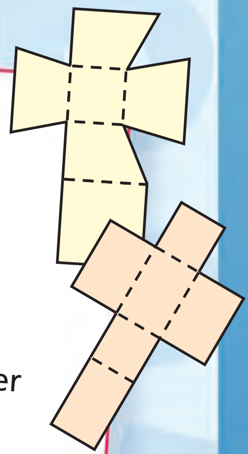
Chapter 12 Three-Dimensional Geometry



Dear Student,

When you were studying geometry in two dimensions, you learned that you could cut apart a figure, rearrange its pieces, and make a new figure with the same area.

You may already know how to find the volume of a three-dimensional figure that is shaped like a brick. In this chapter you will use the cutting-and-rearranging idea to identify rules for finding the volumes of other three-dimensional figures. As in earlier grades, you will use two-dimensional nets to make three-dimensional figures. You will see some new nets as well as some familiar ones, and you will sometimes compare three-dimensional figures by comparing their nets. Which of the two nets shown above looks like it would fold up into a brick-shaped object?

Here's another problem you will learn to solve. Imagine that you want to figure out how much paper it would take to wrap a box. How might you do this?

To solve problems like these, you will use the knowledge that you already have about finding the areas of two-dimensional figures such as parallelograms, triangles, and trapezoids. Onward, into the third dimension!

Mathematically yours,

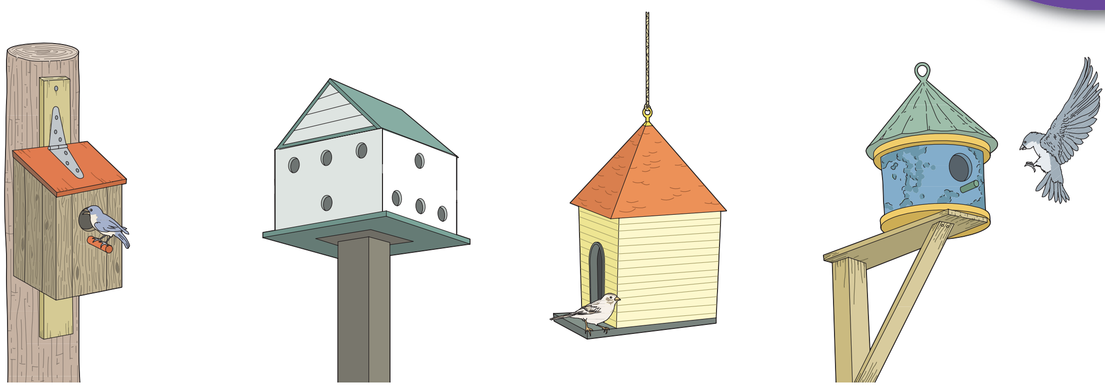
The authors of ***Think Math!***

THE WORLD ALMANAC FOR KIDS

Building a Birdhouse

Birdhouses in backyards or parks attract birds, usually during nesting season. In southern states, February is the best time to put up a birdhouse. In northern states, March is an appropriate time.

Birdhouses come in all shapes and sizes depending on the type of bird it is meant to attract.



Birdhouse A   Birdhouse B   Birdhouse C   Birdhouse D

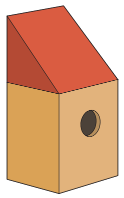
FACT · ACTIVITY 1

Look at the birdhouses and answer the questions.

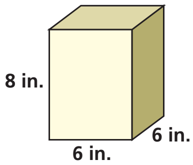
1. What two-dimensional figures can you identify in Birdhouses A, B, and C?
2. Which of the two-dimensional figures in Problem 1 appear to have congruent sides? Explain.
3. What three-dimensional figures do you see in the birdhouses? Identify the birdhouse and the figure.
4. Compare the roofs of Birdhouse B and Birdhouse C. Write the number of faces, vertices, and edges for each birdhouse.

FACT·ACTIVITY 2

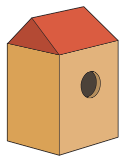
Shanti and Todd are building different model birdhouses using cardboard. Both designs have the same rectangular prism for the bottom of the birdhouse.



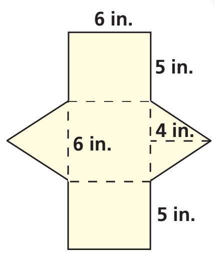
Shanti's birdhouse



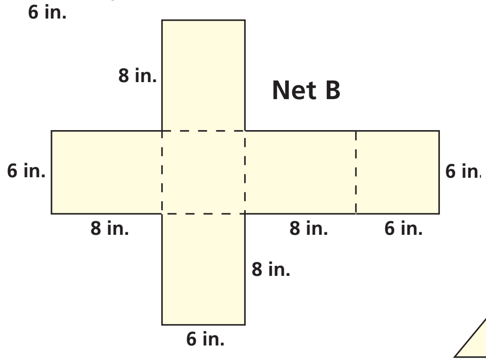
Todd's birdhouse



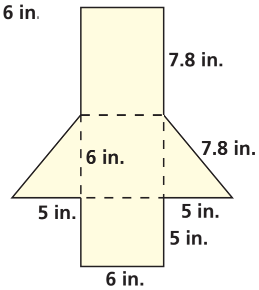
Net A



Net B



Net C



Look at the birdhouses and answer the questions.

1. Which net matches the roof of Shanti's birdhouse? of Todd's bird house? What does the other net show?
2. What is the total surface area of Shanti's birdhouse? of Todd's birdhouse? Remember to include the base and use only the outside dimensions.
3. Find the volume of Todd's birdhouse.

CHAPTER PROJECT

You are going to design a birdhouse and then make a model which is half its actual size.

- Sketch a three-dimensional model and label its sides with measurements. Then draw a net with each side half of the given dimensions in the drawing.
- Trace the net on manila cardboard, cut it out, then fold and tape the sides together. Decorate your model.
- Find the surface area of the model.

ALMANAC Fact

Cowbirds, sometimes known as lazy birds, neither nest nor take care of their young. Instead, the female cowbird lays her eggs in other birds' nests and lets the other birds take care of her young!

Chapter 12
Lesson 2

EXPLORE
Three-Dimensional Figure Search

Find as many three-dimensional figures as possible from your class collection that appear to match each set of attributes.

Group 1

- Same number of faces as vertices
- Either all faces are triangles, or at most one is not
- No parallel faces
- All faces but one share a vertex

Group 2

- Fewer faces than vertices
- Two faces that are parallel, but not congruent
- Other faces are not parallelograms or triangles

Group 3

- More faces than vertices
- One pair of congruent faces that appear parallel, but are twisted in relation to each other
- All other faces are triangles

Group 4

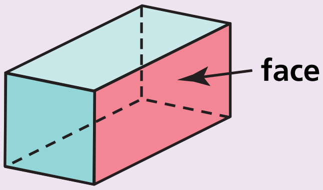
- Fewer faces than vertices
- Two congruent, parallel polygonal faces
- All other faces are parallelograms

Chapter 12
Lesson 2

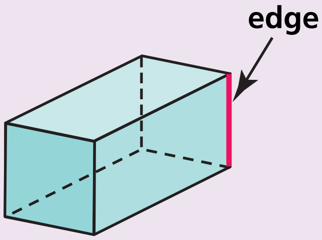
REVIEW MODEL
Faces, Vertices, and Edges

Figures pictured here have faces, edges, and vertices.

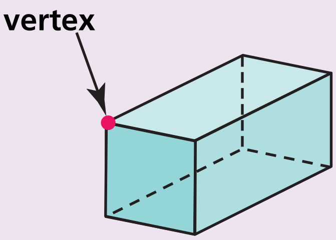
Faces are the flat surfaces that “surround” the insides of a three-dimensional figure.



Edges are the seams where the faces meet.



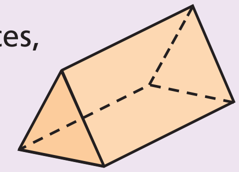
Vertices are the places where more than two faces meet at a point.



You can count the number of faces, vertices, and edges on these three-dimensional figures.

Example 1

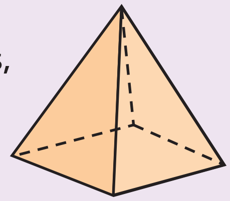
How many faces, vertices, and edges does this three-dimensional figure have?



It has 5 faces, 6 vertices, 9 edges.

Example 2

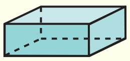
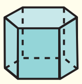
How many faces, vertices, and edges does this three-dimensional figure have?



It has 5 faces, 5 vertices, 8 edges.

Check for Understanding

Copy and complete the table.

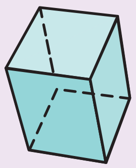
	Three-Dimensional Figure	Number of Faces	Number of Vertices	Number of Edges
1		■	■	■
2		■	■	■
3		■	■	■
4		■	■	■

Chapter 12
Lesson 3

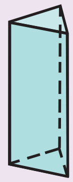
REVIEW MODEL
Classifying Three-Dimensional Figures

A three-dimensional figure with flat faces that are polygons is a *polyhedron.* Polyhedra are named by the polygons that form their bases.

A *prism* is a polyhedron that has two congruent polygons as bases. All other faces of a prism are parallelograms.

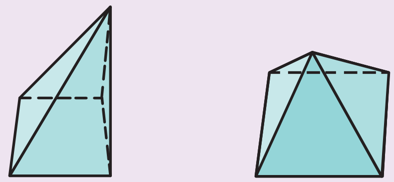


rectangular prism



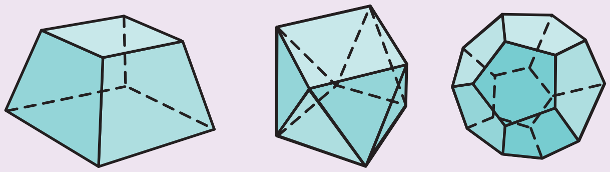
triangular prism

A *pyramid* is a polyhedron with one face that can be any polygon. All other faces are triangles that meet at the same vertex.



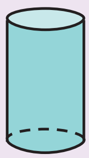
square pyramids

There are *other polyhedra* that are not prisms and not pyramids. Some have parallel bases that are not congruent, some look twisted, and others have flat faces that are not parallelograms or triangles.

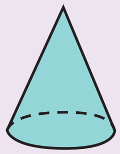


other polyhedra

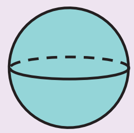
A three-dimensional figure with a curved face is *not* a polyhedron.



cylinder



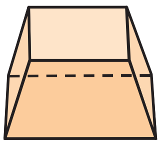
cone

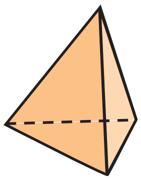


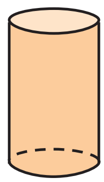
sphere

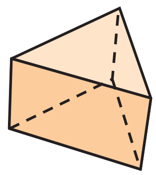
Check for Understanding

Classify each three-dimensional figure. Write *prism, pyramid, other polyhedron,* or *not a polyhedron.*

1 

2 

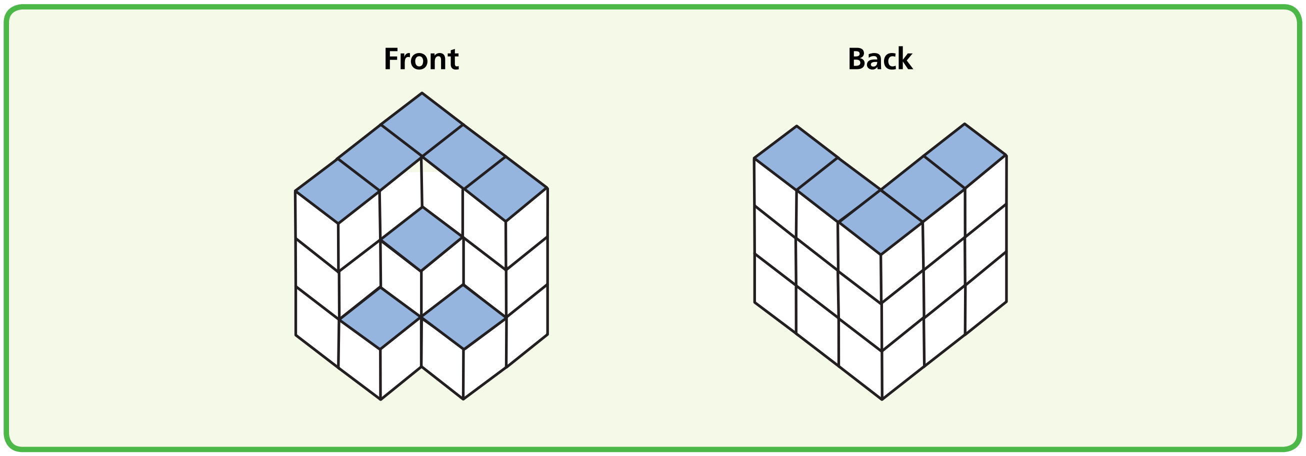
3 

4 

Chapter 12
Lesson 4

EXPLORE
Measuring Three-Dimensional Structures

Build this structure with inch cubes.



1 How many inch cubes does it take to build the structure?

2 How many layers, of floors, does the structure have?

3 How many cubes are in the bottom layer?
Sketch the shape of this floor.

4 How many cubes are in the second layer?
Sketch the shape of this floor.

5 How many cubes are in the top layer?
Sketch the shape of this floor.

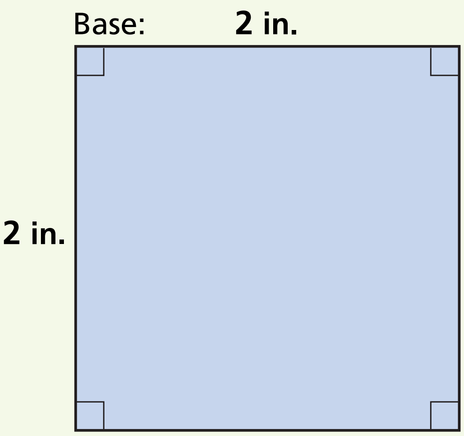
Chapter 12

Lesson 5

EXPLORE
Volume of Prisms

Use the information in each diagram to predict how many inch cubes you will need to build the prism. Build each with cubes and record the volume.

Prism B

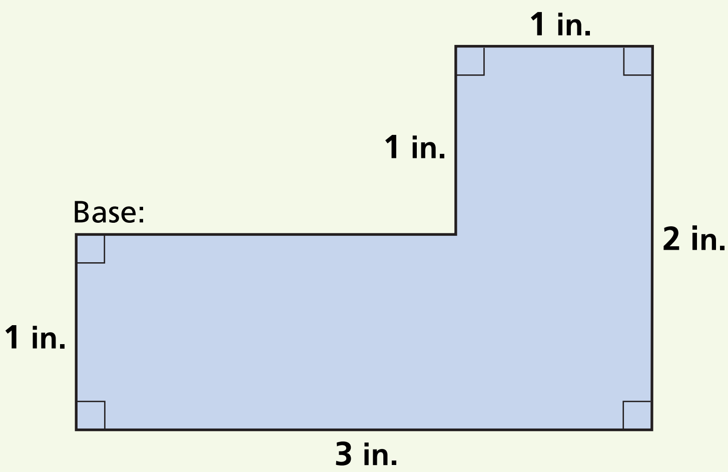


Height of Prism: 3 in.

How many cubes?

What is the volume?

Prism L



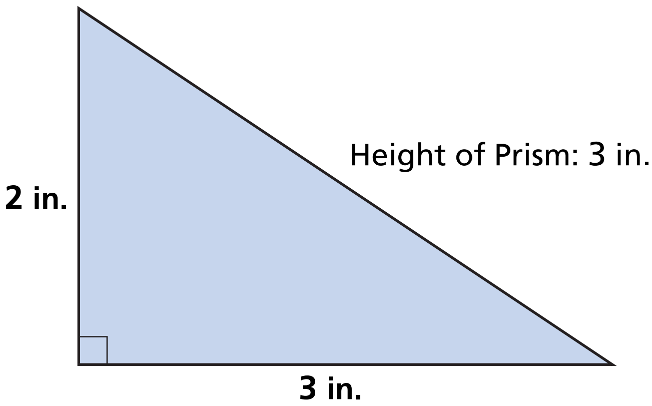
Height of Prism: 1 in.

How many cubes?

What is the volume?

Use the information in the diagram or Figures O and S from the class collection.

How might you find the volume of a triangular prism?



Chapter 12
Lesson 5

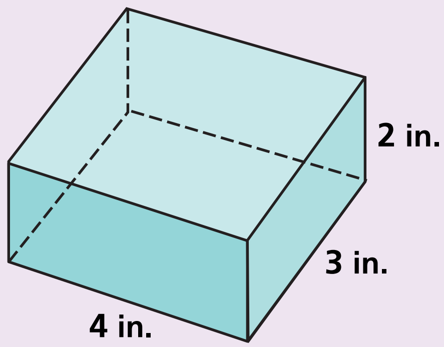
REVIEW MODEL
Volume of a Prism

Volume **is the measure of space a three-dimensional figure occupies.**

Volume is measured in cubic units, such as cubic inches (cu in.), cubic centimeters (cu cm), cubic feet (cu ft), or cubic kilometers (cu km).

You can find the volume of a prism by multiplying its base area by its height.

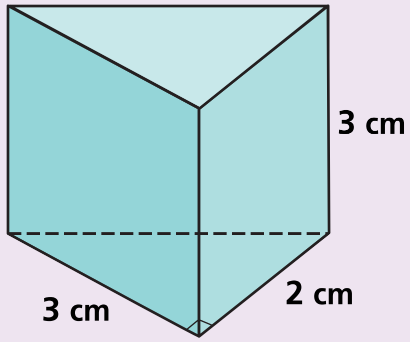
Rectangular Prism



The area of the rectangular base is *base length* × *width*, or $4 \times 3 = 12$; 12 sq in.

The volume of the prism is *base area* × *height*, or $12 \times 2 = 24$; 24 cu in.

Triangular Prism



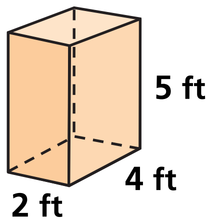
The area of the base of the prism is $\frac{1}{2}$ × *base length* × *height*, or $\frac{1}{2} \times 3 \times 2 = 3$; 3 sq cm.

The volume of the prism is *base area* × *height*, or $3 \times 3 = 9$; 9 cu cm.

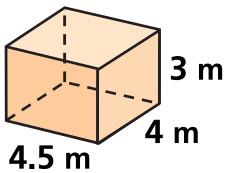
Check for Understanding

Find the volume of each rectangular prism.

1.

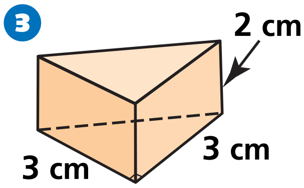


2.

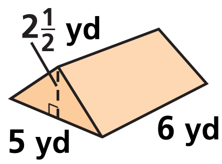


Three of the faces of these triangular prisms are rectangles. Find the volume of each prism.

3.



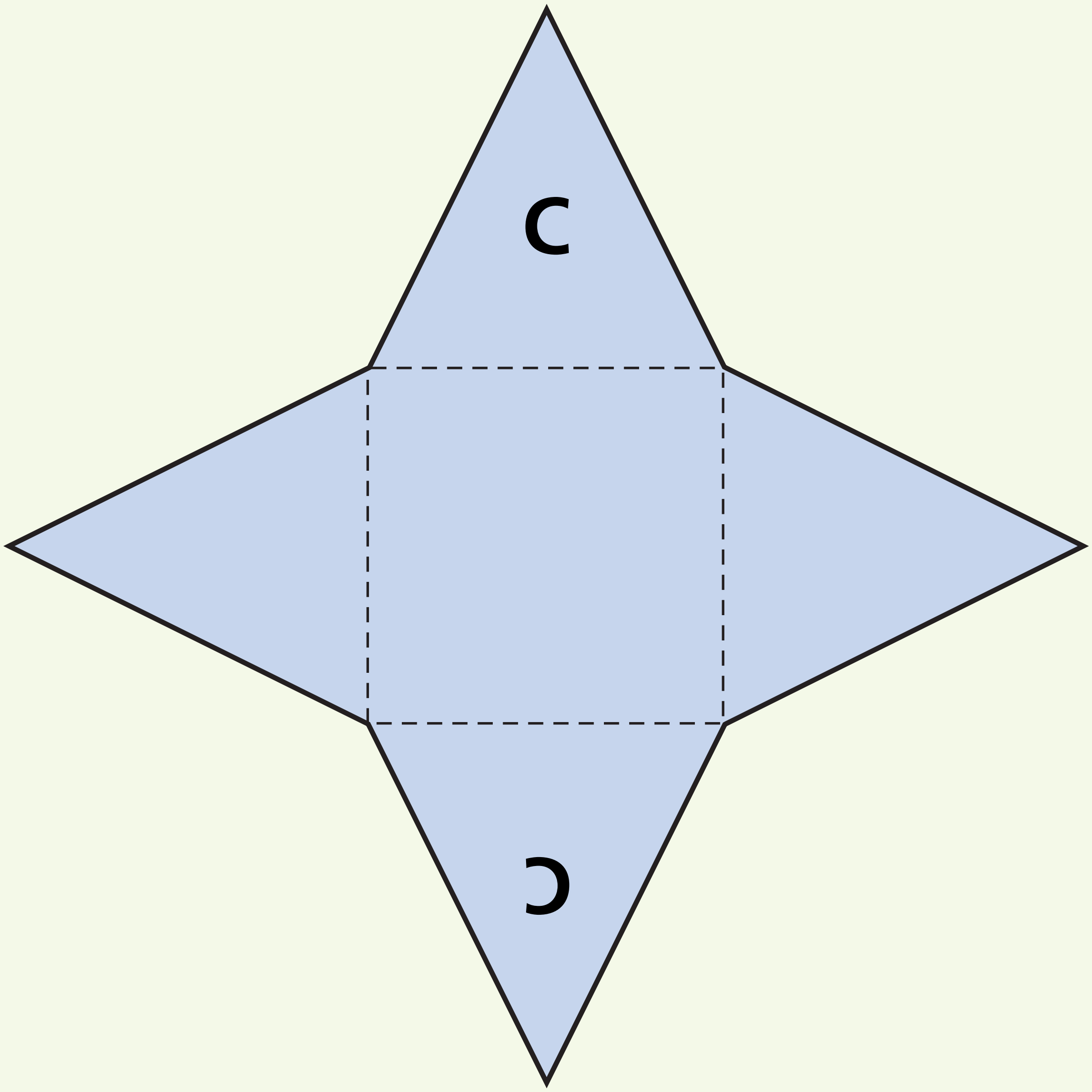
4.



Chapter 12
Lesson 6

EXPLORE
How Much Paper Was Used?

Look at Figure C in your class collection. Use the net to help you figure out how much paper was used to make that polyhedron.



Chapter 12

Lesson 7

REVIEW MODEL
Surface Area

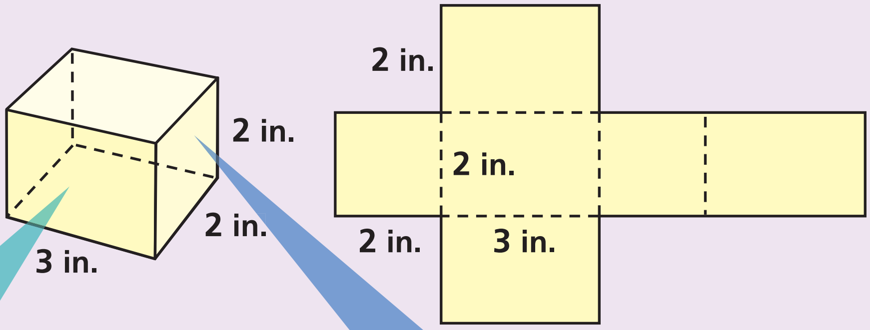
Surface area **is the amount of area on the surface of a three-dimensional figure.**

Surface Area, like all area, is measured in square units, such as square inches (sq in.), square centimeters (sq cm), square feet (sq ft), or square kilometers (sq km).

You can find the surface area of a polyhedron by finding the area of each face of the polyhedron and then finding the sum of the areas. You can use a net of the three-dimensional figure to help you find the area of each face.

Rectangular Prism

There are 4 large congruent rectangles and 2 congruent squares, so the surface area of the prism is $(4 \times 6) + (2 \times 4)$; 32 sq in.



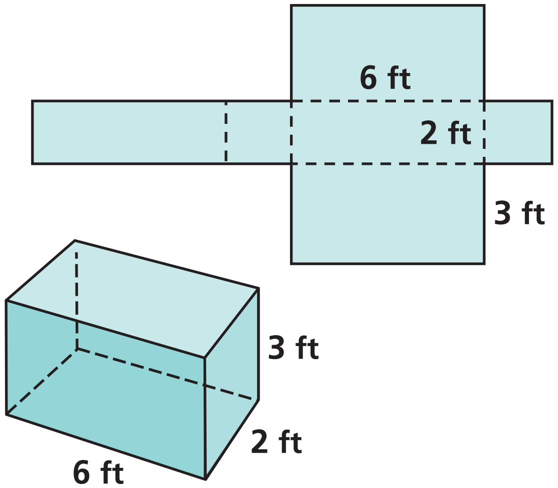
The area of each of the larger rectangular faces is *length* × *width*, or $2 \times 3 = 6$; 6 sq in.

The area of each square face is *length* × *width*, or $2 \times 2 = 4$; 4 sq in.

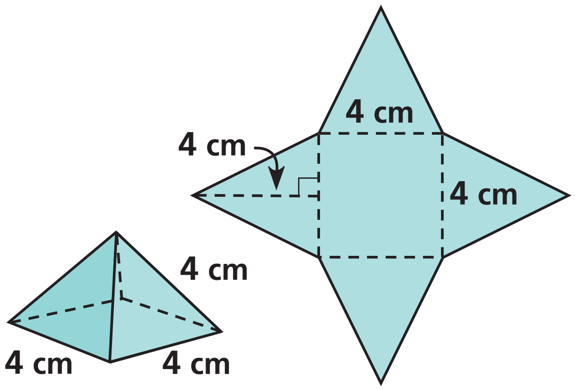
Check for Understanding

Find the surface area of each three-dimensional figure.

1 rectangular prism



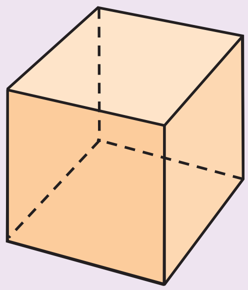
2 square pyramid



Chapter 5
Lesson 12

REVIEW MODEL
Problem Solving Strategy
Guess and Check

William wants a cube-shaped box that has a volume of about 150 cubic inches to hold his marble collection. About how many inches long should each edge of the box be?



Strategy: Guess and Check

Read to Understand

What do you know from reading the problem?

William wants a cube-shaped box that has a volume of about 150 cubic inches.

What do you need to find out?

the edge length of a cube-shaped box that has a volume of 150 cubic inches

Plan

How can you solve this problem?

I can *guess and check* by trying different numbers for the length.

Solve

How might you *guess and check* to solve the problem?

Try an edge length and multiply it by itself three times. If the volume is less than the target volume, increase the guess. If the volume is greater than the target volume, decrease the guess.
So, the edge is about 5.3 inches.

Edge n	Volume n^3	$<$ or $>$ target
3	27	< 150
5	125	< 150
6	216	> 150
5.5	166.375	> 150
5.4	157.464	> 150
5.3	148.877	< 150

Check

Look back at the problem. Did you answer the questions that were asked? Does the answer make sense?

Problem Solving Practice

Use the strategy *guess and check* to solve.

1. Jeanne wants to build a square patio with an area of about 30 square meters. To the nearest tenth of a meter, how long should she make each side of the patio?

2. Tickets for the concert cost $14.50 for one child and one adult. The adult's ticket costs $3.00 more than the child's ticket. What is the cost of each ticket?

Problem Solving Strategies

- ✔ Act It Out
- ✔ Draw a Picture
- ✔ **Guess and Check**
- ✔ Look for a Pattern
- ✔ Make a Graph
- ✔ Make a Model
- ✔ Make an Organized List
- ✔ Make a Table
- ✔ Solve a Simpler Problem
- ✔ Use Logical Reasoning
- ✔ Work Backward
- ✔ Write an Equation

Mixed Strategy Practice

Use any strategy to solve. Explain.

3. Emily bought three items at the grocery store. After paying $4.20, $6.75, and $9.40 for the items, she had $2.50 left. How much money did she start with?

4. Marti joined an exercise club. On the first day she exercised for 5 minutes, on the second day for 10 minutes, and on the third day for 15 minutes. If this pattern continued, for how many minutes did she exercise on the tenth day?

5. A train has 9 cars, each seating 56 people. What is the total number of people who can be seated on the train?

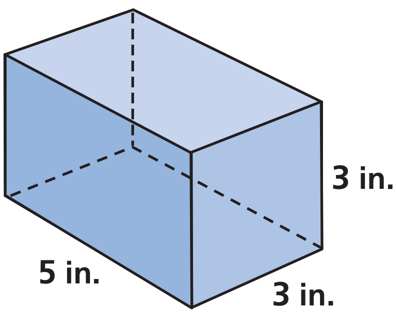
6. Hal has 3 more baseball cards than Abby. Kristen has 5 more cards than Abby. If Hal has 12 cards, how many cards does Kristen have?

For 7–8, use the diagram.

Ms. Blackstone has a collection of number cubes in her classroom. She found a box and measured its dimensions. For 7 and 8, use the diagram of her box.

7. If she has 50 number cubes that are each 1 cubic inch, will they all fit in the box? How do you know?

8. If she decides to put wrapping paper on all the faces of the box, how much paper will she need?



Chapter 12 Vocabulary

Choose the best vocabulary term from Word List A for each definition.

1. A(n) __?__ is a three-dimensional figure that has two congruent polygon-shaped bases and other faces that are all rectangles.
2. The __?__ is the sum of the areas of all the surfaces of a three-dimensional figure.
3. Area is measured using __?__.
4. The __?__ of a three-dimensional figure are line segments formed when two faces meet.
5. The polygons that are flat surfaces of a three-dimensional figure are its __?__.
6. The __?__ of a three-dimensional figure are the points where three or more of its edges intersect.
7. A three-dimensional figure with flat faces that are polygons is called a(n) __?__.
8. A(n) __?__ is a three-dimensional figure with a polygon base and faces that are triangles that meet at a common vertex.

Word List A

base
congruent
cubic units
edges
faces
height
length
parallel faces
perpendicular
polyhedron
prism
pyramid
square units
surface area
vertices
volume
width

Complete each analogy using the best term from Word List B.

9. Cylinder is to __?__ as prism is to pyramid.
10. Banana is to banana peel as __?__ is to surface area.

Word List B

base
cone
sphere
volume

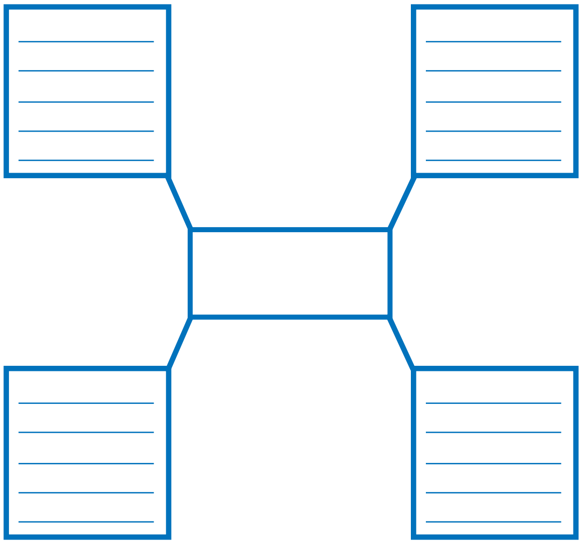
Talk Math

Discuss with a partner what you have learned about three-dimensional figures. Use the vocabulary terms *base, faces,* and *height.*

11. How can you tell the difference between a cone and a cylinder?
12. How can you find the volume of a triangular prism?
13. How can you find the surface area of a prism?

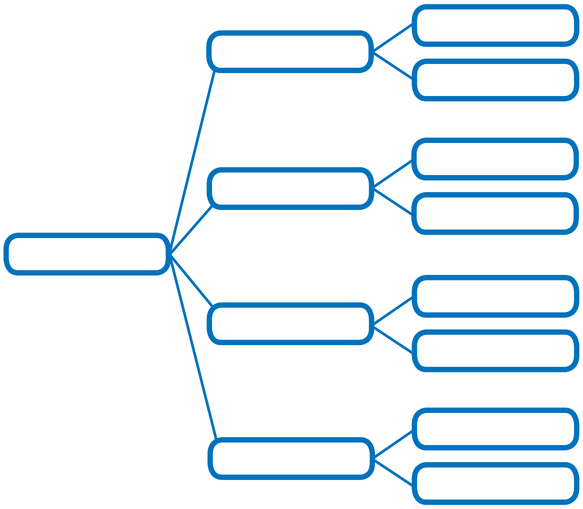
Word Web

14 **Create a word web for the term *base.***



Tree Diagram

15 **Create a tree diagram using the terms *cone, cylinder, prism,* and *pyramid.* Use what you know and what you have learned about solid figures.**



What's in a Word?

VOLUME In daily life, *volume* has many meanings. It is the level of loudness of music or other sounds. An encyclopedia *volume* is one book from a set. The sales *volume* of a store is how many items were sold. Some stores offer *volume* discounts—which means you pay less if you buy a lot of something. But in math, *volume* is a measure. It is the amount of space taken up by a three-dimensional figure. It is determined by the number of unit cubes—whole or part—that fit inside a space.

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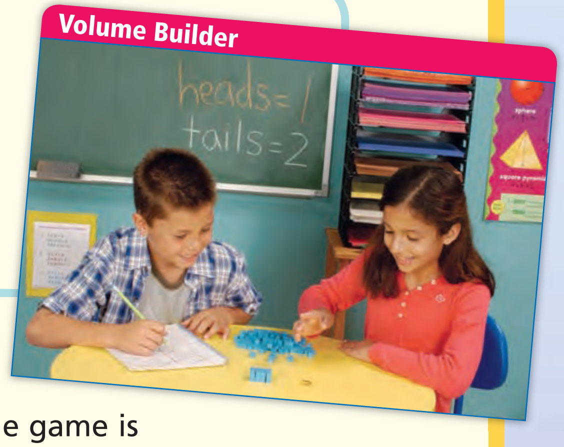
GAME

Volume Builder

Game Purpose
To practice estimating and finding volume of a rectangular prism

Materials
- Cubes
- Coin



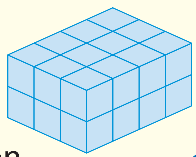
How To Play The Game

1. This is a game for 2 players. The object of the game is to extend the dimension of a rectangular prism in a way that will give the greatest volume. The greater the volume, the more points you score.

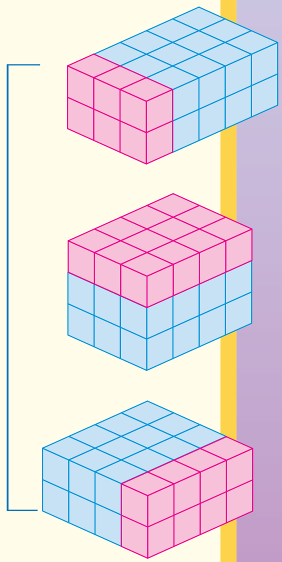
2. Start with a cube placed between you and your partner. The volume of this prism is 1 cubic unit. Decide who will go first. Then take turns.

3. Toss the coin. Extend one of the three dimensions depending on the result of your coin toss. Heads means extend by 1 unit. Tails means extend by 2 units.
   - The figure must remain a rectangular prism after the extra cubes have been added.
   - Decide which dimension to increase. Try to picture the result and estimate the new volumes. Decide which choice would result in a prism of the greatest volume.

   Example: You get heads, and this is the prism so far.

   Heads means you add 1 to any dimension. You could extend the prism in one of three ways:

   - Calculate the new volume. Record that number as your point score for that round.

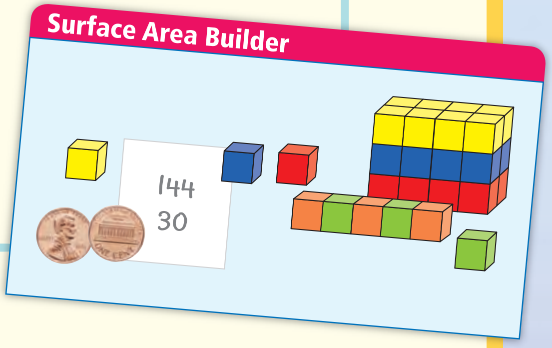
4. Play until one player reaches or goes past 200 points.

GAME

Surface Area Builder

Game Purpose
To practice estimating and finding surface area of a rectangular prism

Materials
- Cubes
- Coin



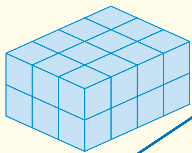
How To Play The Game

1. This is a game for 2 players. The object of the game is to extend the dimension of a rectangular prism in a way that will give the greatest surface area. The greater the surface area, the more points you score.

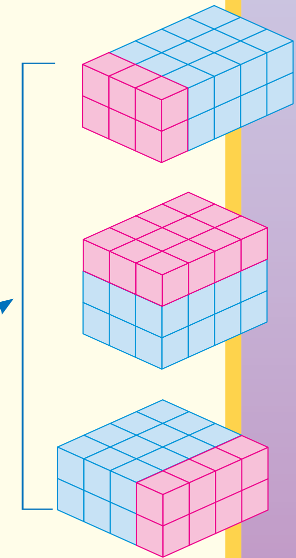
2. Start with a cube placed between you and your partner. The surface area of this prism is 6 square units. Decide who will go first. Then take turns.

3. Toss the coin. Extend one of the three dimensions depending on the result of your coin toss. Heads means extend 1 unit. Tails means extend 2 units.
   - The figure must remain a rectangular prism after the extra cubes have been added.
   - Decide which dimension to increase. Try to picture the result and estimate the new surface areas. Decide which choice would result in a prism with the greatest surface area.

   Example: You get heads, and this is the prism so far.

   Heads means you add 1 to any dimension. You could extend the prism in one of three ways:

   - Calculate the new surface area. Record that number as your point score for that round.

4. Play until one player reaches or goes past 200 points. That player wins.

CHALLENGE

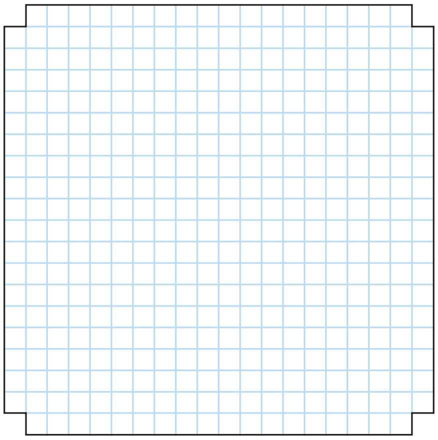
The Ever Changing Volume

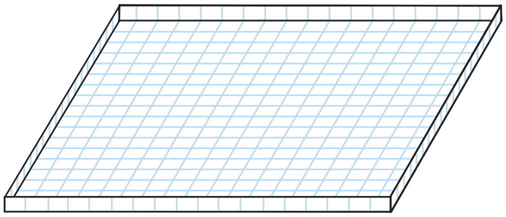
You can experiment to see how volume changes as dimensions change.

You will need: 6 sheets of centimeter grid paper cut into squares that are 20 squares by 20 squares, scissors, tape

Directions

1. Cut out a 1×1 square from each corner of one sheet of the centimeter grid paper. Fold the edges and tape the corners to make an open box. What is the volume of the box in cubic units?





2. Use a new sheet of grid paper. Cut out a 2×2 square from each corner, and make a box. What is the volume?
3. Use a new sheet of grid paper. Cut out a 3×3 square from each corner, and make a box. What is the volume?
4. Use a new sheet of grid paper. Cut out a 4×4 square from each corner, and make a box. What is the volume?
5. Use a new sheet of grid paper. Cut out a 5×5 square from each corner, and make a box. What is the volume?
6. Use a new sheet of grid paper. Cut out a 6×6 square from each corner, and make a box. What is the volume?
7. For which size cut-out corner is the volume of the box the greatest? How do you know?