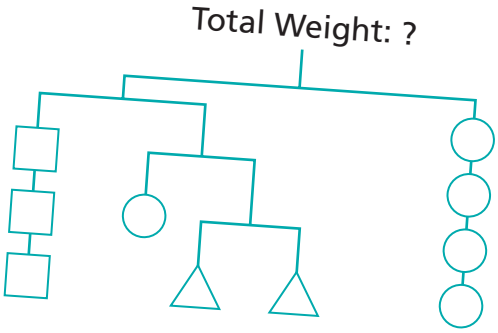
Chapter

13 Fun with Algebra

Dear Student,

In this chapter, you'll start by exploring mobiles. Look at the mobile below. Can you use the key of shape weights at the right to figure out the mobile's total weight?

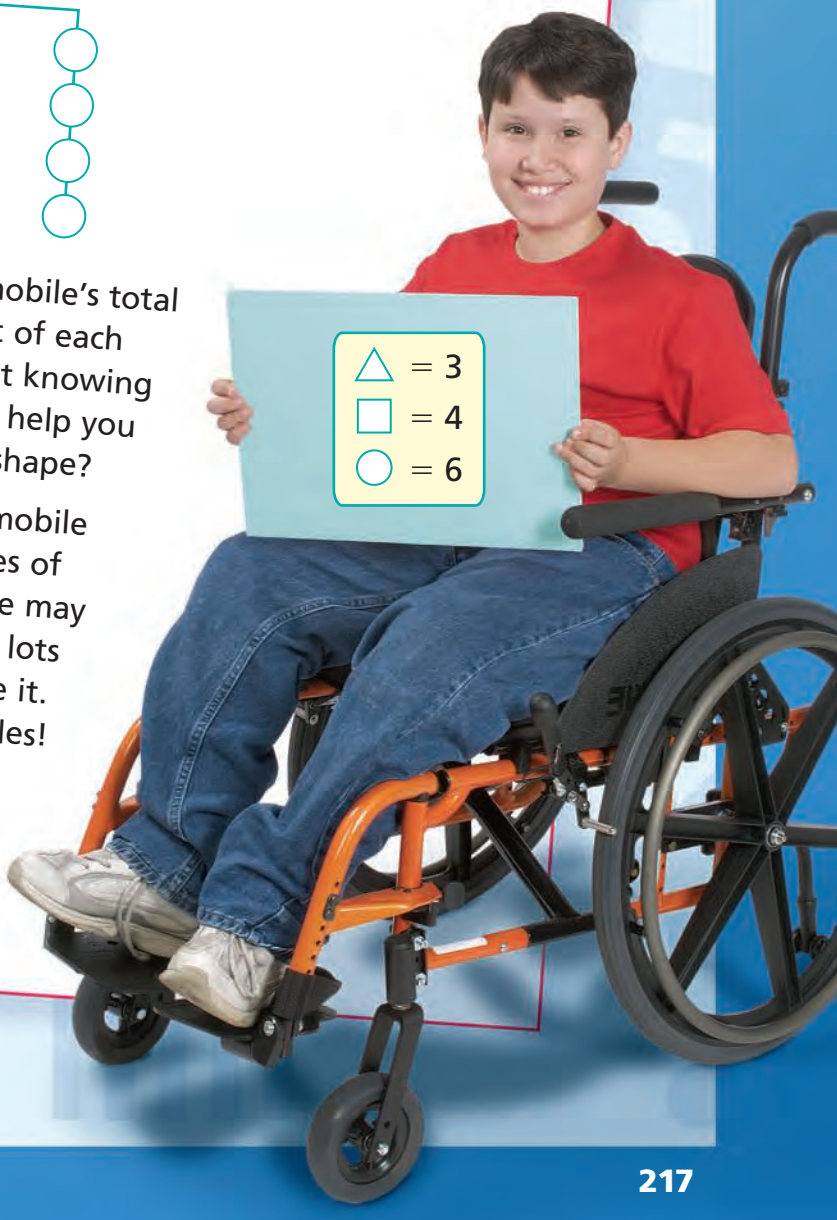


What if you only knew a mobile's total weight, but not the weight of each different shape? How might knowing that the mobile is balanced help you to find the weight of each shape?

In this chapter, you'll solve mobile puzzles as well as other types of puzzles. Whatever the puzzle may be, you'll find that there are lots of ways to describe and solve it. We hope you enjoy the puzzles!

Mathematically yours,

The authors of ***Think Math!***



Balancing Act

Tightrope walking acts have been performed in front of crowds for centuries. Some of the first tightrope walkers were in ancient Egypt and China. The act was called "rope dancing" and the performers walked over knives. In modern circuses, tightrope walkers have a net under them for safety and use a balancing pole to help them perform fantastic tricks. The longer and heavier the pole, the steadier the tightrope walker will be.

FACT·ACTIVITY 1

A balancing pole can weigh up to 31 pounds and be as long as 39 feet. A clown is putting items in the baskets on his pole to help him balance.

Use the items to the right to answer the questions.

3 ounces

7 ounces

6 ounces

1. In his first act, the clown walks across the tightrope with 2 juggling balls in each basket. What is the total weight of the balls in both baskets?
2. During the second act, the clown replaces the left basket with 2 top hats and 1 juggling ball. How many more juggling balls are needed in the right basket to make his shoulder pole balanced?
3. What other combination of objects can the clown put in the left and right baskets so that his shoulder pole is balanced?
4. An equation which describes the items in the clown's shoulder pole in Problem 3 is $2b + c = c + h$. What does the b represent?

FACT·ACTIVITY 2

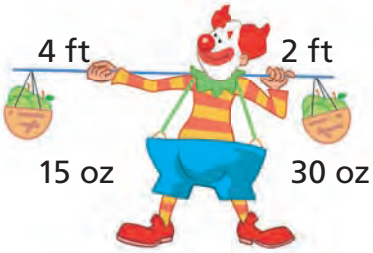
What happens if the weights of the baskets are not the same? This clown knows physics too! To stay balanced on the rope, the clown moves the pole so that one basket is closer to his body than the other. In other words, the body acts as a balance point.

The pictures show 3 ways the clown balances the pole on the tightrope. Use these pictures to answer the questions.

1. Write an equation using only variables to represent the objects and distances shown. Explain what each variable represents.
2. Suppose the clown has 3 hats in the left basket and 3 juggling balls in the right basket. Where should the clown balance the 6-ft pole? Explain.
3. Suppose the clown has 2 juggling balls in the left basket and 2 hats in the right basket. Where should the clown balance the 6-ft pole? Explain.

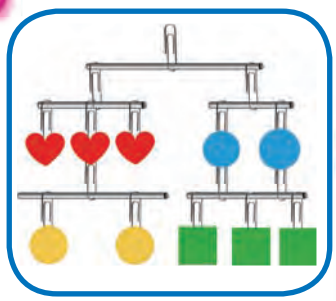
$15 \times 1 = 3 \times 5$

$15 \times 3 = 15 \times 3$



$15 \times 4 = 30 \times 2$

CHAPTER PROJECT



Materials: straws, paper clips, construction paper

Make your own mobile and experiment with balancing its different layers.

Slide 3 paper clips over a horizontal straw as shown in the model, one on each end and one in the middle. Repeat the first step to extend your mobile to 2 or more layers. Cut out 3 or 4 shapes from construction paper. Then trace and cut out 15 copies of each shape. Each layer of your mobile will only use 1 type of paper shape. Clip the shapes to the paper clips. You will need to add or subtract paper clips to balance your mobile.

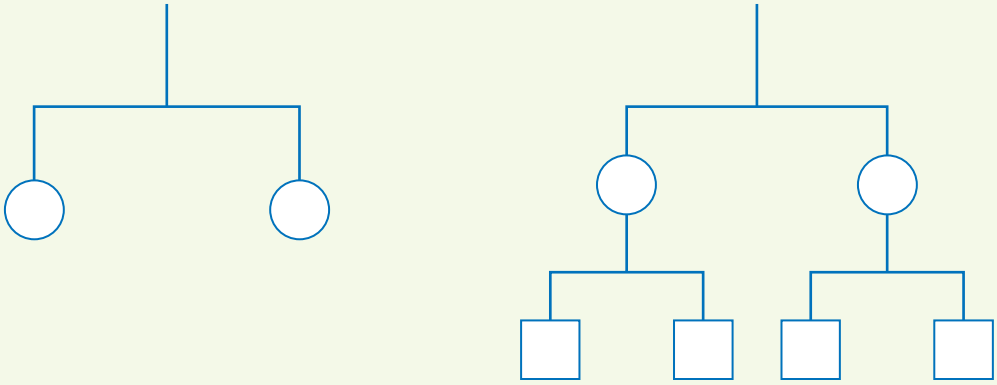
- Describe how you balanced your mobile.

ALMANAC Fact

Jean François Gravelet walked across a tightrope over Niagara Falls between Canada and the United States in 1895. Later, he cooked and ate an omelette while crossing.

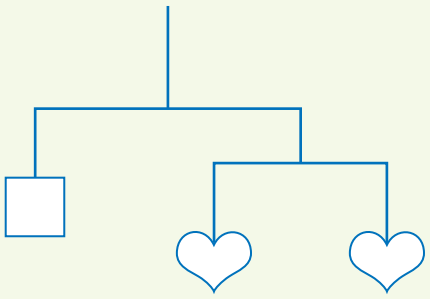
EXPLORE
Balancing Mobiles

Martina makes mobiles that balance perfectly! Her secret is to make sure that every arm has the same total weight on each end.

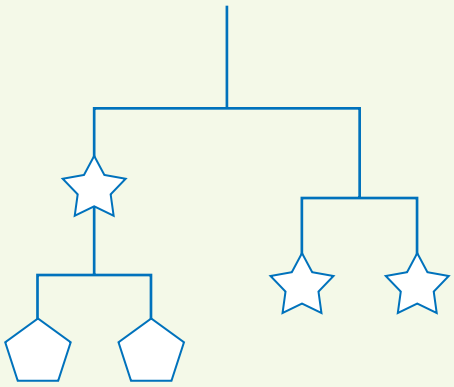


The different shapes she uses each weigh different amounts. That makes it trickier to balance mobiles that combine different shapes, but Martina is an expert. She made these balanced mobiles.

A Total Weight: 20



B Total Weight: 24

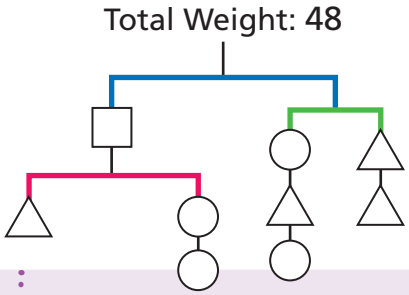


Find the weight of each shape in the two mobiles above.

Chapter 13

Lesson 3

REVIEW MODEL
Writing an Equation for a Mobile



The shapes on the two sides of an arm of a mobile must have the same weight to balance.

You can use words and you can use equations to describe relationships shown on the mobile.

On the red arm—
- 1 triangle weighs the same as 2 circles.

$1t = 2c$

On the green arm—
- 2 circles and 1 triangle weigh the same as 2 triangles.

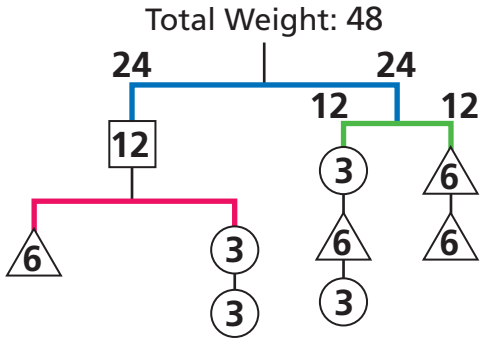
$2c + 1t = 2t$

On the blue arm—
- 1 square, 1 triangle, and 2 circles weigh the same as 2 circles and 3 triangles.

$1s + 1t + 2c = 2c + 3t$

To find the weight of each shape, you can write the halved numbers at each side of an arm and the weights inside or near each shape.

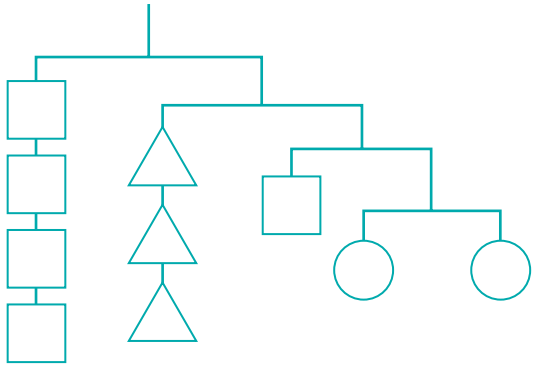
△ = 6
○ = 3
□ = 12



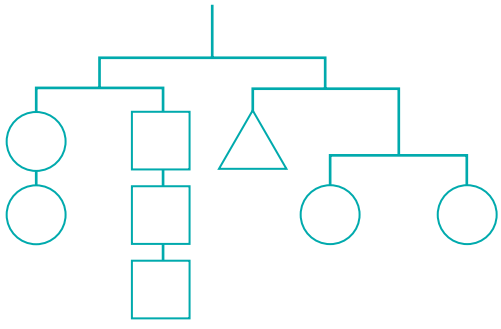
Check for Understanding

Write two equations to agree with each mobile. Find the weight of each shape.

1. Total Weight: 48



2. Total Weight: 24

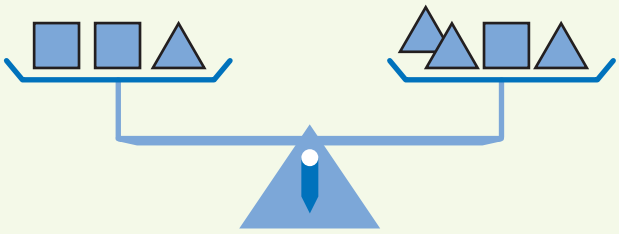


Chapter 13

Lesson 4

EXPLORE
Exploring Balance Puzzles

Brenda had two kinds of blocks. She put some on a balance scale and the two sides weighed exactly the same.



Brenda realized that if she knew the weight of one block, she could figure out the weight of the other!

1 If a ▲ weighs 3 ounces, how much does a ■ weigh?

Explain your answer.

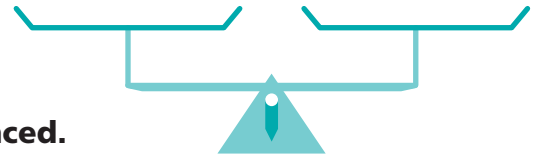
2 If a ■ weighs 15 ounces, how much does a ▲ weigh?

Chapter 13

Lesson 4

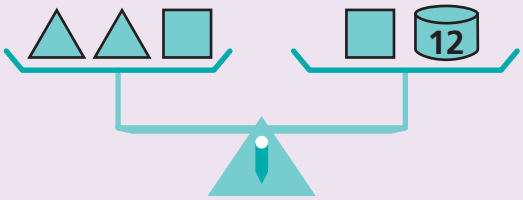
REVIEW MODEL
Balancing Weights

For a balance scale to balance, the two pans must each have the same total weight. If you add the same weights or take away the same weights from both pans, the pans remain balanced.



You can use equations to describe relationships shown on balance scales.

Write an equation for the balance scale.



Step 1 Record what you see on the pans.

△ + △ + □ = □ + 12

or

$t + t + s = s + 12$

or

$2t + s = s + 12$

2 △ and 1 □ on the left weigh the same as 1 □ and 12 on the right.

Step 2 Simplify.

Remove 1 □ from each pan.

△ + △ = 12

or

$t + t = 12$

or

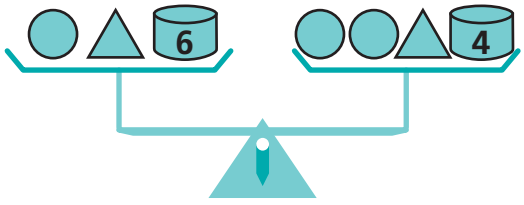
$2t = 12$

2 △ on the left weigh the same as 12 on the right.

So, 1 △ = 6, or $t = 6$

Example

Write an equation for the balance scale.



$c + t + 6 = 2c + t + 4$, or

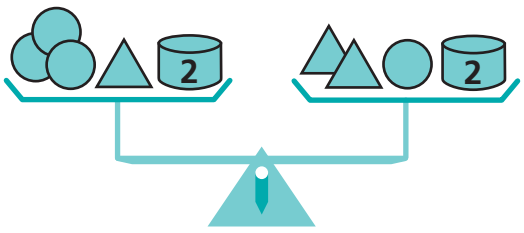
$6 = c + 4$, or

$2 = c$

Check for Understanding

Write an equation for each balance scale.

1



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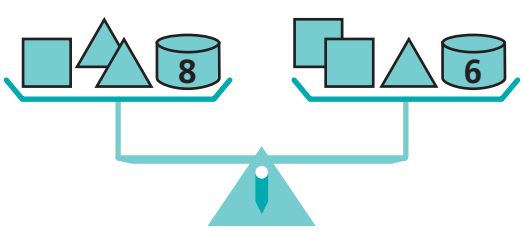


Chapter 13

Lesson 6

REVIEW MODEL
Drawing a Diagram for a Situation

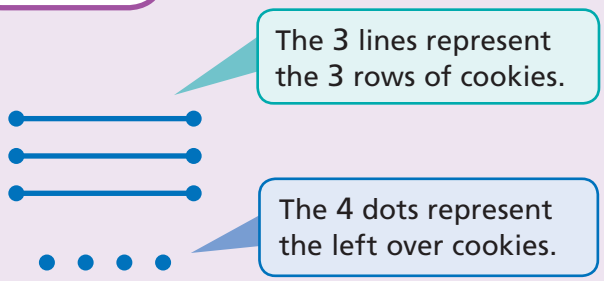
Sometimes you can use diagrams to describe situations and help you find unknown numbers. You have to be flexible with these diagrams. You cannot rely on sizes and lengths to help you figure out the unknowns.

Situation:

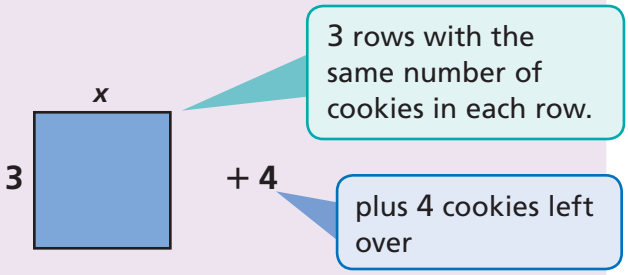
Sophia baked some cookies. She put them on a platter in 3 rows with the same number of cookies in each row. She had 4 cookies left over.

Draw a diagram to show the number of cookies she had.

One Way



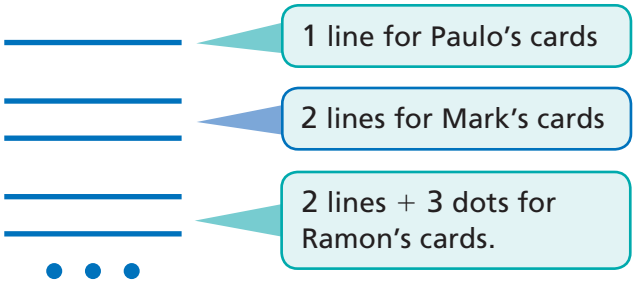
Another Way



Example Ramon has 3 more baseball cards than Mark. Mark has twice as many cards as Paulo.

Draw a diagram to show all the baseball cards.

This is a possible diagram.



Check for Understanding

Draw a diagram to describe each situation.

1. Alex lined up his 8 toy cars end-to-end. Then Troy added his cars to the line.

   Draw a diagram to show all the cars.

2. Julia has 4 more books than Lisa. Lisa has 3 times as many books as Rachel.

   Draw a diagram to show how many books Julia has.

Chapter 13

Lesson 7

REVIEW MODEL
Writing an Equation for a Situation

You can write an equation to describe a situation.

Jason earns the same amount of money each day for walking his neighbor's dog. He earns an extra $10 once each week for giving the dog a bath. Write an equation to describe the total amount Jason earns in one week.

Step 1 Choose a letter to stand for the amount Jason earns each time he walks the dog.

Use *w* to stand for "walks".

Choose a different letter for the total amount he earns each week.

Use *t* to stand for "total amount".

Step 2 Use words to describe a rule for the situation. The total amount Jason earns in one week **is** 7 times the amount he earns each day walking the dog **plus** $10 for giving the dog a bath.

Step 3 Use an equation to describe a rule for the situation. $t = 7w + 10$

Example Natalie bought some salads and some small pizzas at the snack bar. The salads cost $5 each and the pizzas cost $3 each. Describe the total amount Natalie spent.

Use words to describe the situation.
The total amount is $5 times the number of salads she bought plus $3 times the number of pizzas she bought.

Use an equation to describe the situation.
t = 5s + 3p

Use *t* to stand for the total cost, *s* to stand for the number of salads, and *p* to stand for the number of pizzas.

Check for Understanding

Use words to describe a rule for each situation. Then write an equation for the situation.

1. Marcus bought some markers for $2 each and some notebooks for $6 each. Describe the total amount he spent. Use *m* for the number of markers, *n* for the number of notebooks, and *t* for the total amount he spent.

2. Tami has 3 pockets with the same number of dimes in each pocket. She also has 8 dimes in her hand. Describe the total number of dimes she has. Use *p* for the number of dimes in each pocket and *t* for the total number of dimes.

Chapter 13

Lesson 8

REVIEW MODEL

Problem Solving Strategy

Work Backward

Drew played a number trick with his friend, Amy. He told Amy to choose a number, add 4, and multiply the result by 6. Then he told her to subtract 8 and divide that number by 2. Amy said the result was 20. Drew told Amy her starting number. What was Amy's starting number?

Strategy: Work Backward

Read to Understand

What do you know from reading the problem?

Amy chose a mystery number, performed operations on the number, and had a result of 20.

What do you need to find out?

the starting number

Plan

How can you solve this problem?

You can work backward by starting with the resulting number, 20.

Solve

How can you work backward to solve the problem?

You can work backward through the list of steps and undo the operations. First, multiply 20 by 2, add 8, divide the result by 6, and subtract 4.

So, Amy's starting number was 4.

Check

Look back at the problem. Did you answer the question that was asked? Does the answer make sense?

Problem Solving Practice

Use the strategy *work backward* to solve.

1. Ryan has three younger sisters. His sister, Jen, is half the age of their sister, Ashley. Ashley is 10 years older than their sister, Amanda. Amanda is half Ryan's age. If Ryan is 24 years old, how old are his sisters?

2. Mrs. McCarthy bought tickets for her family to go to the art museum. She bought two adult tickets and two student tickets for a total of $42.50. If the adult tickets were $12.50 each, how much was each student ticket?

Problem Solving Strategies

- ✔ Act It Out
- ✔ Draw a Picture
- ✔ Guess and Check
- ✔ Look for a Pattern
- ✔ Make a Graph
- ✔ Make a Model
- ✔ Make an Organized List
- ✔ Make a Table
- ✔ Solve a Simpler Problem
- ✔ Use Logical Reasoning
- ✔ **Work Backward**
- ✔ Write an Equation

Mixed Strategy Practice

Use any strategy to solve. Explain.

3. 104 students showed up to participate in an after-school sports program. There can be up to 10 teams and there can be no more than 15 students on a team. If all teams have the same number of students and no students are left over after the teams are made, how many teams will there be? How many students will be on each team?

4. Mr. Dawson bought deli meat for the class picnic. One package of turkey weighed 2.75 pounds. A package of ham weighed 1.85 pounds, and a package of roast beef weighed 0.94 pound. Did he buy more turkey than the ham and roast beef together? Explain.

5. Kayla has 14 pages of stamps in her collection. There are 24 stamps on each page. How many stamps does Kayla have in her collection?

6. Samantha counted 46 tiles around the walls in her kitchen. The tiles go in a pattern of blue, yellow, green, and white. If the first tile is blue, what color is the 46th tile?

For 7–8, use the table.

7. How much time did Cody spend in the computer lab on Monday and Tuesday?

8. How much longer did Cody spend in the computer lab on Tuesday than on Friday?

Time Cody Spent in Computer Lab

Day	Time
Monday	$\frac{1}{2}$ hr
Tuesday	$\frac{3}{4}$ hr
Wednesday	$\frac{1}{3}$ hr
Thursday	$\frac{5}{12}$ hr
Friday	$\frac{1}{4}$ hr

Chapter 13 Vocabulary

Choose the best vocabulary term from Word List A for each definition.

1. Art with hanging parts that move easily is called a(n) ___?___.
2. To ___?___ a mobile, both sides of an arm must have the same weight.
3. An algebraic or numerical sentence that shows two quantities are equal is a(n) ___?___.
4. A(n) ___?___ has two pans and pivots freely on a fulcrum to compare weights.
5. A letter or symbol that stands for one or more values is called a(n) ___?___.
6. When you use algebra to represent word problems, you are ___?___.
7. When you use pictures to solve word problems, you are ___?___.

Word List A

- balance
- balance scale
- describing situations with diagrams
- describing situations with equations
- equation
- mobile
- variable

Complete each analogy using the best term from Word List B.

8. Seat is to seesaw as pan is to ___?___.
9. Word is to sentence as ___?___ is to equation.

Word List B

- balance scale
- equation
- mobile
- variable

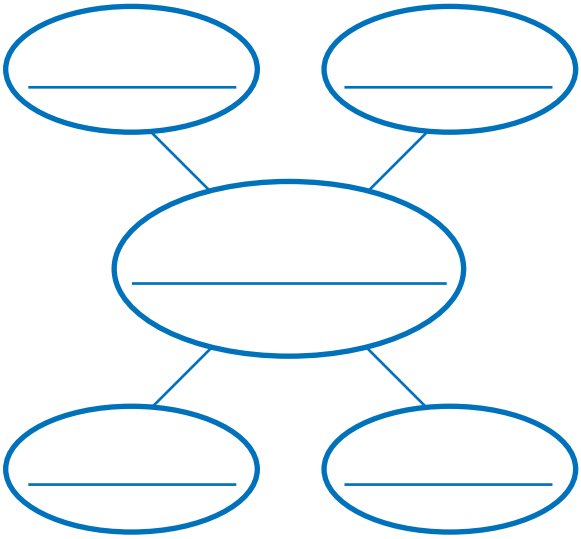
Talk Math

Use the vocabulary terms *equation*, *balance*, and *variable* to discuss with a partner what you have just learned about algebra.

10. How are a mobile and an equation alike?
11. How are a balance scale and an equation alike?
12. How can you write an equation from a diagram?

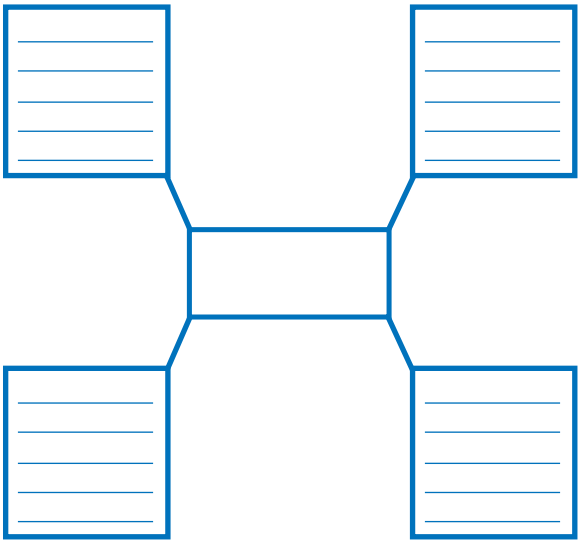
Concept Map

13 **Create a concept map for the term *balance.* Include the words *mobile, balance scale,* and *equation.***



Word Web

14 **Create a word web using the word *variable.***



BALANCE *Balance* comes from a Latin word that means "a scale having two pans." An object was placed in one pan. Weights of known measure were placed in the other pan until the pans *balanced.* The weight of the object was the sum of the weights. Today, we use *balance* in many other ways as well. You *balance* your checkbook to know how much money you have in the bank. You *balance* yourself by standing without wobbling. You might use a *balance* beam in gym class. A *balanced* meal has just the right number of elements.

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GAME

The Balance Puzzle

Game Purpose

To practice relating balance puzzles to equations and solving the puzzles

Materials

- Activity Master 137: *The Balance Puzzle* Scale and Spinner
- Activity Master 138: *The Balance Puzzle* Game Pieces
- Paper clip

How To Play The Game

1. This is a game for 2 players. Cut out all the game pieces. To use the spinner, put a pencil through the paper clip. Put the pencil point on the center of the spinner. Then spin the paper clip around the pencil. Decide who will be the Puzzler and who will be the Solver.

2. The Puzzler secretly picks two numbers from 4 to 9 to be the weights of the blocks. Record the weights. Show which is for the square and which is for the triangle.

3. The Solver spins and puts that many game pieces in the left pan of the scale.

4. The Puzzler fills the right pan with game pieces—using the secret weights—until the pans balance. Use as few pieces as possible.

5. The Solver guesses the weight of the square or the triangle.
   - A correct guess earns as many points as the number of pieces in the right pan.
   - An incorrect guess means use the same weights on your next turn. Record this puzzle so you can use it again. If your next spin gives the same pieces for the left pan, spin again.

6. Trade roles. Play until someone gets 6 points and wins the game!

GAME

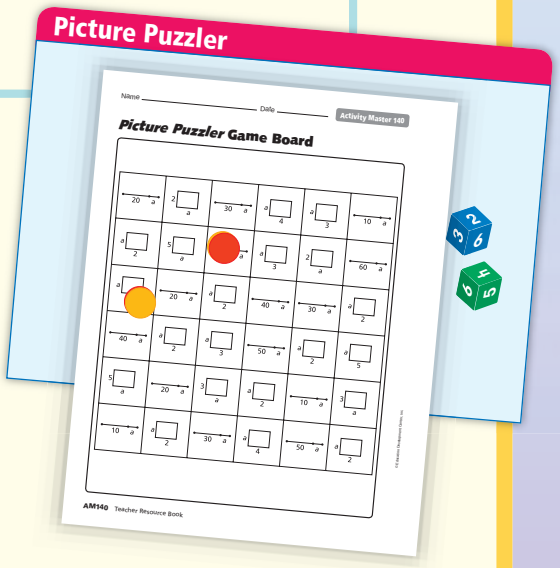
Picture Puzzler

Game Purpose

To select diagrams to match given sums or products

Materials

- Activity Master 140: *Picture Puzzler* Game Board
- 40 two-color counters
- 2 number cubes, each a different color



How To Play The Game

1. This is a game for 2 players. Decide which color cube will represent tens and which will represent ones. Each player will need 20 counters. Choose your colors. Roll one number cube to see who goes first. Then take turns.

2. Toss the cubes. Make your target number according to the colors.

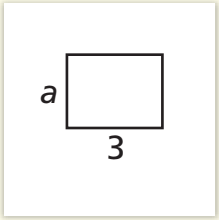
3. Find a diagram that can represent your target number. Any counting number can be used for ***a***.

4. Name the value for ***a*** so your partner can be sure that your diagram represents your target number
   - If your diagram is correct, put a counter in the square.
   - If your diagram is not correct, try one more time.

   Example: Your target number is 63.

   You choose this square and say that $a = 21$.

   $21 \times 3 = 63$, so you can put a counter on the square.



5. The winner is the first player to fill a 2-by-2 block of squares.

   Variation: You can choose which of the rolls to use for the tens and ones each time you toss the cubes.

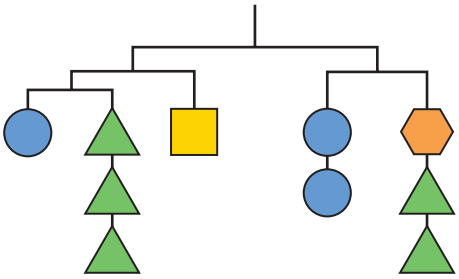
CHALLENGE

Balance Please!

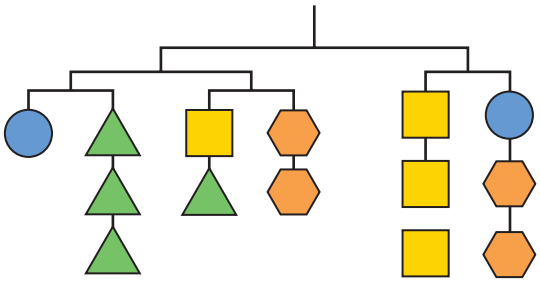
For each mobile:

- **Find the weight of each shape for the total weight given.**
- **Choose a new total weight so that it is possible to balance the mobile with weights that are still whole numbers.**
- **Determine the new weight of each shape so that the mobile balances for the new total weight. Remember, in any one mobile, the same shape always has the same weight.**

1 Total Weight = 72



2 Total Weight = 48



3 Total Weight = 28