Chapter

3 Factoring and Prime Numbers

Dear Student,

Factories make various products. We use very similar words—factors and products—when we are talking about how some numbers can be multiplied together to make other numbers. The numbers we multiply are called factors, and the result of the multiplication is the product. In this chapter, you will learn more about putting numbers together and taking them apart using multiplication.

To begin exploring making and breaking numbers with multiplication, you will solve Mystery Number Puzzles with clues such as those at the right.

Puzzle

- ☑ I have 4 factors
- ☑ I am a 1-digit number
- ☑ I am not divisible by 4

Can you guess what the number is? Don't worry if you can't yet. By the end of the chapter you will know what each of these clues mean and be able to quickly solve the puzzle.

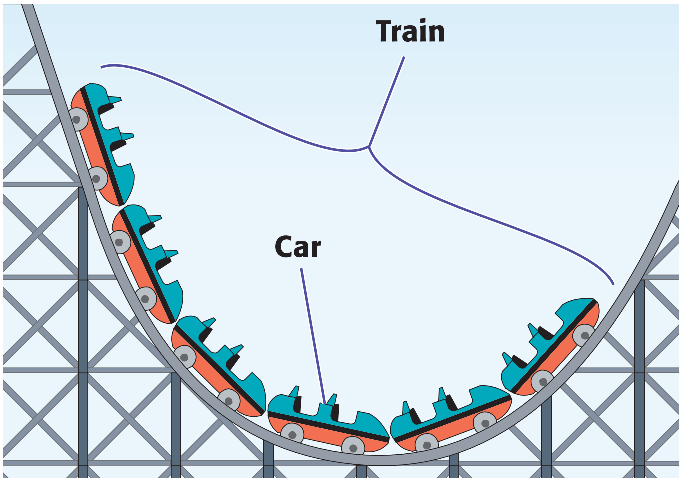
Mathematically yours,
The authors of ***Think Math!***

THE WORLD ALMANAC FOR KIDS

Wild Rides

FACT ACTIVITY 1

Modern roller coasters have train cars linked together, each carrying a number of people. Their hills can be hundreds of feet high.

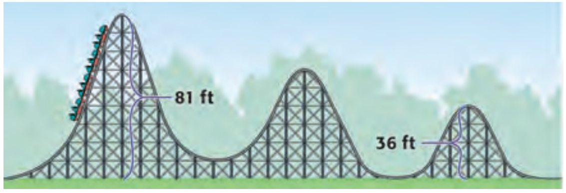


Number of Passengers in Some Roller Coasters	
Name of Roller Coaster	**Number of Passengers**
Kingda Ka	18
Superman Ride of Steel	36
Titan	30
Scream	32
Thunderbolt	24

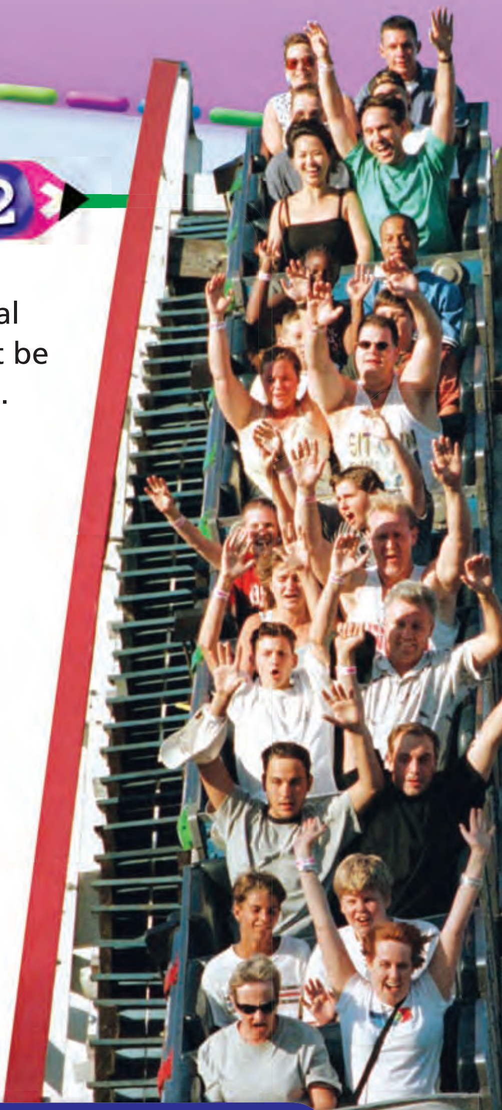
1. The *Scream* is a train of 8 cars. How many passengers does each car carry?
2. Which ride in the table could use trains made of cars that carry 5 passengers each?
3. Suppose you were designing a new 18-passenger train for *Kingda Ka*. Think about how many different-sized cars could be used for 18 passengers if you want the same number of passengers to ride in each car. What are the different numbers of cars and passengers that could be used for the train?
4. Suppose you are designing a new 30-passenger train for *Titan*. Which of the cars you designed for *Kingda Ka* could be used for this train? (Hint: Think about factors of 30.)

FACT·ACTIVITY 2

This roller coaster design is supported by segments of vertical metal beams under the hills. To be cost effective, all hills must be built from segments of metal beams that are the same length.



1. What are the lengths of beam segments that could be used for the 81-ft support? What lengths could be used for the 36-ft support?
2. Which factors do these two numbers have in common? What is the largest segment of beam length that could be used for both supports?
3. The middle hill will use the same size beam segments used in Problem 2. Remember the middle hill is less than 81 ft but greater than 36 ft. List the possible number of beam segments for the middle hill and the possible heights of the hill.



CHAPTER PROJECT

With your group, build a model roller coaster train. Use 3 cardboard egg cartons. Each space for an egg represents a seat on your roller coaster. Cut and arrange the cartons to make trains. Each car in the train must have the same number of seats and the train must have a total of 36 seats.

- How many different car sizes will let 36 people ride the roller coaster train?
- Explain how these different car sizes are related.
- Write a description of another design with 60 seats. Explain the arrangement of cars and seats you choose. Write multiplication sentences to show your arrangement.

Materials

- string/yarn to connect the crates (cars)
- egg cartons
- needles
- scissors
- glue
- paint
- brushes

ALMANAC Fact

Cedar Point, in Sandusky, Ohio, had 16 roller coasters in 2006. That was the most of any amusement park in the U.S.

Chapter 3
Lesson 1

EXPLORE
Mystery Number Puzzles

The boxes to the right of the clues show you the number of digits in the solution.

Make a list of numbers that match the first clue.

Use the other clues to help you eliminate numbers and cross them off your list.

1 Puzzle A **Clues**

- ☑ Multiple of 11 less than 100
- ☑ Odd
- ☑ Sum of the digits is 6 ■ ■

2 Puzzle B **Clues**

- ☑ A factor of 12
- ☑ An odd number
- ☑ Greater than 1 ■

3 Puzzle C **Clues**

- ☑ Square number less than 8×8
- ☑ Even
- ☑ Product of its digits is 18 ■ ■

Chapter 3

Lesson 2

EXPLORE
Finding Factors

Make a list of numbers from the first clue.
Use the other clues to eliminate some numbers.

1 Puzzle A **Clues**

- A factor of 36
- An odd number
- A square number

2 Puzzle B **Clues**

- A factor of 48
- A multiple of 12
- Has 6 factors

Chapter 3
Lesson 2

REVIEW MODEL
Factors of a Number

One whole number is a factor of another whole number if, when you divide the second by the first, the quotient is also a whole number and the remainder is 0.

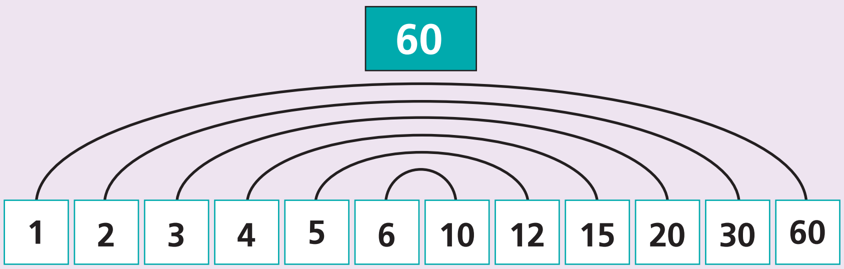
You can use a diagram as a way to organize your list to be sure that you have found all the factors.

Example **Draw a diagram to find all the factors of 60.**

Step 1 Write the pair of factors that consists of 1 and the number itself. Leave space for other factors between them.

Step 2 Decide if 2 is a factor. If 2 is a factor, name the other member of the pair and write this pair inside the first pair.

Step 3 Continue filling in the factor pairs, listing the factors in increasing order from left to right. [As one factor increases, the other decreases, so they keep moving toward the center of the diagram.]



After each step, you only have to check for factors between the two closest numbers. For example, after we write 6 and 10, we check 7, 8, and 9 and then we know we have found all the factors.

Check for Understanding

Draw a diagram to find all the factors of each number.

1. 18

2. 100

Chapter 3

Lesson 3

REVIEW MODEL
Common Factors

Factors of a number are the whole numbers that, multiplied by another whole number, make that particular product.

24	24	24	24
↑	↑	↑	↑
1×24	2×12	3×8	4×6

Factors ⟶ 1, 24   2, 12   3, 8   4, 6

Common factors of two or more numbers are the factors that the numbers share.

Example **Find the common factors of 80 and 100.**

Step 1 List the factors of both numbers.

80: 1, 2, 4, 5, 8, 10, 16, 20, 40, 80
100: 1, 2, 4, 5, 10, 20, 25, 50, 100

Step 2 Find the numbers that are in both lists.

80: **1**, **2**, **4**, **5**, 8, **10**, 16, **20**, 40, 80
100: **1**, **2**, **4**, **5**, **10**, **20**, 25, 50, 100

So, 1, 2, 4, 5, 10 and are the common factors of 80 and 100.

Check for Understanding

Find common factors for each pair of numbers.

1. 12 and 18
2. 15 and 45
3. 22 and 28
4. 9 and 20

Chapter 3

Lesson 5

REVIEW MODEL
Prime and Composite Numbers

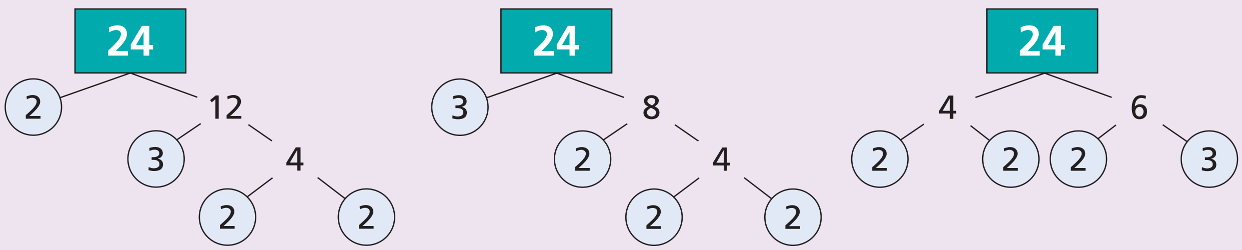
Numbers that have exactly two factors, 1 and the number itself, are prime numbers. Numbers that have more than two factors are composite numbers. 1 is neither prime nor composite. It has exactly one factor, 1.

Number	Prime or Composite?	Reason
2	Prime	1 and 2 are the only factors of 2.
6	Composite	1, 2, 3, and 6 are factors of 6.

A composite number can be expressed as a product of prime numbers. You can use a factor tree to help you find the prime factors of a number.

Example **Write 24 as the product of prime factors.**

Begin with any two pairs of factors for 24. Circle any prime factors, and continue to factor any composite numbers. The prime factors are always the same, no matter how you find them.



So, $24 = 2 \times 2 \times 2 \times 3$.

Check for Understanding

Tell if each number is prime or composite.

1. 3
2. 15
3. 11
4. 18
5. 24

Use a factor tree. Write each number as the product of prime numbers.

6. 12
7. 36
8. 40

Chapter 3
Lesson 7

REVIEW MODEL
Divisibility Rules

A whole number is divisible by another whole number when the quotient is also a whole number and the remainder is zero.

Some numbers have a divisibility rule. Look at the rules in the table.

A number is divisible by		Divisible	Not Divisible
2	if the ones digit is an even number.	94	91
3	if the sum of the digits is divisible by 3.	51	52
5	if the ones digit is 0 or 5.	45	54
6	if the number is even and divisible by 3.	642	651
9	if the sum of the digits is divisible by 9.	729	971
10	if the ones digit is 0.	400	555

Example **Determine if 18 is divisible by 2, 3, 5, 6, 9, or 10.**

18 is divisible by:

2 because the last digit is an even number.
3 because the sum of the digits is divisible by 3.
6 because the number is even and divisible by 3.
9 because the sum of the digits is divisible by 9.

Check for Understanding

Determine if each number is divisible by 2, 3, 5, 6, 9, or 10.

1. 42
2. 200
3. 75
4. 324
5. 360
6. 144
7. 309
8. 96

9. Write a number that is divisible by 3 and 5. Explain how you know the number is divisible by 3 and 5.

Chapter 3
Lesson 8

REVIEW MODEL
Problem Solving Strategy
Guess and Check

Katie wrote clues for a Mystery Number Puzzle. What is the solution to her puzzle?

Katie's Mystery Number Puzzle
- ☑ I am a 2-digit multiple of 3.
- ☑ I am an odd number.
- ☑ My tens digit is greater than my ones digit.
- ☑ The sum of my digits is 15.

Strategy: Guess and Check

Read to Understand

What do you know from reading the problem?

I know that the number has to match all the clues that Katie gave.

What do you need to find out?

I need to find a number that is a 2-digit multiple of 3, is an odd number, has a tens digit greater than its ones digit, and has a digit sum that is 15.

Plan

How can you solve this problem?

I can use the first clue to make a systematic list of guesses for the number and then continue to check with each clue to see if the numbers still fit the clues.

Solve

How can you use guessing and checking to solve the problem?

~~12~~	~~21~~	~~30~~	~~42~~	~~51~~	~~60~~	~~72~~	~~81~~	~~90~~
~~15~~	~~24~~	~~33~~	~~45~~	~~54~~	~~63~~	~~75~~	~~84~~	~~93~~
~~18~~	~~27~~	~~36~~	~~48~~	~~57~~	~~66~~	~~78~~	87	~~96~~
		~~39~~			~~69~~			~~99~~

The choice of what clue to begin with can change the process. Here is one example: I can make a list of the numbers that match the first clue (2-digit multiple of 3). Then I look at each of the remaining clues and cross off any numbers that do not match until only one number remains.

Only 87 remains. So, the solution to the puzzle is 87.

Check

Look back at the problem. Did you answer the questions that were asked? Does the answer make sense?

Problem Solving Practice

Guess and check to solve.

1. An art teacher has 85 markers to distribute equally among students. No fewer than 2 markers and no more than 10 markers were given to any student. After the markers are distributed, there are no markers left over. How many students received markers? How many markers did each student receive?

2. Christopher bought two books about sharks at the aquarium gift shop. He spent $15.75 for the two books. One book cost $0.25 more than the other book. How much did Christopher pay for each book?

Problem Solving Strategies

- Act It Out
- Draw a Picture
- **Guess and Check**
- Look for a Pattern
- Make a Graph
- Make a Model
- Make an Organized List
- Make a Table
- Solve a Simpler Problem
- Use Logical Reasoning
- Work Backward
- Write an Equation

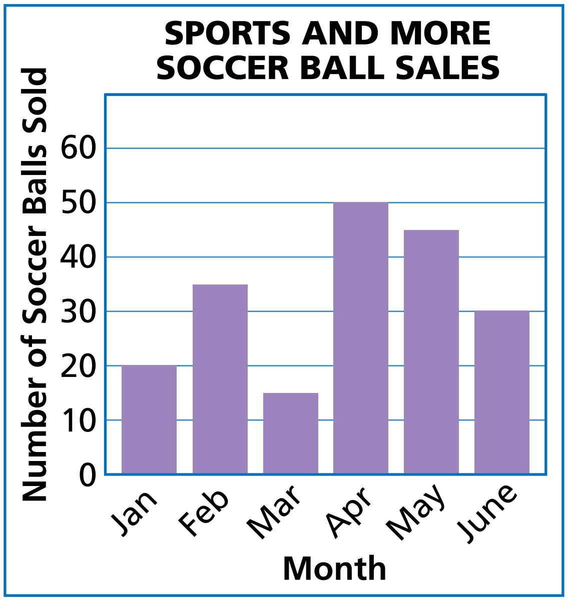
Mixed Strategy Practice

Use any strategy to solve. Explain.

3. Sarah cleans the hamster cage every fifth day. She cleans the bird cage every third day. If she cleans both cages today, in how many days will she clean both cages on the same day again?

4. Matt has a jar full of pennies. He puts the pennies in 29 stacks with 12 pennies in each stack. He has 3 pennies left over. How many pennies were in Matt's jar?

For 5–7, use the graph.



5. Soccer balls cost $15. How much did Sports and More take in on the sale of soccer balls in January and February?

6. In all, how many soccer balls did Sports and More sell during the first six months of the year?

7. How many more soccer balls were sold in the month when the most balls were sold than in the month when the fewest balls were sold?

Chapter 3 Vocabulary

Choose the best vocabulary term from Word List A for each sentence.

1. A number that is __?__ 10 ends in 0.
2. A number is a(n) __?__ number if it has just two factors.
3. A number is divisible by 6 if it is a(n) __?__ multiple of 3.
4. A __?__ is a number that is multiplied by another number to find a product.
5. A number is a(n) __?__ number if it has more than two factors.
6. Some of the __?__ of 4 and 6 are 12, 24, 36, and 48.
7. The __?__ of all prime factors of 30 is 30.

Word List A

- common factors
- common multiples
- composite
- divisibility
- divisible by
- even
- factor
- factoring
- factors
- multiple
- multiples
- odd
- prime
- product
- square number

Complete the analogy using the best term from Word List B.

8. Even is to odd as prime is to __?__ .
9. Product is to __?__ as sum is to addend.

Word List B

- composite
- factor
- multiple

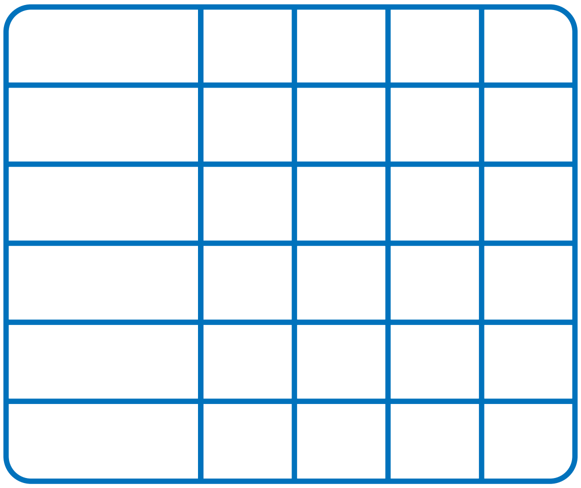
Talk Math

Discuss with a partner what you have learned about factors and multiples. Use the vocabulary terms *factors, divisible by,* and *composite.*

10. How can you find whether a number is prime or not?
11. How can you find common factors of two numbers?
12. How can you tell whether a number is a multiple of 3?

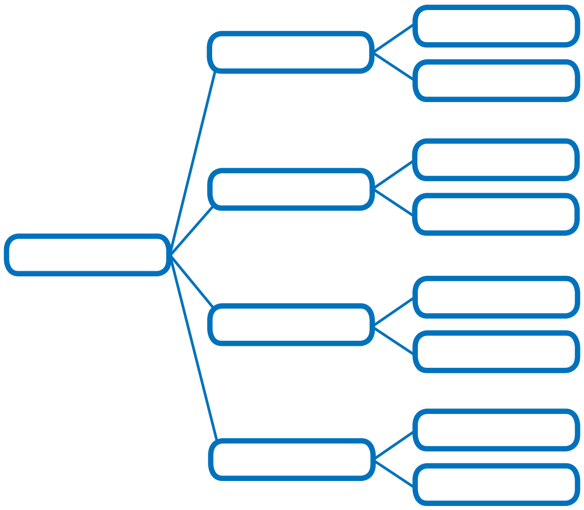
Analysis Chart

13 **Create an analysis chart. Use what you know and what you have learned about *factors* and *multiples.***



Tree Diagram

14 **Create a tree diagram about numbers. Use what you know and what you have learned about *composite* numbers, *prime* numbers, *factors*, and *multiples.***



What's in a Word?

PRIME The word *prime* is a common word that is used in many ways. A person in his or her *prime* is one who is expected to accomplish more in the present than in the past or future. *Prime* can also mean a very good cut of meat, such as *prime* rib. It can mean first in importance, such as a *prime* example. It could mean having the greatest value, such as *prime* real estate. In mathematics, the word *prime* means that a number has exactly two factors, 1 and itself.

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GAME

Factor Trees

Game Purpose

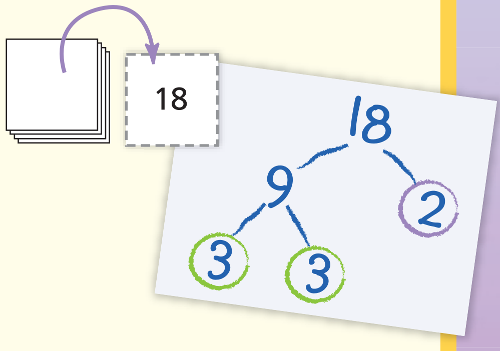
To practice finding prime factors of a number

Materials

- Activity Masters 10–13: *Factor Search* Cards
- 2 different colors of pencils or crayons
- several sheets of blank paper
- scissors

How To Play The Game

1. Play this game with a partner. Cut out all the *Factor Search* Cards. Remove all the prime number cards, and set them aside. They are not used in this game.
2. Mix up the cards. Place them face down in a pile. Decide who will play first.
3. Player 1 turns over the top card and writes that number at the top of a sheet of paper.
4. Player 2 begins a factor tree for the number and circles any prime factors using one color.
5. Player 1 continues the factor tree, if possible, by writing another factor pair and circles any prime factors using the other color.

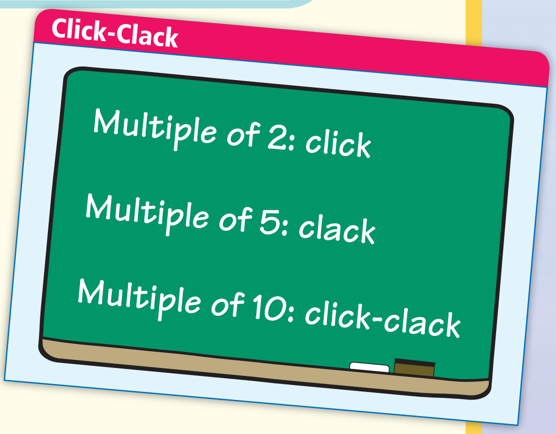


6. Take turns until all of the prime factors for the number have been found and circled. The player who has circled more prime factors is the winner.
7. Turn over a new card, and play the game again.

GAME

Click-Clack

Game Purpose
To practice naming numbers that are multiples of 2, 5, and 10

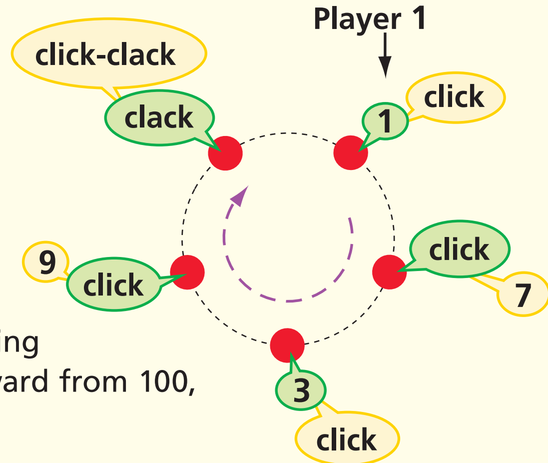


How To Play The Game

1. Four, five, or six players can play this counting game. Sit in a circle. Decide who will go first.

2. To start the round, Player 1 says "1." Continue counting around the circle in order. But if the number is
   - a multiple of 2, say "click" instead.
   - a multiple of 5, say "clack" instead.
   - a multiple of 10, say "click-clack" instead.

   Example: Starting at 1, the first 10 turns would be 1, click, 3, click, clack, click, 7, click, 9, click-clack.



3. If you make a mistake, you are out of the round.

4. When you reach 100, begin counting over at 1. Or you can count backward from 100, replacing multiples of 2, 5, and 10 with click, clack, and click-clack.

5. Play until there is only one player left. That player wins the round and 1 point. Play as many rounds as time allows. The player with the most points wins the game.

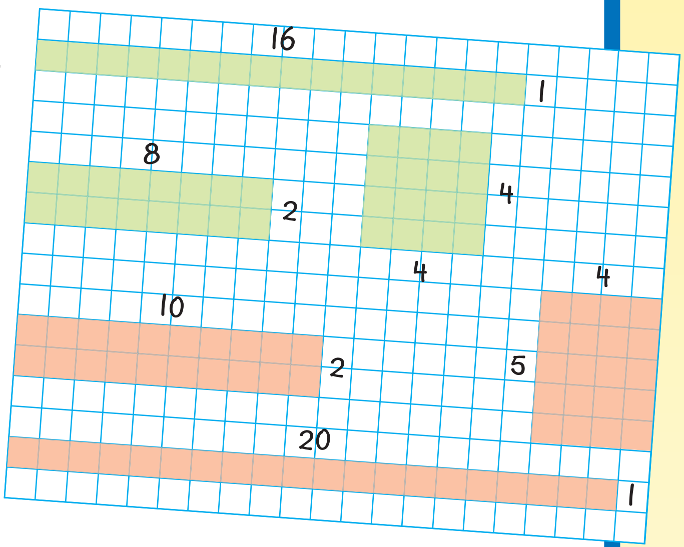
CHALLENGE

Greatest Common Factor Rectangles

What is the greatest common factor of 16 and 20? Follow these steps to find the answer. You will need 30 square tiles or small cubes.

Step 1 Build all possible rectangles using 16 and 20 tiles.

Step 2 Copy the rectangles onto grid paper and write their dimensions.



Step 3 Use the same color to shade rectangles that have the same area.

Step 4 What is the greatest length that is part of both the 16 square unit and the 20 square unit rectangles?

Step 5 What is the greatest common factor of 16 and 20?

Follow the steps above to find the greatest common factor of these pairs of numbers.

1. 9 and 12
2. 10 and 16
3. 14 and 10
4. 14 and 27
5. 18 and 24
6. 20 and 24
7. 11 and 15
8. 20 and 30
9. 16 and 28
10. 21 and 18
11. 28 and 14
12. 24 and 15