough lemonade to fill the larger cooler? Explain your reasoning.

Comparing Units of Capacity

The *capacity* of a three-dimensional object is the amount that it can hold.

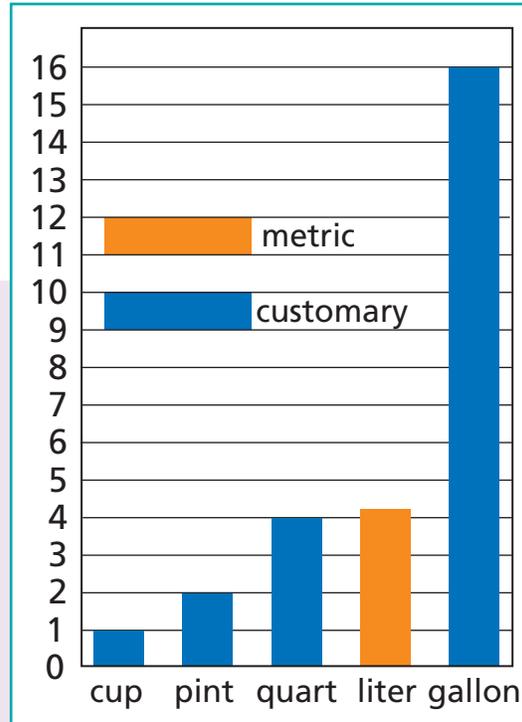
The four most common units of capacity in the customary system of measurement are **cups**, **pints**, **quarts**, and **gallons**.

The two most common units of capacity in the metric system of measurement are the **milliliter** and the **liter**.

In the diagram at the right, the lengths of the bars indicate the relative sizes of five of the six basic units—the cup, the pint, the quart, the liter, and the gallon.

The sixth unit, the milliliter, is too small to appear on the diagram. It is only $\frac{1}{236}$ as big as a cup.

Notice that a liter is slightly bigger than a quart.



Examples

A How many pints are in a gallon?

The gallon bar is 8 times the height of the pint bar, so there are 8 pints in a gallon.

B Which is bigger, a gallon or 4 liters?

Since a liter is bigger than a quart, 4 liters is bigger than 4 quarts. Since there are 4 quarts in a gallon, 4 liters is bigger than a gallon.

C Which is bigger, a pint or 500 milliliters?

A pint is 2 cups. There are 236 milliliters in a cup. So a pint is $2 \times 236 = 472$ milliliters, which is smaller than 500 milliliters.

Check for Understanding

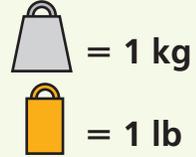
Solve.

- How many cups are in a quart?
- Which is bigger, 3 pints or a liter?
- How many quarts are in a half-gallon?
- Which is bigger, 900 milliliters or 4 cups?
- Which is bigger, 5 cups or a liter?

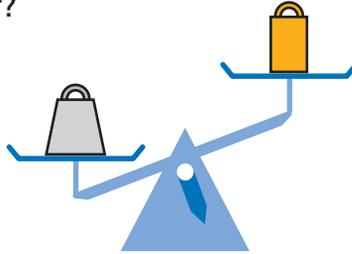
EXPLORE

Comparing Pounds
and Kilograms

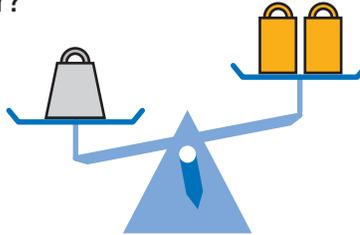
Jean wanted to figure out how kilograms compare with pounds. To do this, she put various weights on opposite sides of a balance scale.



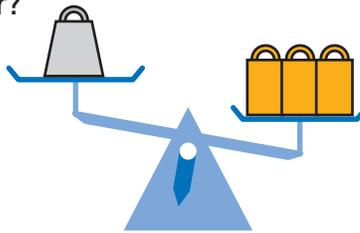
- 1 What does this scale tell her?



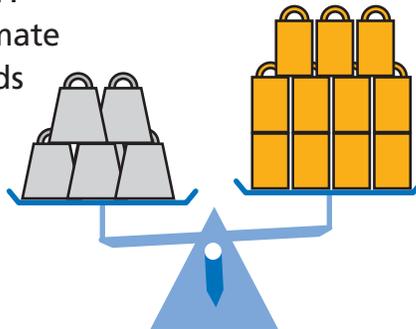
- 2 What does this scale tell her?



- 3 What does this scale tell her?



- 4 What does this scale tell her?
Use a calculator to approximate the relation between pounds and kilograms.



EXPLORE

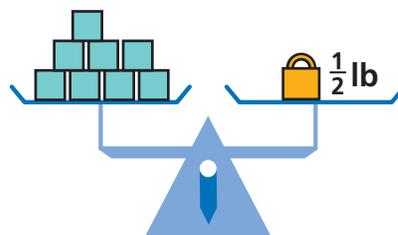
Mystery Bags

Mr. Crepsi hid weights in boxes and bags. To figure out which weight is in a box and which weight is in a bag, he gave students these clues:

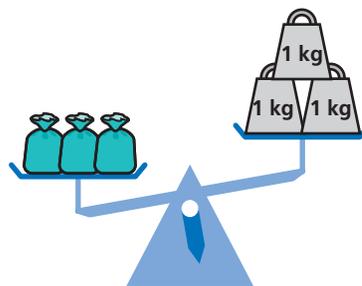
- 1 8 boxes balance a $\frac{1}{2}$ -pound weight.

What weight could be in each box?

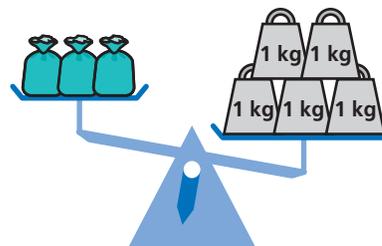
Why do you think so?



- 2 3 bags are heavier than 3 kilograms.



3 bags are a little lighter than 5 kilograms.



What weight could be in each bag?

Why do you think so?

REVIEW MODEL

Writing Equations and Inequalities

You can write equations and inequalities to represent the relation between weights on a scale.

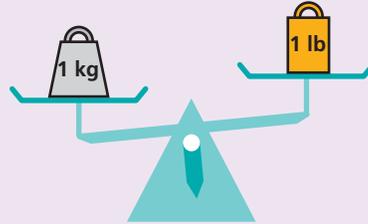
If the scale is in balance . . .



write an equation:

$$8 \text{ oz} = \frac{1}{2} \text{ lb}$$

If the scale is not in balance . . .

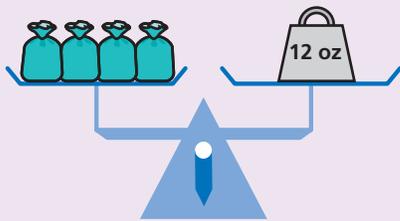


write an inequality:

$$1 \text{ kg} > 1 \text{ lb}$$

If you don't know the weight of an object on the scale, use a variable to represent the weight. You can use any letter or symbol for a variable, but x is the most common.

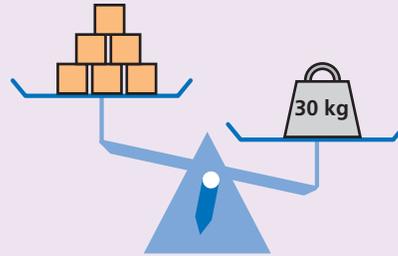
If the scale is in balance . . .



write an equation:

$$4x = 12 \text{ oz}$$

If the scale is not in balance . . .



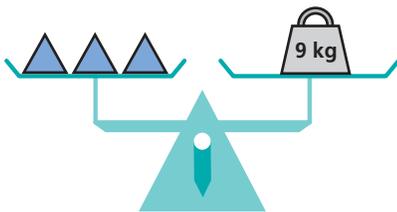
write an inequality:

$$6x < 30 \text{ kg}$$

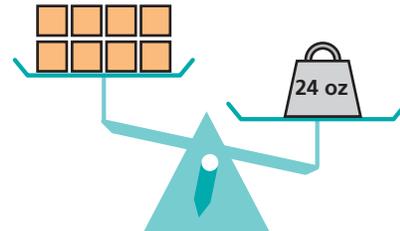
✓ Check for Understanding

Write an equation or inequality to represent the picture.

1



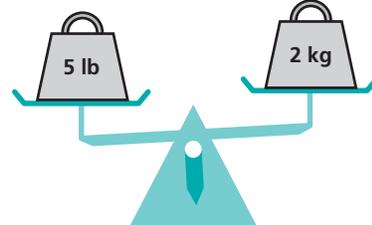
2



3



4



REVIEW MODEL

Problem Solving Strategy

Act It Out

Gina has forgotten her three-digit locker combination. She remembers that the first digit is 5, the second digit is odd, and the third digit is either 7 or 8. How can she find all the possible locker combinations?

Strategy: Act it Out

Read to Understand

What do you know from reading the problem?

The first digit of Gina's three-digit locker combination is 5, the second digit is odd, and the third digit is either 7 or 8.

What do you need to find out?

all possible three-digit combinations

Plan

How can you solve this problem?

You could make cards representing the digits. Then you could act out the process of looking for all the possible combinations.

Solve

How can you *act it out* to solve the problem?

Make cards like these:

| first | second | third |
|-------|-----------|-------|
| 5 | 1 3 5 7 9 | 7 8 |

Then arrange them to make all the combinations of digits you can find: 517, 518, 537, 538, 557, 558, 577, 578, 597, 598. There are ten possible combinations that Gina must try.

Check

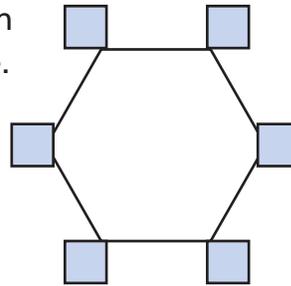
Look back at the problem. Did you answer the questions that were asked? Does the answer make sense?

The answer makes sense because there is 1 possible first digit, 5 possible second digits, and 2 possible third digits. I know from Chapter 10 that I can use multiplication to find numbers of attributes, and $1 \times 5 \times 2 = 10$.

Problem Solving Practice

Use the strategy *act it out* to solve.

- 1 Six houses were arranged in a hexagon shape. One person lived in each house. One day, each person visited the house of every neighbor except the neighbors on either side of his or her house. How many house visits were made?



- 2 In a word game, Beth drew the letters O, P, S, and T. She had to make a 4-letter word from the letters. How many different 4-letter combinations can she make from the letters?

Problem Solving Strategies

✓ Act It Out

- ✓ Draw a Picture
- ✓ Guess and Check
- ✓ Look for a Pattern
- ✓ Make a Graph
- ✓ Make a Model
- ✓ Make an Organized List
- ✓ Make a Table
- ✓ Solve a Simpler Problem
- ✓ Use Logical Reasoning
- ✓ Work Backward
- ✓ Write an Equation

Mixed Strategy Practice

Use any strategy to solve. Explain.

- 3 From April 1 to May 1, the price of tomatoes doubled. From May 1 to June 1 it dropped \$1, to \$3 per pound. What was the price of tomatoes on April 1?
- 4 On a balance scale, 6 quarters balance with 3 half dollars. Five dimes balance with 1 half dollar. How many dimes will balance with 2 quarters?
- 5 A sofa is manufactured in 3 different styles, 3 different colors, and 2 different fabrics. How many style-color-fabric combinations does a buyer have to choose from?
- 6 Toni rented a car for two days for \$80. The charge the first day was \$10 more than the second day. What was the charge the first day?
- 7 Ben has 51 baseball cards and Jeff has 15 baseball cards. At the first Card Club meeting and every meeting thereafter, Ben sold 6 cards to Jeff. After which meeting did the two have equal numbers of cards?
- 8 Ira bought three steaks, all priced the same, two loaves of bread, each costing \$3, and a melon costing \$2. The total cost of the items was \$26. How much did each steak cost?

Choose the best vocabulary term from Word List A for each sentence.

- 1 Numbers that are easy to compute mentally are ___?___ numbers.
- 2 The number of square units needed to cover a surface is the ___?___ of the surface.
- 3 The distance around a figure is the ___?___ of the figure.
- 4 A metric unit for measuring capacity is the ___?___.
- 5 A customary unit for measuring weight is the ___?___.
- 6 A number sentence that shows that two quantities are equal is called a(n) ___?___.
- 7 The ___?___ is the measure of the amount of space a solid figure occupies.
- 8 The amount of matter in an object is its ___?___.
- 9 The ___?___ of an object tells how heavy it is.

Complete each analogy using the best term from Word List B.

- 10 Quart is to ___?___ as kilogram is to mass.
- 11 Ruler is to inch as ___?___ is to pound.

Word List A

area
capacity
compatible
equation
inequality
kilogram
liter
mass
perimeter
quart
pound
round
scale
volume
weight

Word List B

capacity
scale
inequality

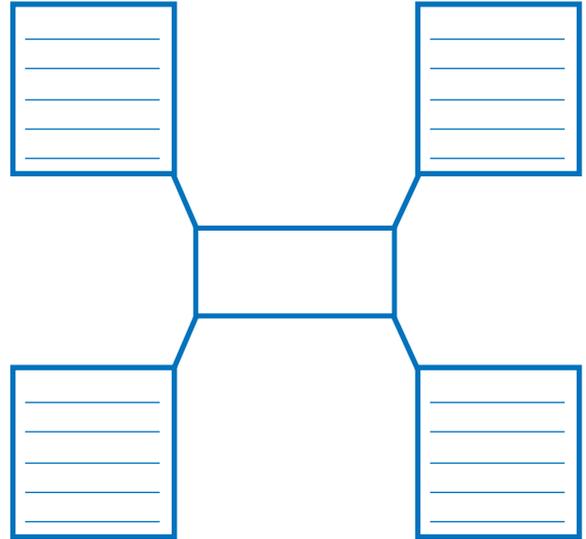
Talk Math

Discuss with a partner what you have learned about estimation. Use the vocabulary terms *compatible* and *round*.

- 12 Ken's living room is a rectangle. How can you estimate its perimeter and area?
- 13 How can you use a liter to estimate a capacity in quarts?
- 14 How can you use a kilogram to estimate a weight in pounds?

Word Web

- 15 Create a word web for the word *round*.



Word Line

- 16 Create a word line using the terms *cup*, *gallon*, *liter*, *milliliter*, and *quart*. Use what you know and what you have learned about estimation and measures of capacity.

Words:

Sequence:

What's in a Word?



SCALE This word has several different meanings. A *scale* is used on diagrams, such as maps and floor plans. It shows the relationship between the actual distance and the distance on the diagram. On a bar graph or line graph, a *scale* is a set of numbers placed at fixed distances to help label the graph. A balance *scale* is used to compare two weights. *Scales* are also part of the skin of a fish or reptile. A musical *scale* is a set of notes that go in order by pitch. A climber *scales* a mountain or cliff.



Technology

Multimedia Math Glossary

www.harcourtschool.com/thinkmath

GAME

The Closest Estimate: Weight

Game Purpose

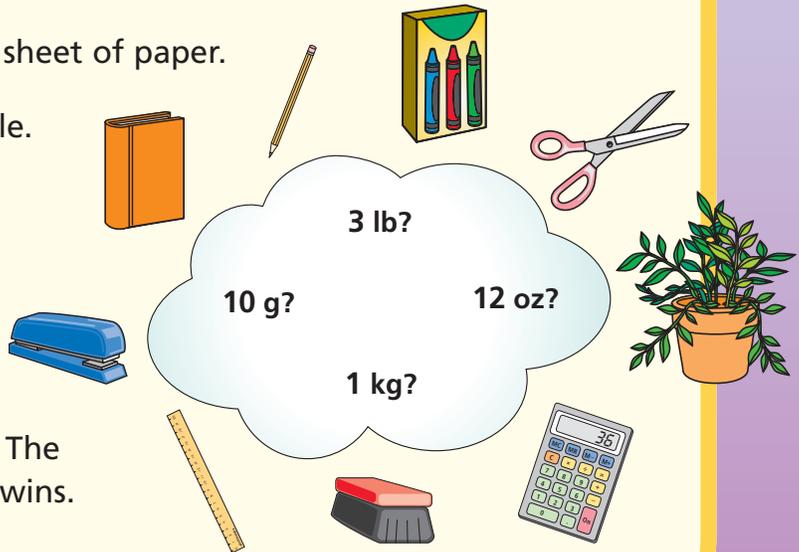
To practice estimating weight

Materials

- classroom objects (various)
- scale (that measures in grams/kilograms/ounces/pounds)
- clock

How To Play The Game

- 1** This is a game for 3 to 4 players. The goal is to look at various objects and estimate their weights. You score 1 point for each estimate that is the closest.
- 2** Choose several objects from your classroom. The objects must be able to be weighed on the scale. As a group, you may set a time limit, such as 30 seconds or 1 minute, for making each estimate. But a time limit is not necessary.
- 3** Display an object.
 - Estimate the weight of the object in grams, ounces, pounds, or kilograms.
 - Record your estimate on a sheet of paper.
- 4** Weigh the object on the scale. The player with the closest estimate gets 1 point.
- 5** Repeat steps 3 and 4 with the rest of the objects.
- 6** After you have used all the objects, add up your points. The player with the most points wins.



GAME

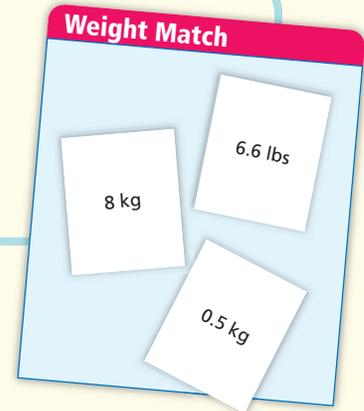
Weight Match

Game Purpose

To practice applying the relationship between pounds and kilograms

Materials

- Activity Masters 155–156
- scissors



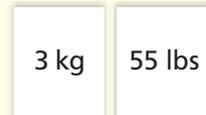
How To Play The Game

- 1** This is a game for 2 to 3 players. The object is to match pounds and kilograms. Remember, 1 kilogram is about 2.2 pounds.
- 2** Cut out all of the weight cards. Mix them up. Choose one player to deal all the cards to all players in the group.
 - If you hold cards that name matching weights, place them face up on the table in front of you.
 - Everyone should verify that the cards show approximately the same weights.
 - If you made an incorrect match, take back your cards.
- 3** After all correct matches have been made, the player to the left of the dealer picks a card from another player's hand.
 - If the card matches a card already in the player's hand, he or she places the matching pair face up on the table.
 - Everyone should verify the match.
 - If the match is correct, the player chooses again. The same player continues to choose until he or she cannot make a match. Then it is the next player's turn.
- 4** Play until all cards have been matched and placed face up. The player with the most matches is the winner.

A Correct Match



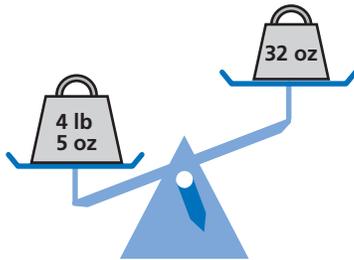
Not a Match



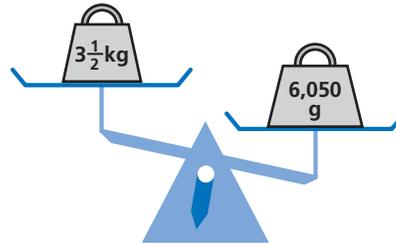
CHALLENGE

Each of the scales shown is unbalanced. What weight should be added to balance each scale?

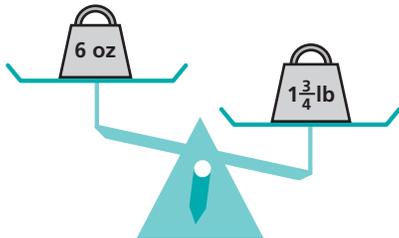
1



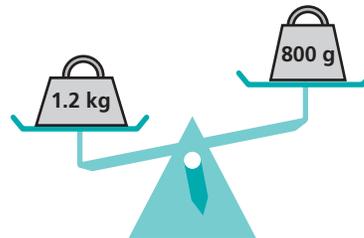
2



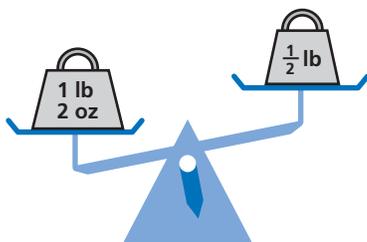
3



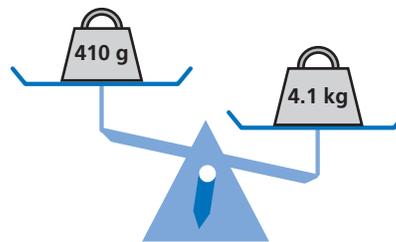
4



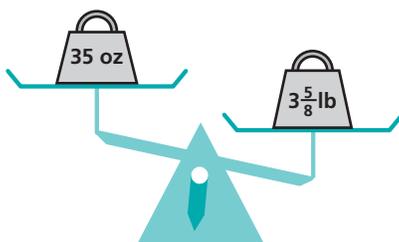
5



6



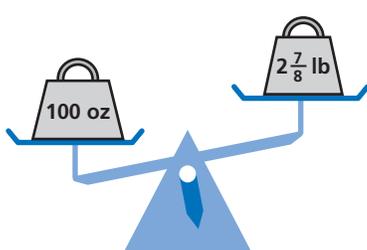
7



8



9



10

