

2 Multiplication and Large Numbers

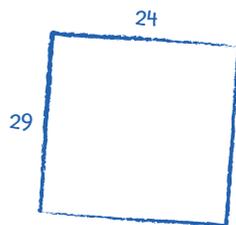
Dear Student,

This chapter is titled “**Multiplication and Large Numbers**” because you already know quite a bit about multiplication and are ready for some new ideas that will help you to multiply large numbers.

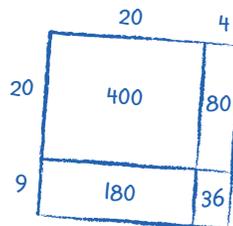
We begin by looking for patterns in a multiplication table. Noticing patterns in columns and rows can help you develop new strategies for multiplying larger numbers.

There are many ways to solve a problem such as 24×29 .

One way is to think about it as an area model with 29 rows and 24 columns and draw a quick sketch.



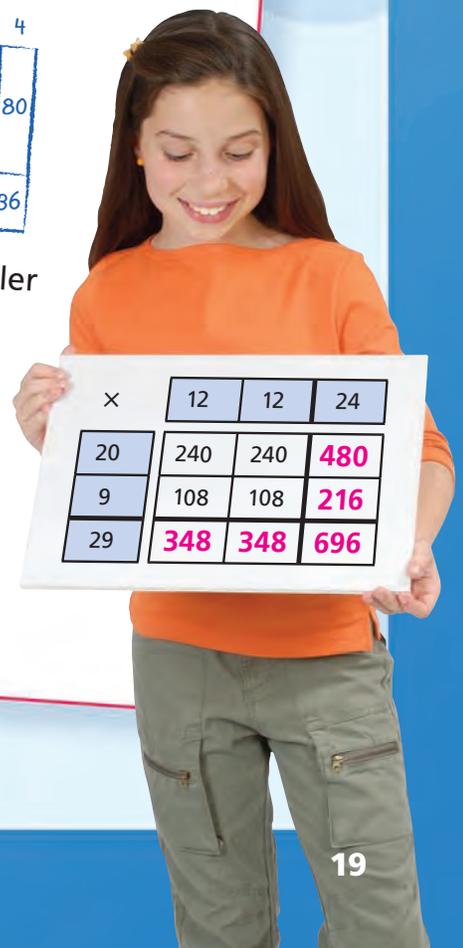
By splitting up the area model into smaller parts, we can use four simpler multiplications to solve the one harder one. Here's one way:



Or we can split up the numbers and multiply simpler ones in a puzzle like this.

We hope that by the end of this chapter you will have seen how patterns, pictures, and puzzles can help you use simpler multiplication problems to do more difficult ones.

Mathematically yours,
The authors of *Think Math!*



×	12	12	24
20	240	240	480
9	108	108	216
29	348	348	696

Watt's That?

FACT • ACTIVITY 1

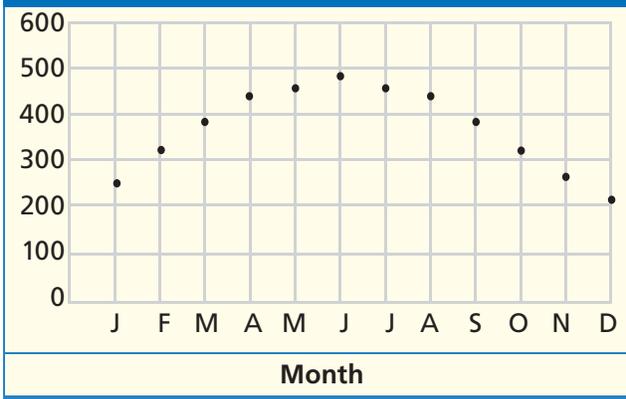
Solar energy is a renewable resource. One way to collect and use the energy in sunlight is to place large arrays of solar modules (called photoelectric cells) on rooftops. Photoelectric cells change light directly into electricity. The graph below shows the approximate potential amount of solar energy that reaches Houston, Texas, each month of the year as measured in watts per square meter.



- 1 About how many watts per square meter of solar power does Houston receive in January?
- 2 Write a number sentence to show about how much solar power a 10-square meter solar module will receive in January.
- 3 About how many watts of solar power would an 8-square meter solar module receive during October?
- 4 During which month(s) does Houston receive about 2 times as much solar power as in December? Explain.

Solar Power (watts/square meter)

Potential Solar Energy Reaching Houston, Texas



- 5 If a solar module receives a total of 75,000 watts of solar energy in January, what could be the area of the solar module? First, write a multiplication sentence that will help you figure out the area of the solar module. Then find the area.

FACT • ACTIVITY 2

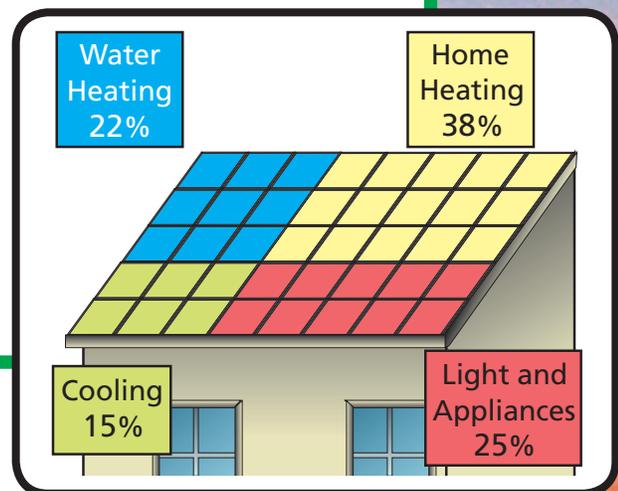
Photoelectric solar modules generate electricity when struck by sunlight. These modules may be grouped into an array that covers most of a home's roof. Homes use energy to heat air and water, for air conditioning and refrigeration, and for lights and other appliances. The picture below is a representation of how solar energy might be used in a typical house.

- 1 Write multiplication number sentences to show how many modules are in each of the four sections (each home use.)
- 2 Each module generates 90 watts of electricity. How many watts are used for heating water? How many watts are generated by all the solar modules?
- 3 How many modules would be needed to build 6 of these arrays?

CHAPTER PROJECT

You can cook with solar power, too. Get a pot (preferably black) and a box that the pot can fit into with room to spare. Line the inside of all 6 sides of the box with aluminum foil to make them reflective. Put some water in the pot, and leave it outside for about an hour. Use a thermometer to test the temperature. Now, put the pot inside the box and place it in a spot where there is sunlight. Check the temperature after another hour to see if the water heats up.

- Write multiplication sentences to help you find the area of each reflective side of your cooker.
- Find the total reflective surface.
- Suppose you built a larger or smaller cooker. Research some home-made solar cookers and explain how they differ from the one you built.
- Do you think more reflective surface will cook the same food faster? Work with your class to select two solar cookers with different total reflective surfaces. Experiment to see which heats the water faster.



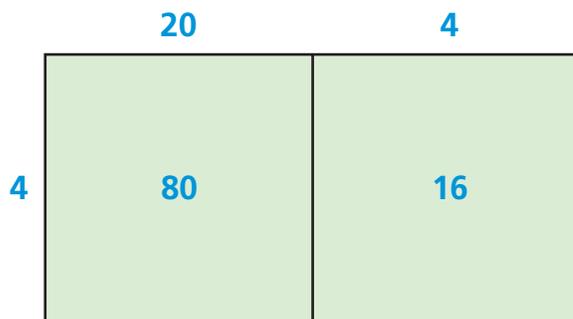
ALMANAC Fact

The Earth is about 93 million miles from the sun. Light travels about 183,000 miles per second. It takes about 8 minutes and 20 seconds for light from the sun to reach Earth.

EXPLORE

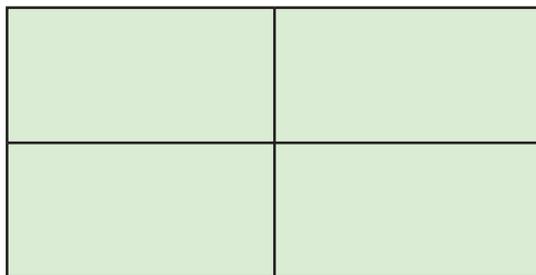
Splitting Area Models

You can split an area model and use simpler multiplication problems to solve more difficult ones.



$$24 \times 4 = (20 \times 4) + (4 \times 4) = 80 + 16 = 96$$

Or, you can split an area model more than once.



PROBLEM: With a partner, draw and split an area model to show 18×22 . Find the product.

REVIEW MODEL

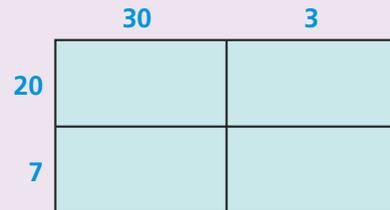
Splitting an Area Model to Multiply

You can split an area model, write new factors, and then complete a matching puzzle to help you multiply.

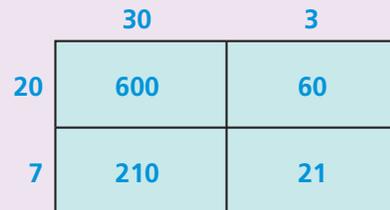
Example: Find 27×33 .

Step 1 Draw an area model.

Split the area model to show factors that are easier to multiply.



Step 2 Write the partial products in the sections of the area model.



Step 3 Complete a Cross Number Puzzle to match.

So, $27 \times 33 = 891$

×	30	3	33
20	600	60	660
7	210	21	231
27	810	81	891

Check for Understanding

Draw and split an area model and then complete a matching Cross Number Puzzle to multiply.

1 14×16

2 21×18

3 38×22

4 13×47

REVIEW MODEL

Doubling and Adding to Multiply

You can use the strategy doubling and adding to help find products.

Think about the products in a row of a multiplication table.

First, fill in the products for 17×1 , 17×2 , 17×3 , 17×4 , and 17×10 .

×	1	2	3	4	5	6	7	8	9	10
17	17	34	51	68						170
	↑ You know	↑ Double 17	↑ Add 17 + 34	↑ Double 34						↑ Multiply by 10

Then, use those products to fill in the other products.

×	1	2	3	4	5	6	7	8	9	10
17	17	34	51	68	85	102	119	136	153	170
				↑ Halve 170	↑ Double 51	↑ Add 51 + 68		↑ Double 68	↑ Add 68 + 85	

You can use these same ideas to multiply some two-digit numbers.

Example 1 Find 13×16 .

- A** Think: $10 \times 16 = 160$
- B** Think: $1 \times 16 = 16$ and $2 \times 16 = 32$, so $3 \times 16 = 16 + 32$, or 48.
- C** Add: $160 + 48 = 208$
So, $13 \times 16 = 208$.

Example 2 Find 21×43 .

- A** Think: $10 \times 43 = 430$
- B** Double 430 to get 20×43 .
 $2 \times 430 = 860$
- C** Add one more 43. $860 + 43 = 903$
So, $21 \times 43 = 903$.

Check for Understanding

Use the strategy doubling and adding to find the products.

1	×	1	2	3	4	5	6	7	8	9	10
24		■	■	■	■	■	■	■	■	■	■

2 24×16

3 12×24

4 15×8

5 14×16

EXPLORE

Greater Multiples

Find these products.

1 $10 \times 10 = ?$

2 $10 \times 100 = ?$

3 $100 \times 100 = ?$

Solve this problem with a partner.

- 4 Patrick and Jaimee want to estimate $4,162 \times 3,321$ by multiplying 4,000 and 3,000. Patrick thinks that $4,162 \times 3,321$ is about 12,000. Jaimee thinks that this product is a lot larger—about 12,000,000!

What do you think and why?

EXPLORE

A Huge Number of Marbles

Jorge counted all the marbles in his collection and found he had 151. He thought that if all 211 fifth graders in his school had the same number of marbles that he had, that would be a huge number of marbles!

Without calculating it, what do you already know about the product?

REVIEW MODEL

Estimating Products

You can estimate products of whole numbers by rounding the factors or by using compatible numbers for the factors. In multiplication, compatible numbers are factors that are easy to multiply.

Here are four different ways to estimate 23×38 .

- 1 Round both factors up. $23 \times 38 \rightarrow 30 \times 40 \rightarrow 1,200$
- 2 Round both factors down $23 \times 38 \rightarrow 20 \times 30 \rightarrow 600$
- 3 Round each factor to the nearest ten . . $23 \times 38 \rightarrow 20 \times 40 \rightarrow 800$
- 4 Use factors that are close to 27 and 38 and are easy to multiply. $23 \times 38 \rightarrow 25 \times 40 \rightarrow 1,000$

The way you adjust the factors should be tied to the type of estimate you need.

- A way to find an estimate that is greater than the actual product is to round both factors up.
- A way to find an estimate that is less than the actual product is to round both factors down.
- A way to find an estimate that is close to the actual product is to round each factor to the nearest ten or to use factors that are close to the factors and easy to multiply.

Check for Understanding

Estimate each product two ways.

1 62×24

2 52×35

3 127×98

4 77×42

5 17×221

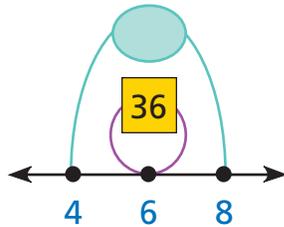
6 119×988

EXPLORE

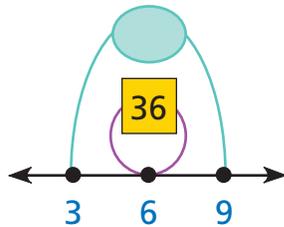
Bigger Steps Away
from Square Numbers

Your teacher will tell you which steps to investigate.

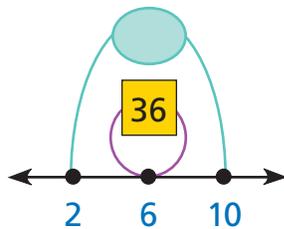
If investigating 2 Steps Away you might try something like this:



If investigating 3 Steps Away you might try something like this:



If investigating 4 Steps Away you might try something like this:



- 1 Try some different numbers and see if you find a pattern.
- 2 Pick a 2-digit number.
 - You may use a calculator to square the number.
 - Use your pattern to predict the product for your number of steps away.
- 3 Did it work? You may check with a calculator.

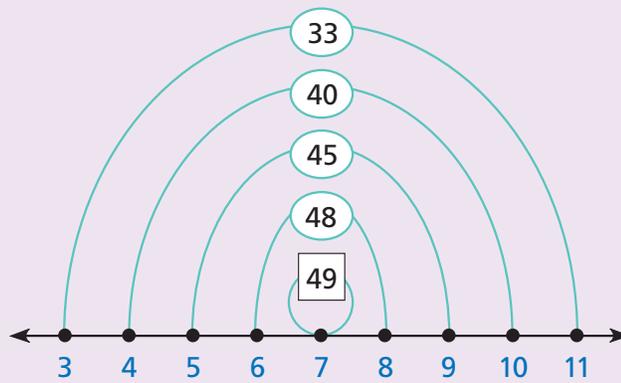
REVIEW MODEL

Using a Pattern to Multiply

You can use the pattern shown below to help you to find the product of factors that are 1, 2, 3, and 4 steps away from a square number.

In the diagram below, the number squared is 7: $7 \times 7 = 49$.

Look at the products for the pairs of factors that are 1, 2, 3, and 4 steps away from 7. Do you see a pattern?



Square number..... 49

- 1 step away $\rightarrow 6 \times 8 = 49 - (1 \times 1) = 49 - 1 = 48$
- 2 steps away $\rightarrow 5 \times 9 = 49 - (2 \times 2) = 49 - 4 = 45$
- 3 steps away $\rightarrow 4 \times 10 = 49 - (3 \times 3) = 49 - 9 = 40$
- 4 steps away $\rightarrow 3 \times 11 = 49 - (4 \times 4) = 49 - 16 = 33$

Check for Understanding

Use a pattern to complete these related number sentences.

1 $10 \times 10 = \blacksquare$

$9 \times 11 = \blacksquare$

$8 \times 12 = \blacksquare$

$7 \times 13 = \blacksquare$

$6 \times 14 = \blacksquare$

2 $12 \times 12 = \blacksquare$

$11 \times 13 = \blacksquare$

$10 \times 14 = \blacksquare$

$9 \times 15 = \blacksquare$

$8 \times 16 = \blacksquare$

3 $50 \times 50 = \blacksquare$

$49 \times 51 = \blacksquare$

$48 \times 52 = \blacksquare$

$47 \times 53 = \blacksquare$

$46 \times 54 = \blacksquare$

REVIEW MODEL

Problem Solving Strategy

Solve a Simpler Problem

Grant works 24 hours a week. How many hours does he work in 31 weeks?

Strategy: Solve a Simpler Problem

Read to Understand

What do you know from reading the problem?

Grant works 24 hours each week.

What do you need to find out?

the number of hours Grant works in 31 weeks

Plan

How can you solve this problem?

I can use the strategy *solve a simpler problem* to find 24×31 .

Solve

How can you solve a simpler problem?

One Way: I can multiply 24 by 30 (which is simpler to compute mentally) and then make an adjustment by adding another 24.
 $24 \times 30 = 720$; $720 + 24 = 744$

Another Way: I can split an area model and complete a puzzle.

	20	4	
30	600	120	
1	20	4	

×	20	4	24
30	600	120	720
1	20	4	24
31	620	124	744

So, Grant works 744 hours in 31 weeks.

Check

Look back at the problem. Did you answer the questions that were asked? Does the answer make sense?

Problem Solving Practice

Solve a simpler problem to solve.

- 1 Pat's Pet Shop sells flea collars for \$17. Last month they sold 23 collars. How much money did they take in on the sale of flea collars?
- 2 Chad earns \$12 an hour. He worked 4 hours in January, 13 hours in February, and 23 hours in March. How much did he earn those three months?

Problem Solving Strategies

- ✓ Act It Out
- ✓ Draw a Picture
- ✓ Guess and Check
- ✓ Look for a Pattern
- ✓ Make a Graph
- ✓ Make a Model
- ✓ Make an Organized List
- ✓ Make a Table
- ✓ **Solve a Simpler Problem**
- ✓ Use Logical Reasoning
- ✓ Work Backward
- ✓ Write an Equation

Mixed Strategy Practice

Use any strategy to solve. Then explain what strategy you used and how you solved the problem on a separate piece of paper.

- 3 Anthony lives 4 blocks from school. How many blocks does he walk to school and back home in a week?
- 4 Michel's family is planning a trip. They can go to either Dallas or San Francisco. They can drive, fly, or take the train to their destination. How many choices do they have?
- 5 Katlin exercises for 2 minutes on Day 1, 4 minutes on Day 2, 8 minutes on Day 3, and 16 minutes on Day 4. If the pattern continues, how many minutes will she exercise on Day 7?
- 6 Students in kindergarten through fifth grade attend Garden Elementary School. There are 4 classes in each grade with about 25 students in each class. About how many students attend Garden Elementary School?

Emma made this table to show some popular dog breeds. Use the table for 7–9.

- 7 Which is the second most popular breed?
- 8 Which breeds are less popular than a Beagle?
- 9 List the breeds in order from most popular to least popular?

Most Popular United States Dog Breeds	
Dog Breed	Number of Registered Dogs
Beagle	45,033
Dachshund	39,473
German Shepherd	43,950
Golden Retriever	52,530
Labrador Retriever	144,934

Choose the best vocabulary term from Word List A for each sentence.

- 1 Multiples of 5 that end in 5 are ____?
- 2 The numbers 4, 8, 12, and 16 are ____? of 4.
- 3 The ____? for 371 is $3 \times 100 + 7 \times 10 + 1 \times 1$.
- 4 You can estimate a product by ____? factors and then multiplying them.
- 5 You can find the value of a number in expanded notation by ____?.
- 6 A(n) ____? is a rectangle of rows and columns.
- 7 You can just round factors or use ____? to estimate a product.
- 8 A(n) ____? has a horizontal line of objects.
- 9 You can add ____? to find the product of two numbers.

Complete the analogy using the best term from Word List B.

- 10 Ones is to thousands as billions is to ____?.
- 11 Horizontal is to vertical as rows are to ____?.

Word List A

adding
products
array
compatible
numbers
doubling
products
estimating
expanded
notation
factors
multiples
odd multiples
partial products
rounding
row
word

Word List B

arrays
billions
columns
trillions

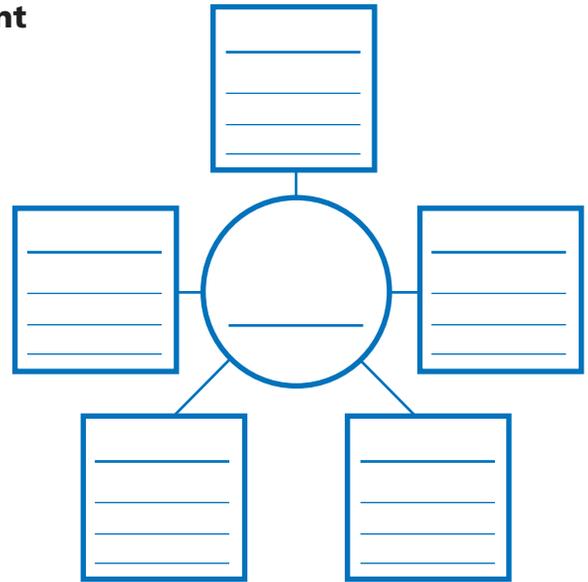
Talk Math

Discuss with a partner what you have learned about multiplying large numbers. Use the vocabulary terms *product*, *square number*, *neighbor numbers*, and *steps away*.

- 12 How can you multiply 41×39 ?
- 13 How can you multiply an odd multiple of 5 by itself?
- 14 How can you multiply 56×64 ?

Concept Map

- 15 Create a concept map that shows different ways to multiply larger numbers.



Analysis Chart

- 16 Create an analysis chart using the vocabulary terms *factors*, *rounding*, *compatible numbers*, and *neighbor numbers*. Use what you know and what you have learned about multiplying.

What's in a Word?



TRILLION Many familiar words begin with *tri-*. Some of them are *triangle*, *tricycle*, *trio*, and *triple*. Can you see what these words have in common? A triangle has three angles, a tricycle has three wheels, a trio is three people, and a triple is something that occurs three times. The prefix *tri-* means "three." The word *trillion* puts together the prefix *tri-* with *-illion* from the word *million* to get *trillion*. So, *million* is the first group of millions, *billion* is the second group of millions, and *trillion* is the third group of millions.



Technology

Multimedia Math Glossary

www.harcourtschool.com/thinkmath

GAME

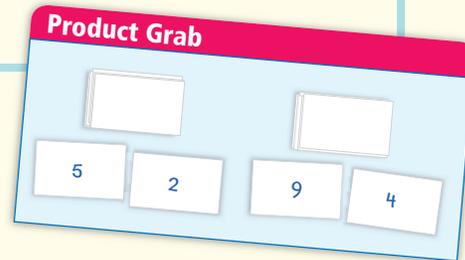
Product Grab

Game Purpose

To review basic multiplication facts through 12

Materials

- 48 index cards
- clock



How To Play The Game

- 1 Play this game with a partner. Use index cards to make 4 sets of number cards, each set numbered 1 to 12.
- 2 One player mixes up the cards, and deals them all face down.
- 3 Players turn over the top two cards from their piles at the same time and state the products.
 - The player whose cards have the greater product grabs all 4 cards and sets them aside.
 - If the products are equal, both players turn over two more cards. The player with the greater product takes all 8 cards.

Example: Danielle's cards Evan's cards



The product of Evan's cards is 35, and the product of Danielle's cards is 12. Since 35 is greater than 12, Evan takes all 4 cards.

- 4 If you run out of cards from your original pile, mix up the cards you have grabbed, and use them to keep playing.
- 5 At the end of 10 minutes, the player with the greater number of cards wins the game. If a player runs out of cards before the 10 minutes are up, the other player wins the game.

GAME

Factor Search

Game Purpose

To practice multiplication and division facts

Materials

- Activity Master 9: *Factor Search Grid*
- Activity Masters 10–13: *Factor Search Cards*
- counters in two different colors • scissors

How To Play The Game

- 1 Play this game with a partner. Each player needs a set of counters and a *Factor Search* grid. Cut out all of the *Factor Search* cards.
- 2 Mix up the *Factor Search* cards, and place them face down in a pile. Each player takes a *Factor Search* card. The player who has the greater number goes first. Put those two cards at the bottom of the pile.
- 3 Take turns. Turn over the top card from the pile.
 - Find a square on your *Factor Search* grid by naming two factors for the product shown on the *Factor Search* card.
 - Put a counter on the grid at the product location.
 - If there is no open square for a product, the player loses a turn.

Example: Emilee turns over this card.

Possible factors: The only possible factors are 9 and 3. The factors cannot be 1 and 27 because 27 is not a factor on the *Factor Search* grid.

Possible moves: There are two squares on the *Factor Search* grid on which Emilee could put a counter: 9 across and 3 down, and 3 across and 9 down.

- 4 The first player to put three counters in a line—horizontally, vertically, or diagonally—wins.



27

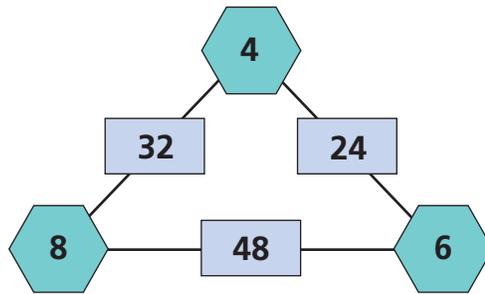
CHALLENGE

Hexagon Loop

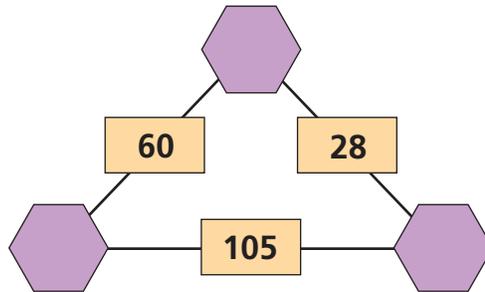
Someone has left some mystery puzzles! Your skills will be tested as you try to solve them. Don't get dizzy!

- Figure out the relationship between the numbers in the rectangles and the hexagons in Puzzle A.
- Use that same relationship to solve Puzzles B and C.

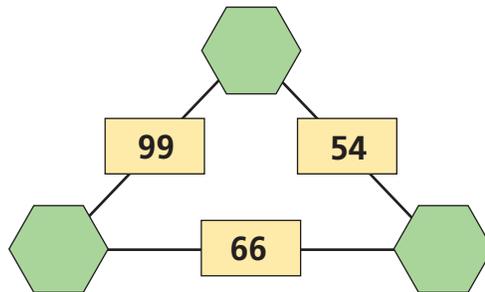
A



B



C



Now that you know what's going on, trace the outline of one puzzle, and make up your own. Challenge a classmate with it.