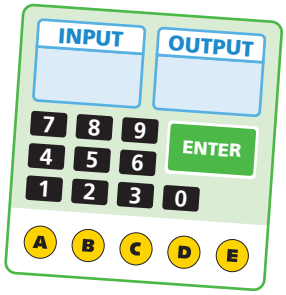
Chapter

1 Algebra: Machines and Puzzles

Dear Student,

When you saw the title of this chapter, "Algebra: Machines and Puzzles," you might have wondered what machines and puzzles have to do with math. You will see a variety of machines, puzzles, and puzzling machines in this program.

Here is an input-output machine that you will see in some of the lessons in this chapter.



What do you think this machine does? Does it look like anything you have seen before? What do you suppose the five buttons at the bottom of the machine do?

This completed puzzle may look familiar to you.

18	11	29
16	15	31
34	26	60

If you have seen it before, how would you describe how it works to a younger student? If it is new to you, try to figure out how it works.

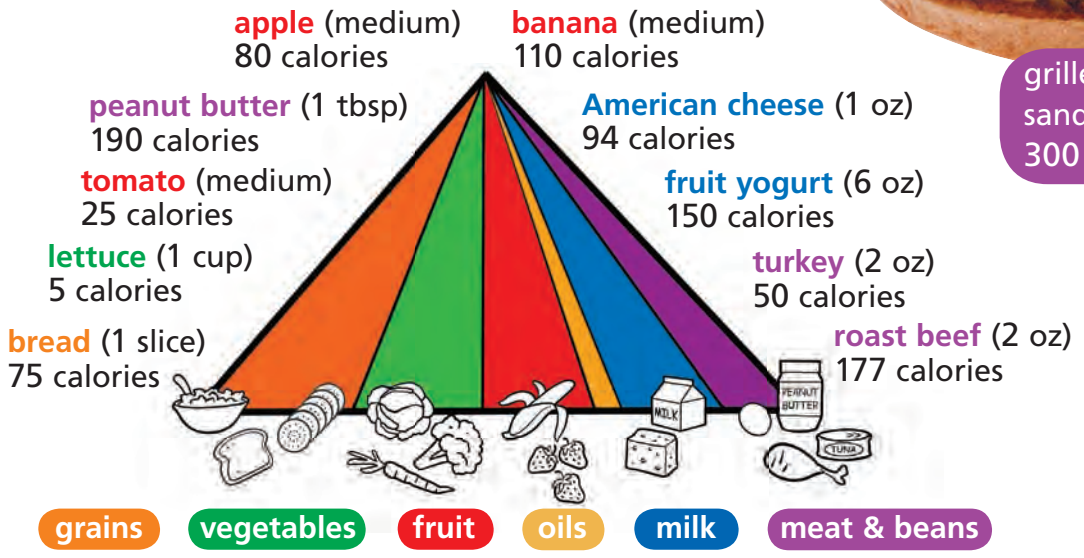
You already know how to add, subtract, multiply, and divide, but did you know that by the end of this first chapter, you will also be doing some algebra? Mathematicians and high school students use algebra, and you can too!

Mathematically yours,
The authors of ***Think Math!***

Food for Thought

Food is like fuel. It stores the energy your body needs to grow, play, and stay healthy. This energy is measured in calories. You need a balance between the number of calories you eat and the energy you use. If you don't get enough calories, you may feel tired. But, eating too many calories can also slow you down.

grilled chicken sandwich: 300 calories



FACT·ACTIVITY 1

Use the food pyramid to answer the questions.

1. Plan a sandwich using any of the ingredients shown above. How many calories are in your sandwich?
2. How do the calories in your sandwich compare to the calories in a ready-made grilled chicken sandwich?
3. Michael is a professional soccer player. He can burn about 1,000 calories during a game. What combination of foods on this page could Michael eat that would be burned up during a soccer game? Create two different menus for Michael to eat.

FACT ACTIVITY 2

The table below shows the approximate number of calories used in 10 minutes for different activities. Use the table to answer the questions.

Calories Used in Various Activities			
Calories used in 10 minutes if your weight is...	50 lbs	100 lbs	150 lbs
Activity			
Sleeping	4	8	12
Standing	8	16	24
Raquetball	30	60	90
Running	44	88	132
Rock climbing	30	60	90

1. How many calories would a 150-pound person burn waiting in line for 10 minutes?
2. How many calories would a 100-pound person burn by sleeping for 1 hour? for 8 hours?
3. How many calories would a 150-pound person use playing racquetball for 30 minutes?

CHAPTER PROJECT

Make choices to balance calories and activities.

- Research the number of calories that two other activities use up in 10 minutes.
- Select an activity from this page or from your research that you would like to do. Estimate the number of calories you would use in one hour of that activity.
- Use the food options from Fact Activity 1 or research the calorie content of other foods. Create two different lunch menus that will balance (be about equal to) the calories you use in one hour of the activity you chose.
- Present your choices on a chart or poster.

ALMANAC Fact

When Sir Ranulph Fiennes and Dr. Mike Stroud walked across Antarctica in 1993, they each used about 12,000 calories per day–about as many calories as are found in 200 apples, 24 burritos, or 5 pounds of chocolate.

Chapter 1
Lesson 1

EXPLORE
Exploring Coin Combinations

1 Here is the kind of question you will not often see in this program.

> **Ashley made a 75¢ purchase.**
> **She paid with a one-dollar bill.**
> **How much change should she get?**

All Ashley needs to know is that she should receive a quarter's worth of change!

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2 In this book, you are likely to find questions like this:

> **How many different combinations of pennies, nickels, and dimes make a quarter's worth of change?**

---

3 List four different combinations of coins that make 25¢. You will list more later.

Chapter 1

Lesson 2

REVIEW MODEL
Cross Number Puzzles

You can practice computation and learn about equations and properties of addition in Cross Number Puzzles. To complete these puzzles, follow the rule: Amounts on both sides of a thick line must be the same.

Look at the numbers in this Cross Number Puzzle.

25	17	42
15	38	53
40	55	95

You can write addition and subtraction number sentences to go with a puzzle.

Some number sentences you might write for this puzzle are:

$25 + 17 = 42$
$53 - 38 = 15$
$40 + 55 = 95$
$95 - 53 = 42$
$(25 + 17) + (15 + 38) = 42 + 53 = 95$

Solve this puzzle by choosing numbers to make amounts on both sides of thick lines the same.

Example Find the missing numbers.

In the second row I see 45 on one side of a thick line and 73 on the other side. I need more on the side with 45. How much more? I can think of ■ + 45 = 73 or 73 − 45 = ■

The missing number is 28.

24	51	
	45	73
52	96	148

24 and 51 are on one side of the thick line. What belongs on the other side?

24 + 51 = ■

75 is the missing number.

Check for Understanding

Copy and write the missing numbers in the Cross Number Puzzles.

1

13	25	■
19	■	73
■	■	■

2

271	■	382
108	■	575
■	■	■

3

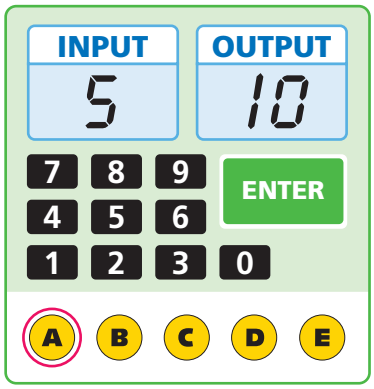
683	■	194
■	100	■
■	■	115

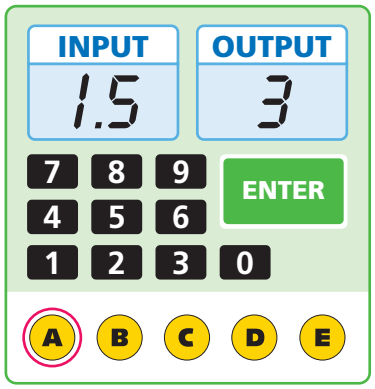
Chapter 1

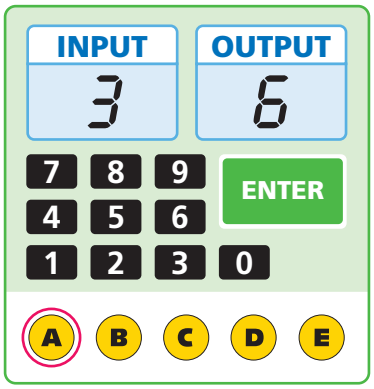
Lesson 3

EXPLORE
Exploring Input-Output Machines

If you press Button A, type in these inputs, and press ENTER, you get these outputs.







1 Find a rule for what the machine is doing when you press Button A.

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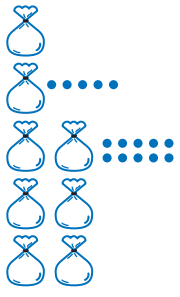
2 Do the numbers in this table work for your rule?

INPUT	2	5	1	3	0	4	12	50
MACHINE OUTPUT	4	10	2	6	0	8	24	100

---

3 This table shows a three-step rule. What do you notice about each output?

INPUT	2	5	1	3	0	4	12	50
Add 5	7	10	6	8	5	9	17	55
Double	14	20	12	16	10	18	34	110
Subtract 10	4	10	2	6	0	8	24	100
MACHINE OUTPUT	4	10	2	6	0	8	24	100



Chapter 1

Lesson 3

REVIEW MODEL
Input-Output Tables

To complete an input-output table, you use a rule, or rules, and an input number.

This table shows a 1-step rule. The rule is *add 6.* Sometimes you have to figure out what the rule is by looking for patterns in input-output pairs you know.

INPUT	6	3	10	0	21	7
Add 6	12	9	16	6	27	■
OUTPUT	12	9	16	6	27	■

Think:

$7 + 6 = 13$

So, the output is 13.

This table shows a 3-step rule. The rules are *multiply by 3,* then *subtract the input,* then *add 2.*

INPUT	5	12	0	8	10	9
Multiply by 3	15	36	0	24	30	■
Subtract the input	10	24	0	16	20	■
Add 2	12	26	2	18	22	■
OUTPUT	12	26	2	18	22	■

Think:

$9 \times 3 = 27$

$27 - 9 = 18$

$18 + 2 = 20$

So, the output is 20.

Check for Understanding

Copy and complete the input-output tables.

1.

INPUT	8	0	10	12
Multiply by 4	32	0	■	■
OUTPUT	32	0	■	■

2.

INPUT	10	20	30	40
Subtract 8	2	■	■	■
OUTPUT	2	■	■	■

3.

INPUT	5	1	11	8
Multiply by 2	10	■	■	■
Add 4	14	■	■	■
Subtract 2	12	■	■	■
OUTPUT	12	■	■	■

4.

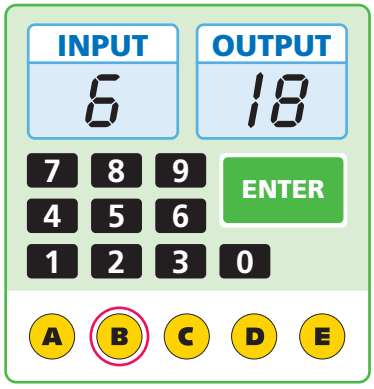
INPUT	10	9	8	7
Multiply by 3	■	■	■	■
Divide by 3	■	■	■	■
Add 10	■	■	■	■
OUTPUT	■	■	■	■

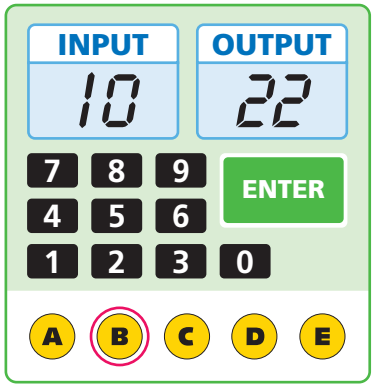
Chapter 1

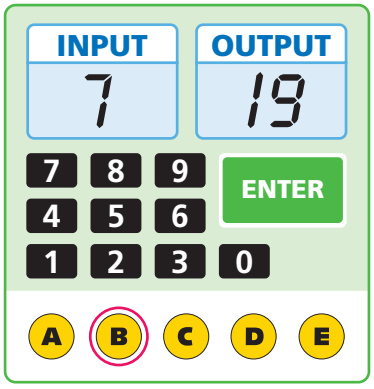
Lesson 5

EXPLORE
Exploring Button B and Button C on the Machine

This is what happens to 6, 10, and 7 when Button B on the machine is pressed.



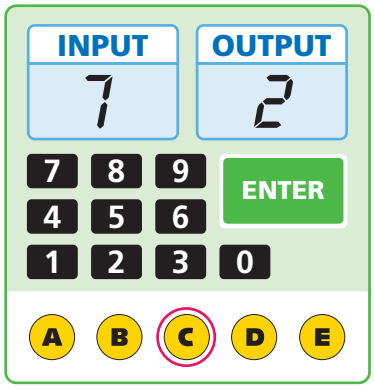


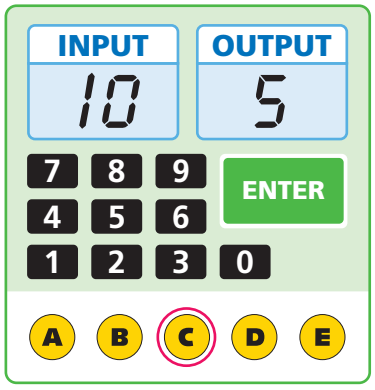


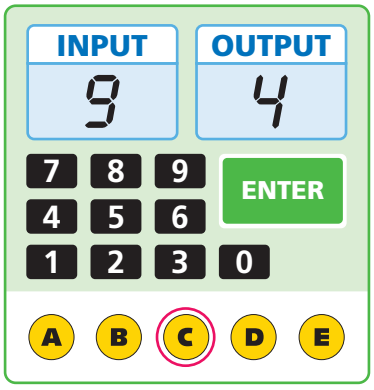
1. What is a rule for Button B?

2. How would you make bag-and-dot drawings for your rule?

This is what happens to three inputs when Button C on the machine is pressed.







3. What is a rule for Button C?

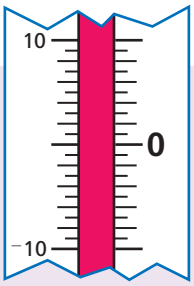
4. What will the OUTPUT be if the INPUT is 4?

Chapter 1

Lesson 5

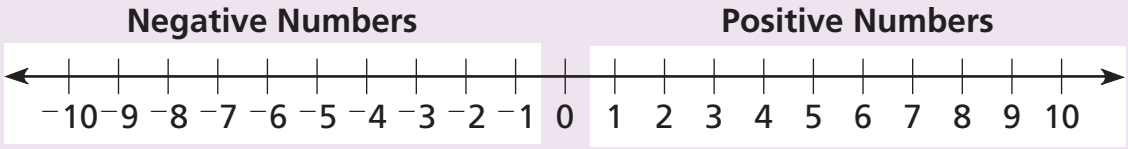
REVIEW MODEL
Negative Numbers

You can use a thermometer or a number line to help you think about negative numbers.



On a thermometer, positive temperatures are above 0. Negative, or minus, temperatures are below 0.

On a number line, positive numbers are to the right of 0. Negative numbers are to the left of 0. 0 is neither positive nor negative.



Negative numbers are always written with a negative sign. $^{-}6$ is read "negative 6."

You can use a thermometer or a number line to help you subtract.

Example

Rule: Subtract 4 or -4.

INPUT	7	6	5	4	3	2	1	0
OUTPUT	3	2	1	0	$^{-}1$	$^{-}2$	$^{-}3$	$^{-}4$

Think: Start at 3. Move 4 spaces left on the number line. End at $^{-}1$.

Check for Understanding

Copy and complete the input-output tables for the subtraction rules.

1. Rule: Subtract 2, or -2.

INPUT	4	3	2	1
OUTPUT	2	1	■	■

2. Rule: Subtract 5, or -5.

INPUT	6	4	2	0
OUTPUT	1	■	■	■

3. Rule: Subtract 8, or -8.

INPUT	12	10	8	6
OUTPUT	4	■	■	■

4. Rule: Subtract 10, or -10.

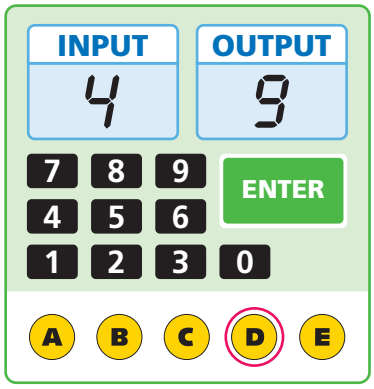
INPUT	20	10	5	0
OUTPUT	■	■	■	■

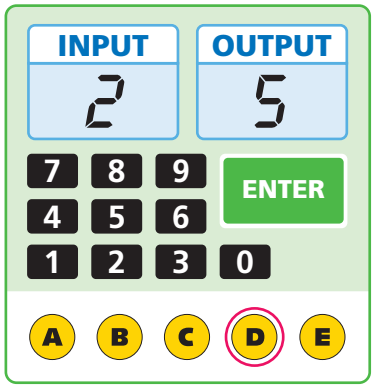
Chapter 1

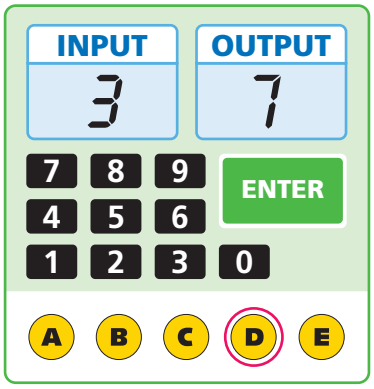
Lesson 6

EXPLORE
Exploring Button D on the Machine

If you press Button D, type in these inputs, and press ENTER, you get these outputs.







1 Find a rule for what the machine is doing.
Try combining operations.

2 Do the numbers in this table work for your rule?

INPUT	6	8	5	10	25	12
OUTPUT	13	17	11	21	51	25

Chapter 1

Lesson 7

REVIEW MODEL
Multiplying Cross Number Puzzles

You can practice multiplication in Cross Number Puzzles. Remember the Cross Number Puzzle rule: Amounts on both sides of a thick line must be the same.

Look at the numbers in Puzzle A and in Puzzle A × 5.

- Each number in Puzzle A × 5 is 5 times the matching number in Puzzle A.
- The new puzzle still works—the amounts on both sides of a thick line are the same.

A

3	7	10
6	9	15
9	16	25

× 5

A × 5

15	35	50
30	45	75
45	8	125

Here are some matching addition and subtraction sentences for these puzzles.

Puzzle A	**Puzzle A × 5**
3 + 7 = 10	15 + 35 = 50
15 − 9 = 6	75 − 45 = 30

When multiplying a puzzle by a number, you multiply all the numbers in that puzzle by the number.

$(3 \times 5) + (7 \times 5) = 10 \times 5$, or $15 + 35 = 50$

$(15 \times 5) - (9 \times 5) = 6 \times 5$, or $75 - 45 = 30$

Check for Understanding

Copy and write the missing numbers in the Cross Number Puzzles.

1

C

3	6	■
2	5	■
■	■	■

× 4

C × 4

12	■	■
■	20	■
■	■	■

2

D

1	16	■
13	7	■
■	■	■

× 2

D × 2

■	32	■
■	■	■
28	■	■

Chapter 1

Lesson 8

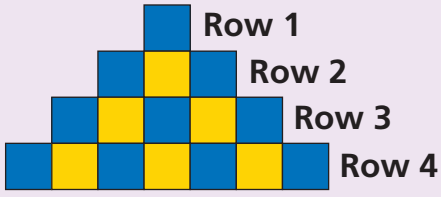
REVIEW MODEL

Problem Solving Strategy

Look for a Pattern

Hailey is making a design with squares. She started at the top and kept adding rows of squares to the bottom.

Complete the table to show the number of squares there will be in rows 5, 6, and 7. Write a rule.



Row	1	2	3	4	5	6	7	r
Number of Squares	1	3	5	7	■	■	■	■

Strategy: Look for a Pattern

Read to Understand

What do you know from reading the problem?

Hailey made a design with squares. The design shows 1 square in Row 1, 3 squares in Row 2, 5 squares in Row 3, and 7 squares in Row 4.

Plan

How can you solve this problem?

You can look for a pattern in the table that tells you how to find the number of squares, if you know the number of rows. Then you can use the pattern to complete the table and to help you write the rule.

Solve

How can you find the pattern?

Look at how the input- and output-pairs relate. Each output is twice the input minus 1. So, a rule for the pattern is $2r - 1$. To complete the table, use the Row numbers in place of r. $(2 \times 5) - 1 = 9$.

Check

Look back at the problem. Did you answer the questions that were asked? Do the answers make sense?

Problem Solving Practice

Look for a pattern to solve these problems.

1. Mike saves 4 pennies on Day 1, 8 pennies on Day 2, 12 pennies on Day 3, and 16 pennies on Day 4. If the pattern continues, how many pennies will he save on Day 8? What is a rule for the pattern?

2. Monica is making picture frames where the length and width are related by a rule. Sizes are shown in inches in the table. What is the length when the width is 8 inches? What is a rule for the pattern?

Width	2	3	5	8	x
Length	4	5	7	■	■

Problem Solving Strategies

- ✔ Act It Out
- ✔ Draw a Picture
- ✔ Guess and Check
- ✔ **Look for a Pattern**
- ✔ Make a Graph
- ✔ Make a Model
- ✔ Make an Organized List
- ✔ Make a Table
- ✔ Solve a Simpler Problem
- ✔ Use Logical Reasoning
- ✔ Work Backward
- ✔ Write an Equation

Mixed Strategy Practice

Use any strategy to solve. Explain.

3. Anthony had 4 boxes with 6 erasers in each box. He gave 2 erasers to each of 3 friends. How many erasers does he have left?

4. Madison earns $10 each week walking a neighbor's dog. She saves half the money she earns. How long will it take her to save $50?

5. In the morning Thomas spent 15 minutes raking and 20 minutes mowing the lawn. In the afternoon, he spent twice as much time raking and twice as much time mowing. How much time did Thomas spend raking and mowing?

6. Jasmine bought a notebook for $5, a ruler for $2, and a set of markers for $4. Heather bought a book bag for $8 and some pencils. They both spent the same amount of money. How much did Heather pay for the pencils she bought?

For 7–9, use the table.

7. How much farther is it from Dallas to New York than from Dallas to Chicago?

8. How far is a round trip between Dallas and Atlanta?

9. Carla made a round trip between Dallas and one of the cities listed in the table. She drove 2,722 miles in all. To which city did Carla drive?

Distance Between Dallas and Some United States Cities

City	Number of Miles to Dallas
Atlanta	782
Chicago	967
Miami	1,361
New York	1,550
San Francisco	1,732

Chapter 1 Vocabulary

Choose the best vocabulary term from Word List A for each definition.

1. Division is a(n) ___?___.
2. Numbers to the right of zero on a number line are ___?___ numbers.
3. Numbers to the left of zero on a number line are ___?___ numbers.
4. To ___?___ a number, just multiply it by 2.
5. An algebraic expression is ___?___ for a rule in a function machine.
6. If two values are the same, then one value ___?___ the other.
7. If the input of an input-output table is 8 and the ___?___ is subtract 2, then the output is 6.
8. One way to solve a problem involving groups of numbers is to make a(n) ___?___.

Word List A

- algebra
- double
- equals
- function
- negative
- operation
- organized list
- output
- positive
- rule
- shorthand
- zero

Complete each analogy using the best term from Word List B.

9. Addend is to sum as ___?___ is to product.
10. Input is to rule as ___?___ is to algebraic expression.

Word List B

- combination
- factor
- inverse
- variable

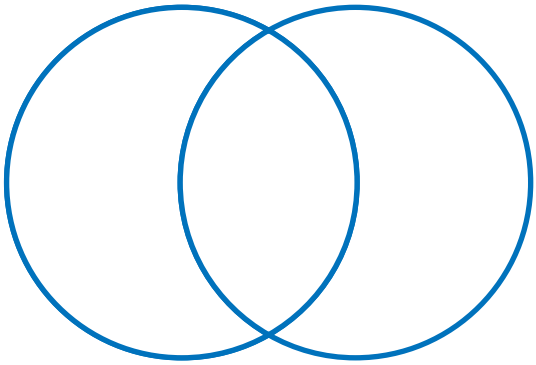
Talk Math

Use the vocabulary terms *operation, rule,* and *inverse* to discuss with a partner what you have just learned about algebra.

11. How can you identify a rule to complete an input-output table?
12. If you know the output of a function machine, how can you find the input?

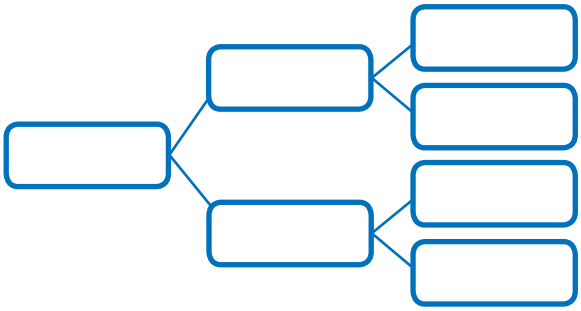
Venn Diagram

13 **Create a Venn diagram for the words *addend, algebraic expression, factor, input, output, product, rule, sum,* and *variable.***



Tree Diagram

14 **Create a tree diagram using the word *function.* Use what you know and what you have learned about algebra and rules.**



COMBINATION The word *combination* has different meanings in the English language. The root of the word *combination* is *combine* which means "to put together." The combination for a combination lock is put together in only one way. If you try other arrangements of the group of numbers, they won't open the lock. In mathematics, however, a *combination* is simply a group of numbers in any arrangement or order. For example, the list of the numbers 5, 13, and 22 is considered to be the same combination as the numbers 22, 5, and 13.

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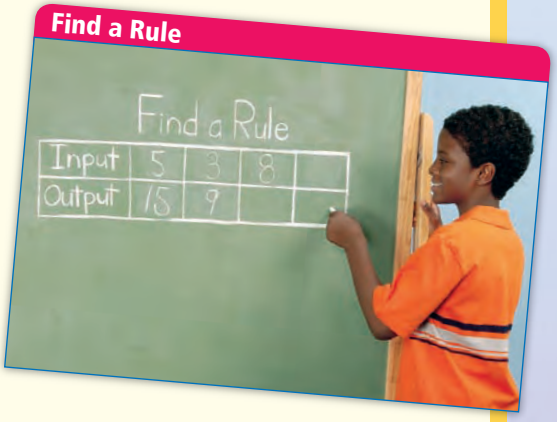
GAME

Find a Rule

Game Purpose
To practice finding rules for pairs of input-output numbers

How To Play The Game



1. Play the game with a small group of students. The group will need pencil and paper or space at the board with chalk or a marker.

2. Decide who will be the rule-maker. The rule-maker draws an input-output table and chooses a secret rule. The rule can use addition, subtraction, multiplication, or division.

   Examples of rules: $+ 9$, $- 8$, $\times 4$, $\div 3$

3. The other players take turns writing input numbers in the input-output table. The rule-maker writes the output for each input number using the secret rule.
   - If the output is not a whole number, the rule-maker writes an X in the box. This might happen if the rule-maker chooses a division rule or a subtraction rule whose output will be less than zero.

4. When a player thinks he or she knows the secret rule, that player asks the rule-maker to give an input.
   - If the player gives the correct output, he or she wins that round. The winner becomes the next rule-maker and chooses a new secret rule.
   - If the player is not correct, the game continues until another player determines the secret rule and wins the round.

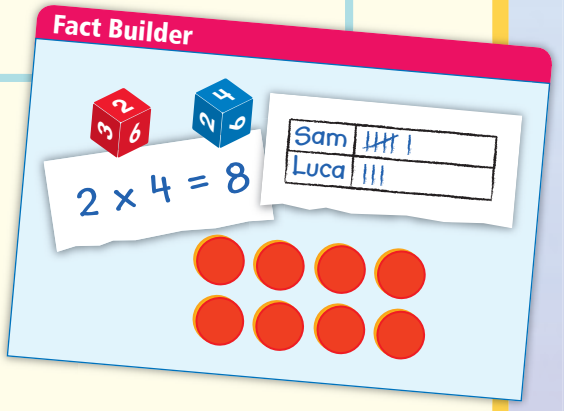
5. Play as many rounds as time allows.

GAME

Fact Builder

Game Purpose
To practice multiplication facts up to 12 × 12

Materials
• number cube (numbered 1–6)



How To Play The Game

1. Two, three, or four players can play. Each player needs a pencil and paper.

2. The first player tosses the number cube twice to get two factors and writes the multiplication fact.
   • Players must agree that the product is correct. To check the product, draw an array or use counters.
   • If the product is correct, the player receives 1 point.

3. The next player tosses the number cube once and adds the outcome to the lesser number to make a new multiplication fact.

4. Continue taking turns, adding the number tossed to the lesser factor, and writing the product.
   • If the toss of the cube causes one factor to be greater than 12, start again by tossing two cubes.

Example of play:

Roll	Outcome	Multiplication Fact
First	3	
Second	5	$3 \times 5 = 15$
Third	4	$5 \times (3 + 4) = 5 \times 7 = 35$
Fourth	6	$7 \times (5 + 6) = 77 \times 11 = 77$

5. The first player to score 10 points wins the round. Play as many rounds as time allows.

CHALLENGE

What's Going On Here?

Derrick and Lea like to make up codes to try to stump one another. After a lesson on input-output tables and rules, they decided to make up their own math codes. Try these math codes to see whether you can keep up with Derrick and Lea.

Write a rule for each symbol.

1 What does the ♥ mean?

INPUT	2 3	4 3	1 4	3 3	0 1
Code	♥	♥	♥	♥	♥
OUTPUT	7	13	5	10	1

2 What does the  mean?

INPUT	8 2	10 5	4 4	9 3	2 1
Code					
OUTPUT	3	1	0	2	1

3 What does the  mean?

INPUT	3 2	1 6	0 2	5 3	1 11
Code					
OUTPUT	10	14	4	16	24

4 What does the ● mean?

INPUT	6 2	8 4	1 7	12 8	0 8
Code	●	●	●	●	●
OUTPUT	2	3	2	5	2

5 Create your own math code. Write some examples using the code. Then challenge a classmate to determine the meaning of your code.