

2 Multiplication

Dear Student,

In this chapter, you will be figuring out the number of dots or tiles in pictures like the ones at the right.

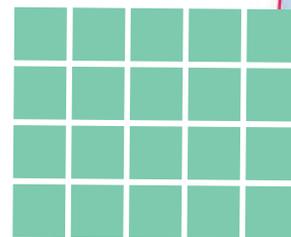
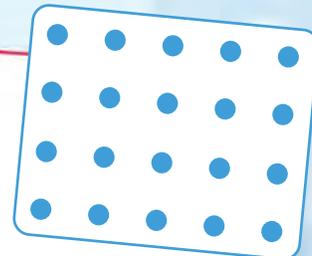
You will develop different strategies—multiplication and more—for finding the number of tiles or dots in these pictures.

Towards the end of the chapter, you will see pictures where you know the total number of tiles, but the rows or columns are not labeled. Your job will be to find the unknown number of columns or rows.

As you go through the chapter, think of times when the strategies you will be developing will be useful. For example, can finding the number of squares in the pictures at the top of this page help you figure out how many cookies to give each of 5 friends when you have 20 cookies to share?

We hope you will enjoy these lessons!

Mathematically yours,
The authors of *Think Math!*



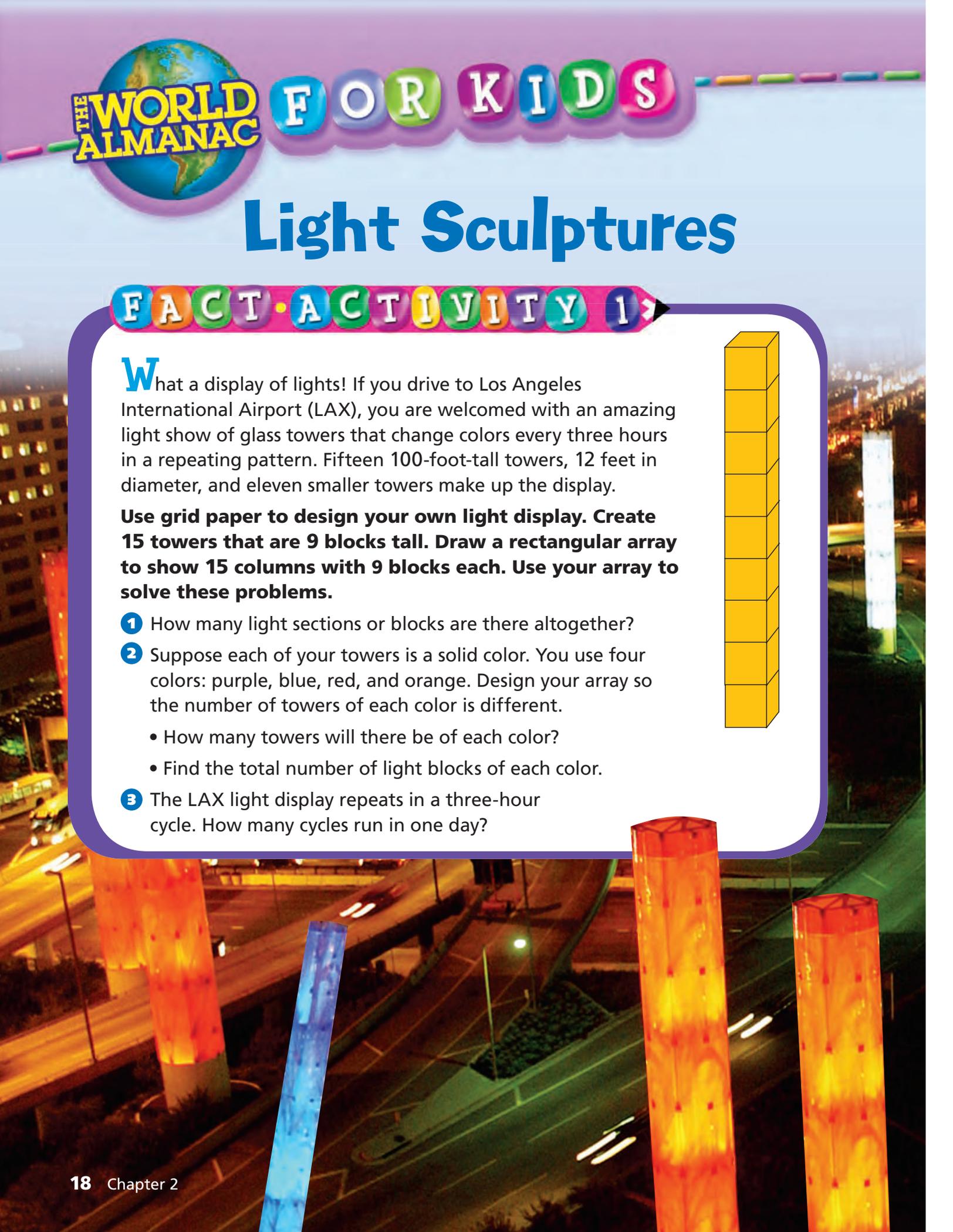
Light Sculptures

FACT • ACTIVITY 1

What a display of lights! If you drive to Los Angeles International Airport (LAX), you are welcomed with an amazing light show of glass towers that change colors every three hours in a repeating pattern. Fifteen 100-foot-tall towers, 12 feet in diameter, and eleven smaller towers make up the display.

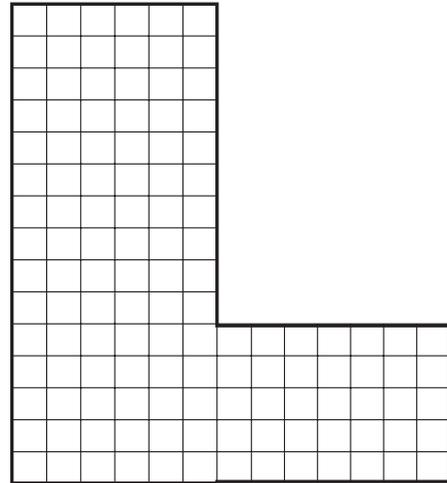
Use grid paper to design your own light display. Create 15 towers that are 9 blocks tall. Draw a rectangular array to show 15 columns with 9 blocks each. Use your array to solve these problems.

- 1 How many light sections or blocks are there altogether?
- 2 Suppose each of your towers is a solid color. You use four colors: purple, blue, red, and orange. Design your array so the number of towers of each color is different.
 - How many towers will there be of each color?
 - Find the total number of light blocks of each color.
- 3 The LAX light display repeats in a three-hour cycle. How many cycles run in one day?



FACT • ACTIVITY 2

Huge lights show the letters *L-A-X* at the airport. Create a model for the letter *L* to design a new light display. Suppose you want to light 3 rectangular sections using red, white, and blue. Copy the *L* grid shown. Divide the grid into three arrays that will represent the 3 lighting sections.



- 1 Write a multiplication sentence to represent each array, and determine the number of lights needed to fill each section.
- 2 What is the total number of lights in the entire display?
- 3 Suppose your *L* design can have 165 light blocks in all. Draw a 15×11 array to represent all the light blocks. Divide it into 4 smaller arrays to verify that the sum of the four products is 165. Hint: Begin with a 10×10 array.

CHAPTER PROJECT

Design your own light display of 100 lights on a square grid.

- Use 4 different colors.
- Draw the arrangement so there are 4 rectangular sections.
- Write a multiplication sentence for each smaller array.
- Show how the number of lights in the four arrays add up to 100.

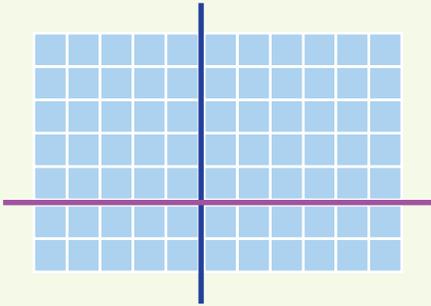


ALMANAC Fact

LAX is one of the world's busiest airports. More than 60 million passengers traveled into or out of LAX in 2005!

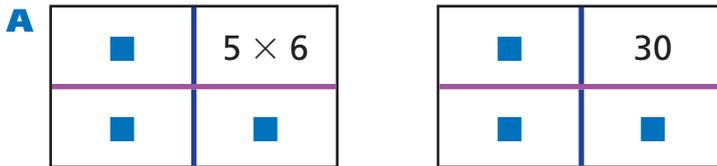
EXPLORE

Array Sections



- 1 Find the number of squares in this array.
- 2 Explain how you found this number.

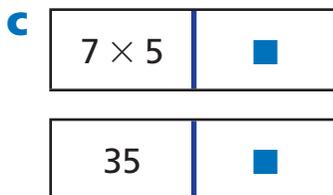
- 3 Copy and complete the diagrams and number sentences to match the array.



$$(\square \times \square) + (\square \times \square) + (5 \times 6) + (\square \times \square) = \square$$



$$(\square \times \square) + (\square \times \square) = \square$$



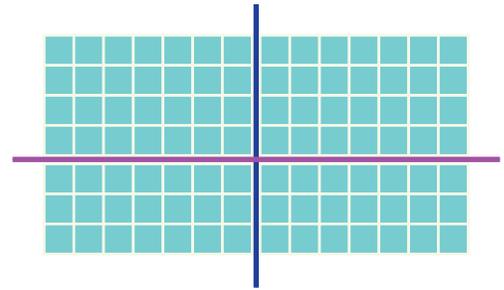
$$(\square \times \square) + (\square \times \square) = \square$$

REVIEW MODEL

Separating an Array in Different Ways

Find the number of squares in this array.

There are many ways to find the number of squares in an array.



1st Way

Step 1

Separate the array into four smaller sections, as is done above. The large array is separated into two 4-by-7 arrays and two 3-by-7 arrays.

Step 2

Complete each table to match the array.

4×7	4×7	28	28
3×7	3×7	21	21

Step 3

Write a number sentence to find the total number of squares in the array.

$$(4 \times 7) + (3 \times 7) + (4 \times 7) + (3 \times 7) = 98$$

There are 98 squares in this array.

Another Way

Step 1

Separate the array with only the horizontal line above. The large array is separated into a 4-by-14 array and a 3-by-14 array.

Step 2

Complete each table to match the array.

4×14	56
3×14	42

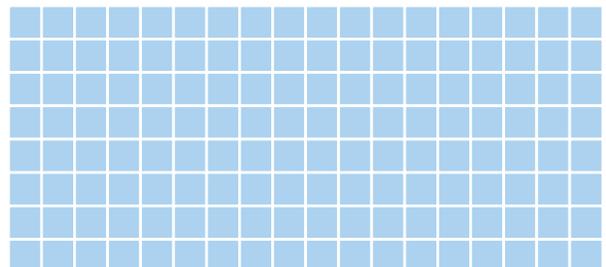
Step 3

Write a number sentence to find the total number of squares in the array.

$$(4 \times 14) + (3 \times 14) = 98$$

✓ Check for Understanding

- 1 Find the number of squares in this array. Show your work.



EXPLORE

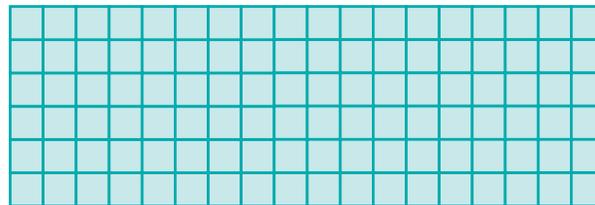
Combining Multiplication Facts

How many squares are in an array with 6 rows and 18 columns?

- 1 Copy and complete this table.

	1	2	3	5	6	8	10	11
$\times 6$	■	■	■	■	■	■	■	■

- 2 Use some of the multiplication facts in the table to separate the array and find the number of squares in each section. Copy and complete the grid and tables in **A** and **B** below.



A

6×10	■	■
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B

60	■	■
----	---	---

- 3 How many squares are in the array?

- 4 What is 6×18 ?

REVIEW MODEL

Using an Array to Explore a Multiplication Shortcut

You can use arrays to model a multiplication shortcut.

How many squares are in an array with 4 rows and 17 columns?

Step 1

Make a table to show multiplication facts you already know about the number of rows or the number of columns in the array.

	1	2	4	5	7	10
$\times 4$	4	8	16	20	28	40

This table is about multiplying by 4 because there are four rows in the array.

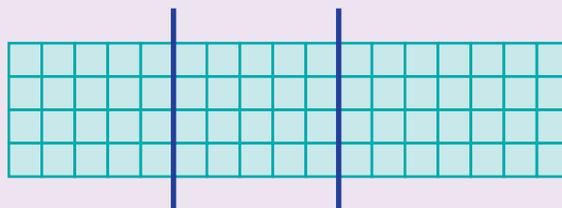
Step 3

Using the table from Step 1, find the number of squares in each section of the array.

$4 \times 5 = 20$	$4 \times 5 = 20$	$4 \times 7 = 28$
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Step 2

Use the facts from the table to decide how to separate your array into smaller sections.



Here the array is separated into three smaller arrays, a 4-by-5 array, a 4-by-5 array, and a 4-by-7 array, since $5 + 5 + 7 = 17$.

Step 4

Find 4×17 . Add the number of squares from each section of the array to find the total number of squares in the array.

$$\begin{aligned} 4 \times (5 + 5 + 7) \\ = (4 \times 5) + (4 \times 5) + (4 \times 7) \\ = 20 + 20 + 28 = 68 \end{aligned}$$

There are 68 squares in an array with 4 rows and 17 columns.

✓ Check for Understanding

Find the number of squares in each array. Show your work.

- 1 How many squares are in an array with 9 rows and 23 columns?

- 2 How many squares are in an array with 7 rows and 19 columns?

EXPLORE

Multiplication Patterns

Copy and complete the multiplication table.

×	1	2	3	4	5	6	7
1							
2							
3							
4							
5							
6							
7							

- How could you use the 5-times column to complete the 6-times column?

- Choose one of the top two rows and double the answers.
What do you notice?

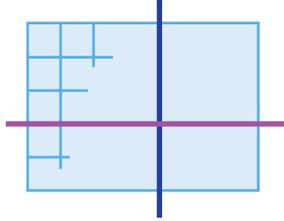
- Choose any two of the top four rows and add the answers.
What do you notice?

- Do you see any other patterns?

EXPLORE

How Many Rows and Columns?

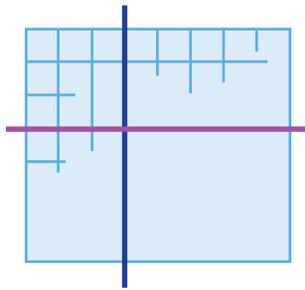
- 1 How many columns are in this array?



$3 \times \blacksquare$	$3 \times \blacksquare$
$2 \times \blacksquare$	$2 \times \blacksquare$

12	9
8	6

- 2 How many rows are in this array?



$\blacksquare \times 3$	$\blacksquare \times 5$
$\blacksquare \times 3$	$\blacksquare \times 5$

9	15
12	20

- 3 Use 15 tiles to make a rectangular array.

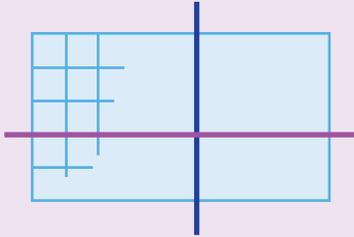
- A** How many rows does your array have?
- B** How many columns does your array have?
- C** Write a multiplication sentence to describe your array.
- D** Write the fact family that matches your array.

REVIEW MODEL

Finding the Number of Rows or Columns in an Array

You can find the missing dimension of an array by finding the missing factor in multiplication sentences.

How many columns are in this array?



$3 \times \blacksquare$	$3 \times \blacksquare$
$2 \times \blacksquare$	$2 \times \blacksquare$

15	12
10	8

Step 1

Because the array is incomplete, you must find the number of columns by using the tables with the multiplication expressions and the total number of squares in each section of the large array.

Make one table by writing multiplication sentences using the corresponding sections of the array and the tables above.

$3 \times \blacksquare = 15$	$3 \times \blacksquare = 12$
$2 \times \blacksquare = 10$	$2 \times \blacksquare = 8$

Step 2

Find the missing factor in each multiplication sentence.

$3 \times 5 = 15$	$3 \times 4 = 12$
$2 \times 5 = 10$	$2 \times 4 = 8$

By finding the missing factor in each multiplication sentence, you find the number of columns in each section of the large array.

Step 3

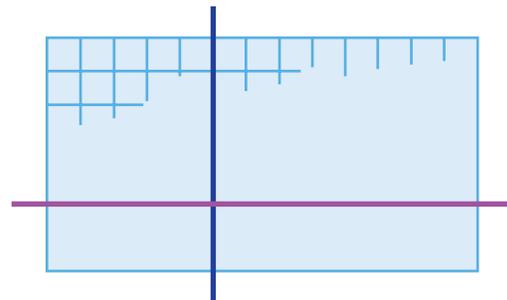
Since $5 + 4 = 9$, there are 9 columns in the large array.

✓ Check for Understanding

1 How many rows are in this array?

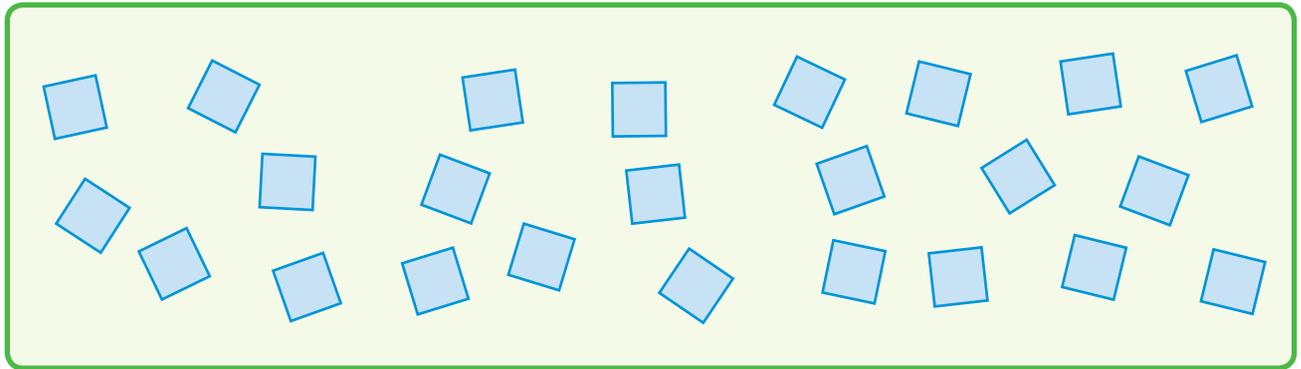
$\blacksquare \times 5$	$\blacksquare \times 8$
$\blacksquare \times 5$	$\blacksquare \times 8$

25	40
10	16

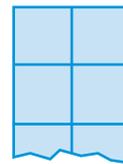


EXPLORE

Arranging 24 Tiles



- 1 Arrange these 24 tiles into an array with 2 columns. How many tiles are in each column?



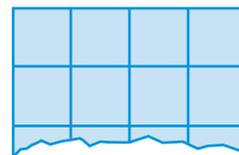
$$24 \div 2 =$$

- 2 Now arrange the tiles into an array with 3 columns. How many tiles are in each column?



$$24 \div 3 =$$

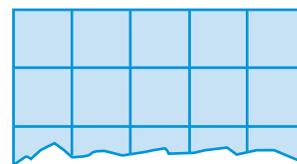
- 3 Now arrange the tiles into an array with 4 columns. How many tiles are in each column?



$$24 \div 4 =$$

- 4 Now arrange the tiles into an array with 5 columns.

A How many tiles are in each column?



B Can you write a number sentence to describe the array?

REVIEW MODEL

Problem Solving Strategy

Solve a Simpler Problem

Halaina read 23 books each month of the year.

How many books did she read in an entire year?

Strategy: Solve a Simpler Problem

Read to Understand

What do you know from reading the problem?

Halaina read 23 books each month of the year.

Plan

How can you solve this problem?

There are 12 months in one year. You can solve several simpler multiplication problems to find out how many books Halaina read in one year.

Solve

How can you solve simpler problems to solve this problem?

Make a 12-by-23 array.

Separate it into smaller sections using multiplication facts you know. For example, you could create 4 sections:

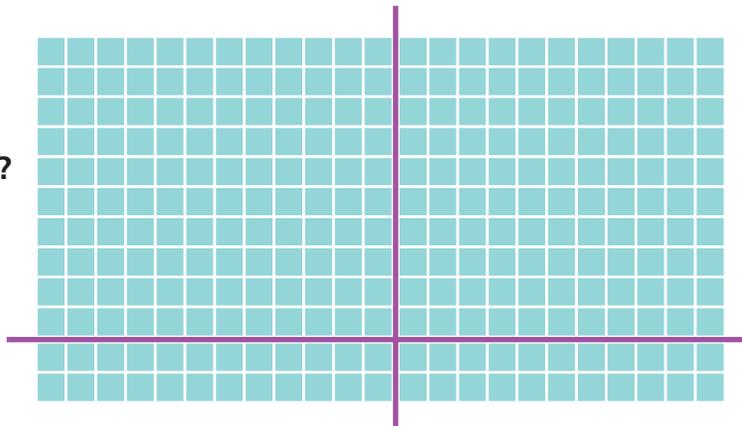
10×12 , 2×12 , 10×11 ,

and 2×11 . Find the number

of squares in each section: 120, 24, 110, and 22.

Add to find the total number of squares in the large array

$120 + 24 + 110 + 22 = 276$. Halaina read 276 books in one year.



Check

Look back at the problem. Did you answer the question that was asked? Does the answer make sense?

Problem Solving Practice

Solve a simpler problem to solve.

- 1 Staci uses 36 beads in each necklace that she makes. She made 11 necklaces. How many beads did she use?
- 2 Rob washes 6 cars each week. How many cars does he wash in 23 weeks?

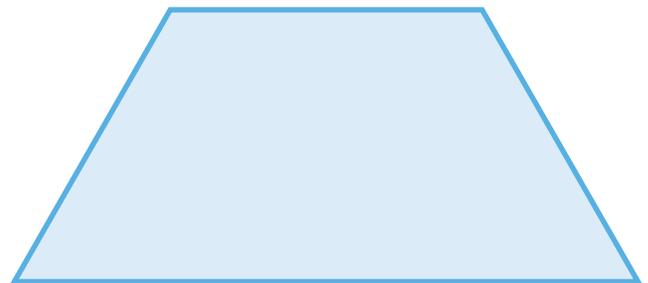
Mixed Strategy Practice

Use any strategy to solve.

- 3 Mrs. Holmes' class made kites. She hung her students' kites in the hallways. She had 2 rows of 7 kites in one hall and 2 rows of 4 kites in another hall. How many kites were displayed in all?
- 4 Todd has baseball practice from 3:30 P.M. to 4:30 P.M. It takes him a half hour to get home. Then he has one hour to eat his dinner before he must start his homework. At what time does he start his homework?
- 5 Adele, Denise, Ron, and Tom are all standing in line in the cafeteria. How many different ways can they arrange themselves to stand in line?
- 6 Aidan won the same number of tickets at each of the 3 games he played at the fair. His sister gave him 5 more tickets. If Aidan then has 23 tickets, how many tickets did he win at each game he played?

Use pattern blocks for Problems 7–8.

- 7 Alycia made a trapezoid using 3 red trapezoids, 1 blue rhombus, and 1 green triangle. What other combination of pattern blocks can be used to make a trapezoid congruent to the one Alycia made?
- 8 Use a different combination of pattern blocks to make another congruent trapezoid.



Problem Solving Strategies

- ✓ Act It Out
- ✓ Draw a Picture
- ✓ Guess and Check
- ✓ Look for a Pattern
- ✓ Make a Graph
- ✓ Make a Model
- ✓ Make an Organized List
- ✓ Make a Table
- ✓ **Solve a Simpler Problem**
- ✓ Use Logical Reasoning
- ✓ Work Backward
- ✓ Write an Equation

Choose the best vocabulary term from Word List A for each sentence.

- 1 A(n) ? problem can be rewritten as a division sentence.
- 2 An operation related to multiplication is ?.
- 3 Multiply ? to find a product.
- 4 In a division problem, the r stands for ?.
- 5 A letter that can stand for a number is called a(n) ?.
- 6 A number that is multiplied by another number to find a product is a ?.
- 7 A column is part of a(n) ?.
- 8 When there are ? tiles, it means that there is a remainder.

Complete each analogy using the best term from Word List B.

- 9 Addend is to sum as ? is to product.
- 10 Horizontal line is to vertical line as row is to ?.

Word List A

array
column
divide
division
factor
factor pairs
horizontal line
leftover
missing factor
remainder
remaining
row
variable
vertical line

Word List B

array
column
factor
variable

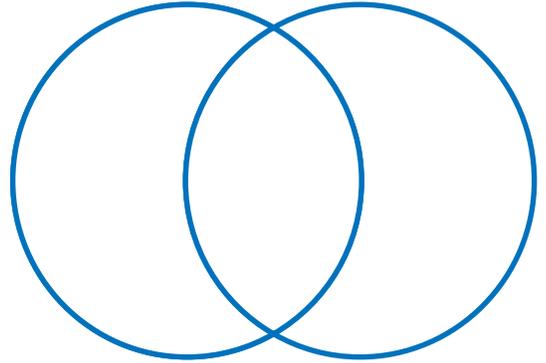
Talk Math

Discuss with a partner what you have learned about multiplication and division. Use the vocabulary terms *array*, *column*, and *row*.

- 11 How can you use an array to model multiplication?
- 12 How can you use an array to model division?
- 13 A large array of dots is separated into two smaller arrays. How can you find the total number of dots?

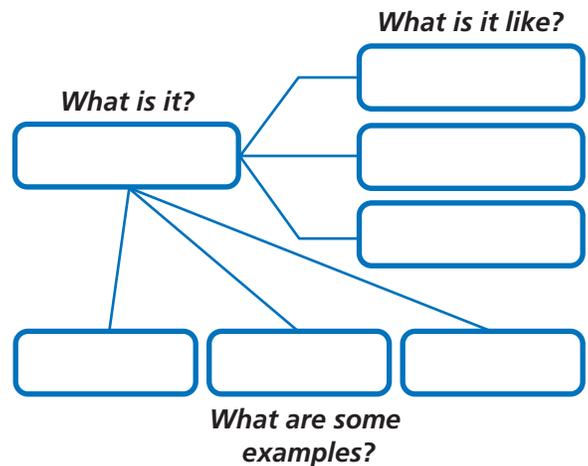
Venn Diagram

- 14 Create a Venn diagram for multiplication terms and division terms. Use the words *array*, *column*, *divide*, *division*, *factor*, *factor pairs*, *leftover*, *missing factor*, *product*, *remainder*, *remaining*, and *row*.



Word Definition Map

- 15 Create a word definition map using the word *division*. Use what you know and what you have learned about multiplication and division.



What's in a Word?



PRODUCT The word *product* can be used in many different situations. The *product* of a farm might be corn, beans, wheat, milk, or beef. Those things are produced on a farm. The *product* of a factory might be cars, marbles, baseball bats, or light bulbs. Those things are produced in a factory. Similarly, in mathematics a product is produced by multiplying two or more numbers.



Technology

Multimedia Math Glossary

www.harcourtschool.com/thinkmath

GAME

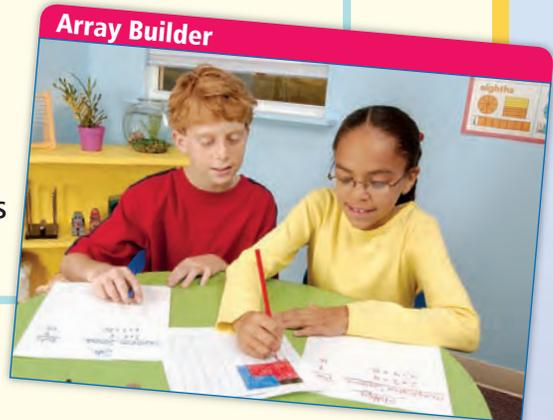
Array Builder

Game Purpose

To practice using arrays as a model for multiplying

Materials

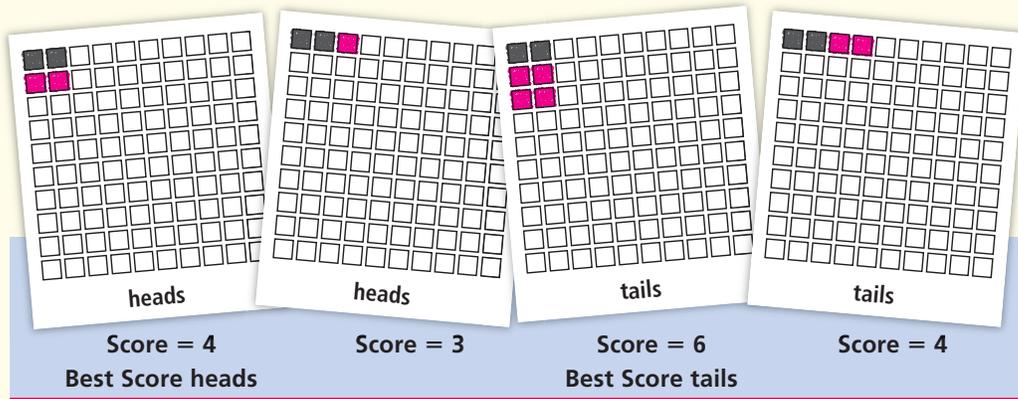
- Activity Master 8: *Array Builder*
- 2 different colors of crayons or pencils
- a coin



How to Play the Game

- 1 Play this game with a partner. Before starting, make a 1×2 array on the *Array Builder* by shading the two upper left squares. Choose your crayon color. Then decide who will play first.
- 2 Player 1 flips the coin.
 - If the coin lands heads up, add 1 row or column to the array.
 - If the coin lands tails up, add 2 rows or columns to the array.
 - Try to make an array that will give the largest product. Your score for that turn is the product for the array.

Example: The first 4 possible plays of the game are shown in red.



- 3 Take turns flipping the coin and making new arrays until there are not enough squares left to make a play.
- 4 Add your points. The player with the most points is the winner.

GAME

Fact Family Fandango

Game Purpose

To practice writing multiplication and division fact families

Materials

- 2 number cubes (labeled 1–6)

How to Play the Game

- 1 Play this game with 3 players. Player 1 tosses the number cubes and records their sum. Player 2 makes a second number the same way. Player 3 uses the two numbers to write a multiplication sentence. All players must agree that the product is correct.

Example:

John tosses these numbers.



Charlie tosses these numbers.



Nancy writes this multiplication sentence.

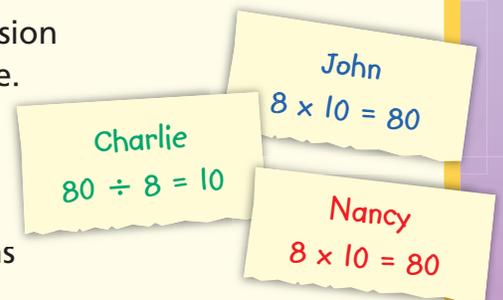
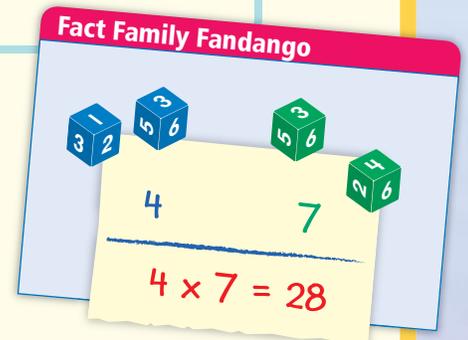
$$8 \times 10 = 80$$

- 2 Next, each player secretly writes another member of the fact family for that multiplication sentence.
- 3 Compare all 3 multiplication sentences. You score 1 point if you wrote a number sentence that no one else wrote.

Example: Here are the multiplication and division sentences that John, Charlie, and Nancy wrote.

So, Charlie scores 1 point.

- 4 Switch roles, and repeat steps 1 through 3. Play until one player scores 10 points and wins the game.



CHALLENGE

Cheryl likes to share. Help solve each of her problems so that she can share evenly with no leftovers.

You may want to use counters, tiles, or coins to make arrays.

Cheryl wants to share her raisins. When she tries to share them with one friend, there is 1 left over. When she tries to share them with 2 friends, there are 2 left. When she tries to share them with 3 friends, there is 1 left. When she tries to share them with 4 friends, there are 4 left. (Remember that Cheryl herself shares with each group.)

- 1 Does Cheryl have an odd number or an even number of raisins? How do you know?
- 2 What is the smallest number of raisins Cheryl could be trying to share?
- 3 What is the smallest number of raisins she should have next time so that she can share them evenly with 1, 2, 3, or 4 friends?

Cheryl has a box of crayons. The table below shows what happens when she tries to share them.



When she tries to share them with	there are (is)
1 friend	none left over
2 friends	1 left over
3 friends	2 left over

- 4 Does Cheryl have an odd number or an even number of crayons? How do you know?
- 5 What is the smallest number of crayons she could be trying to share?
- 6 What is the smallest number of extra crayons she should have next time so that she can share them evenly with 1, 2, or 3 friends?