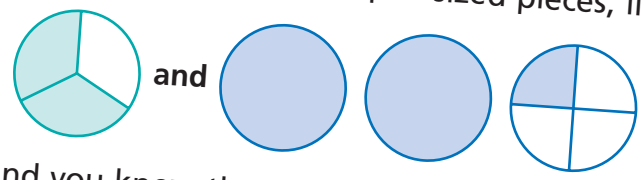
Chapter

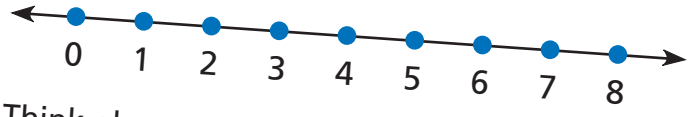
4 Equivalence and Comparison of Fractions

Dear Student,

You already know a lot about fractions. In this chapter, "Equivalence and Comparison of Fractions," you may see fractions in a whole new way! You have certainly seen them represent a number of equal-sized pieces, like this:



And you know that a fraction, like any other number, may be located on a number line.

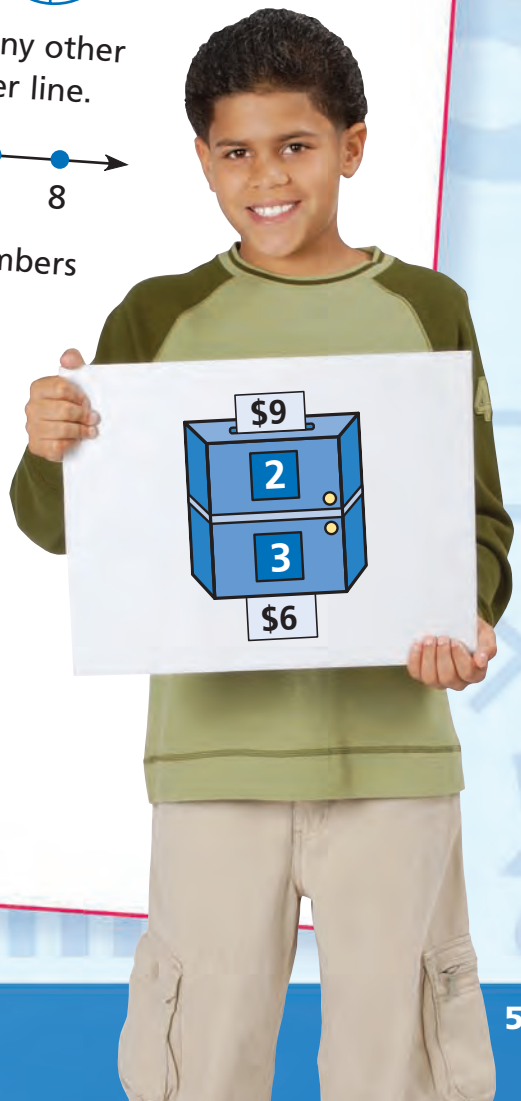


Think about where each of these numbers would be located: $\frac{1}{2}$, $1\frac{3}{4}$, $\frac{1}{3}$, and $7\frac{1}{4}$.

You probably haven't thought about fractions as instructions to multiply and divide numbers by certain amounts. Look at this new machine and its input and output.

Can you figure out how the machine works? You will know this and so much more about fractions by the end of this chapter!

Mathematically yours,
The authors of ***Think Math!***



THE WORLD ALMANAC FOR KIDS

Growing Up, Measuring Up

Great White

All living things grow, but they grow at different rates. In the one year between your birth and first birthday, you probably grew about 10 inches. As incredible as that sounds, many animals grow even faster.

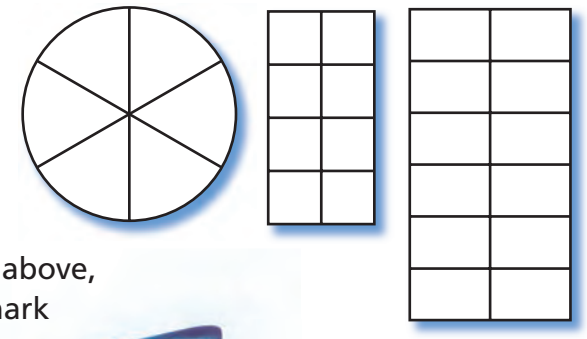
FACT·ACTIVITY 1

The whale shark is only about $\frac{1}{2}$ meter long at birth—about the length of a human baby. In time, the whale shark can grow to more than 18 meters in length, making it the world's largest fish.

Use the table and your answer for 1 to answer 2–4.

Annual Growth Rate of Some Sharks	
Name of Shark	**Growth Rate (meters per year)**
Great White	$\frac{1}{3}$
Sharpnose	$\frac{1}{4}$
Thresher	$\frac{1}{2}$

1. Copy the three shapes below. Shade each shape to show three different ways to represent $\frac{1}{2}$.



2. Is the annual growth of a sharpnose shark greater than or less than the annual growth of a thresher shark? Explain.

3. Write equivalent fractions to help you compare the growth rates of all 3 sharks. Write the shark names in order from the one with the fastest growth rate to the one with the slowest growth rate.

4. Based on the growth rate in the table above, how long does it take a great white shark to grow 1 meter? Explain.

Sharpnose

Thresher

People grow at varying rates. From age 5 to age 10, Lanie grew the same amount, $2\frac{1}{2}$ inches, each year. The chart shows how much she grew in other years.

FACT ACTIVITY 2

Use Lanie's growth chart for 1–4.

1. Write the number of inches Lanie grew between age 4 and age 5 as an improper fraction.
2. Lanie says that she grew more than $2\frac{1}{8}$ inches between age 10 and age 11. Is she right? Explain how you know.
3. Lanie's brother grew $2\frac{7}{8}$ inches between age 3 and age 4. Is that greater than or less than the number of inches Lanie grew at the same age? Explain how you know.
4. Write a mixed number in simplest form that is smaller than any number in the table.

Lanie Grows Up!

Age from Birthday to Birthday	Inches Grown per Year
2–3	$3\frac{1}{4}$
3–4	$2\frac{3}{4}$
4–5	$2\frac{1}{2}$
10–11	$2\frac{1}{4}$
11–12	$3\frac{1}{3}$

CHAPTER PROJECT

Using library resources or the Internet, research domestic pets and choose one that is less than 6 feet tall. Write the breed or animal species on a card. Also write the typical height of the animal in feet, using fractions or mixed numbers. Then in groups of 4 or 5, combine your cards and order the animals' heights from least to greatest. You may have to convert from mixed numbers to improper fractions, use equivalent fractions, and simplify fractions.

ALMANAC Fact

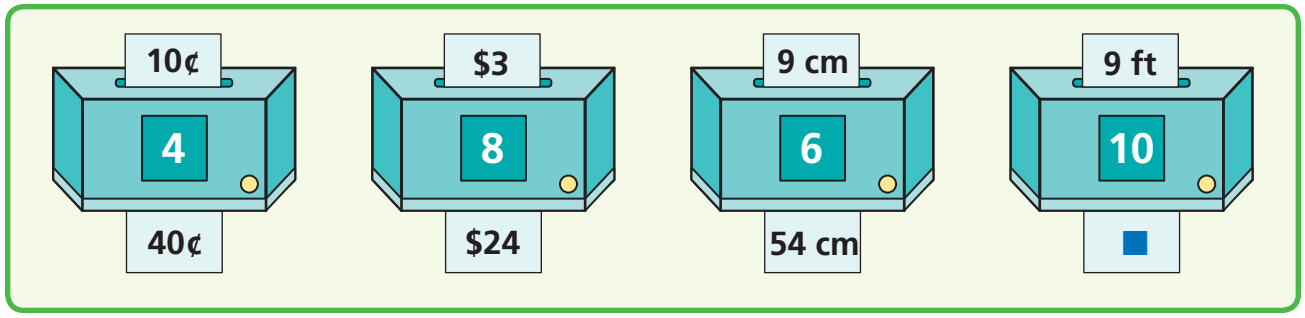
Baby blue whales are 25 feet long when born. Each day for the first seven to eight months, they grow up to one inch in length and gain 200 pounds.

Chapter 4

Lesson 1

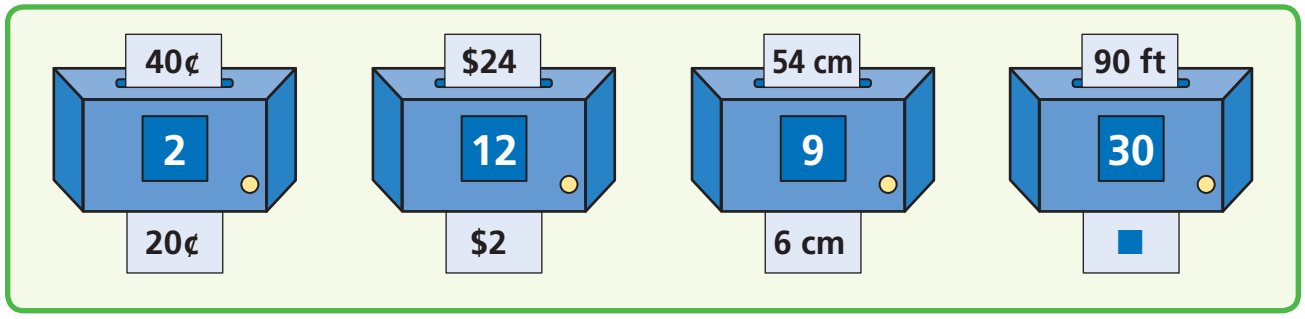
EXPLORE
Two New Machines

Here is the first new machine.



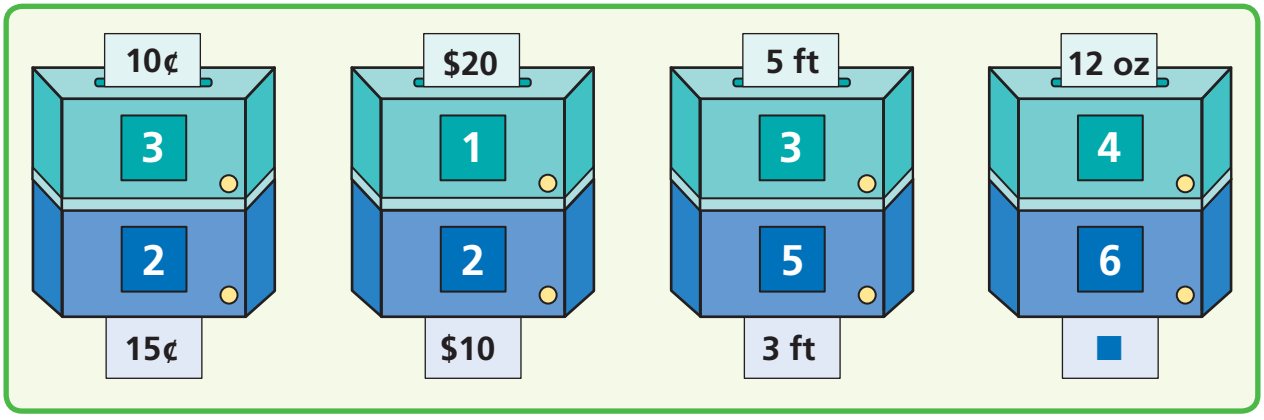
1 What does this machine do to the numbers on the cards that are put into its top?

Here is the second new machine.



2 What does it do to the numbers on the cards that are put into its top?

Now the machines are put together.



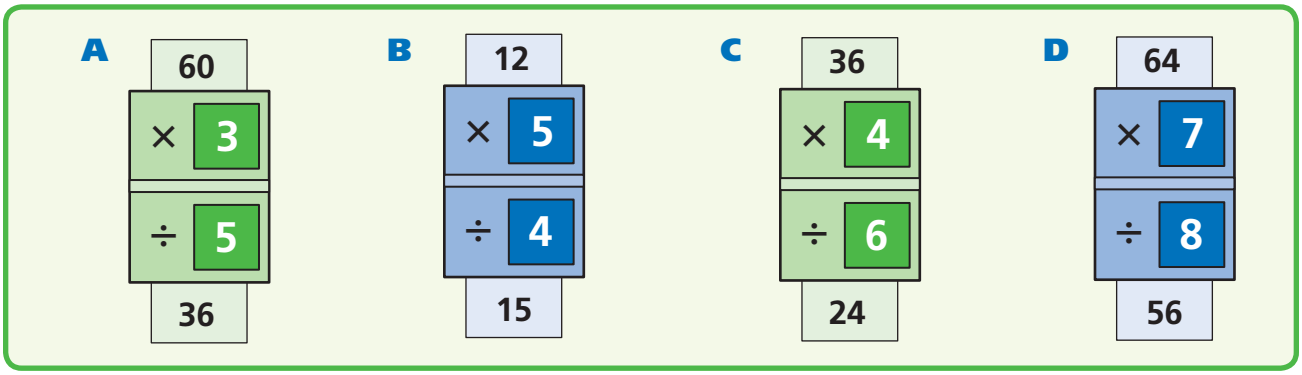
3 What does the combined machine do to the numbers on the cards?

Chapter 4

Lesson 2

EXPLORE
Order of Multiplying and Dividing

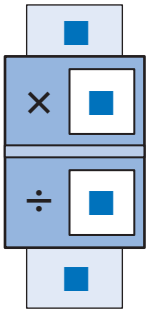
Dakota was experimenting with these fraction machines.



She wanted to divide first, because multiplying first gave large numbers.

1 Try Dakota's experiment. Was she correct that dividing first gives the same answers?

2 Will Dakota's method work for other fractions? Try some other fractions and input numbers. Show your experiments.

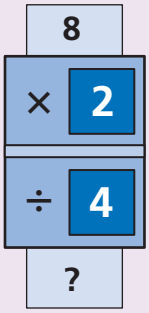


Chapter 4

Lesson 2

REVIEW MODEL
Using Fraction Machines to Multiply and Divide

You can use a model to help you investigate the order of operations for multiplication and division.



First multiply, then divide.

8

8 × 2

(8 × 2) ÷ 4

First divide, then multiply.

8

8 ÷ 4

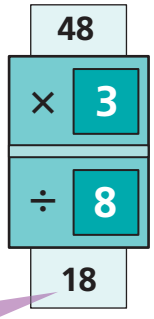
(8 ÷ 4) × 2

You can multiply and then divide, or you can divide and then multiply. The outcomes are the same.

Example 1

The input, 48, is a large number, so you may want to divide first.

48 ÷ 8 = 6
6 × 3 = 18

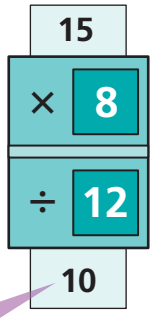


The output is 18.

Example 2

The denominator is not a factor of the input, so you may want to multiply first.

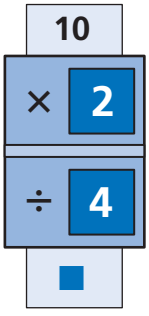
15 × 8 = 120
120 ÷ 12 = 10

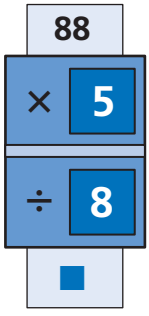


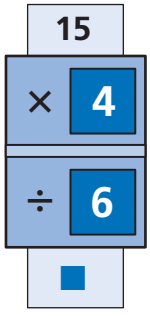
The output is 10.

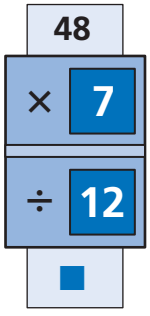
Check for Understanding

Multiply and then divide. Or, divide and then multiply. Write the outputs.

1. 

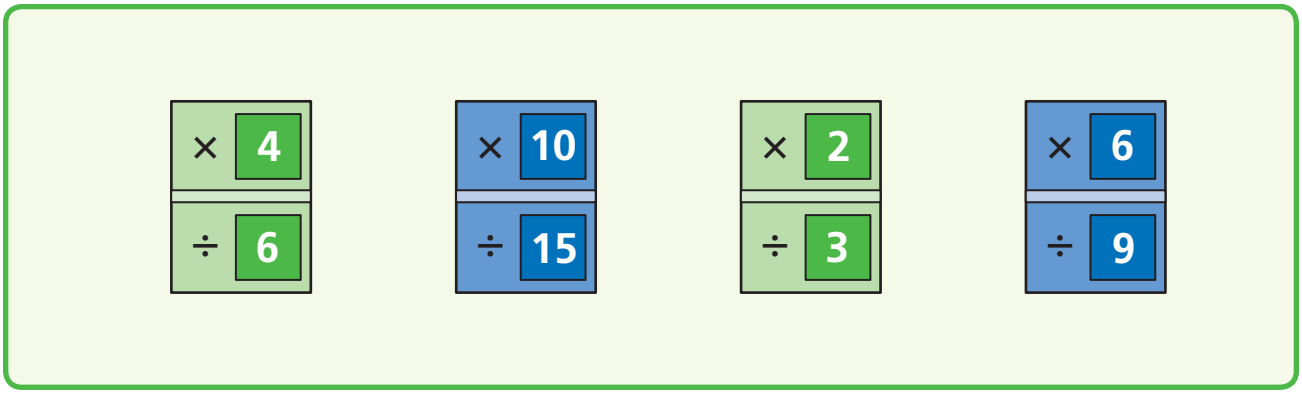
2. 

3. 

4. 

EXPLORE
Fraction Machine Experiment

Here are four fraction machines.



Try this experiment. Be prepared to discuss your results.

1. Choose three multiples of 3 as input numbers.

2. Find out what happens when you put your first number through all four machines.

3. Do the same with your other two numbers.

Chapter 4

Lesson 4

REVIEW MODEL
Writing Equivalent Fractions

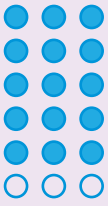
You can use dot sketches to help you write equivalent fractions.

Example 1 **Use dot sketches to find fractions equivalent to $\frac{5}{6}$. Note that all columns for equivalent fractions must be shaded the same way.**

Draw a column of 6 open dots. Shade 5 of the dots to represent $\frac{5}{6}$.

Draw two columns of dots, each exactly like the first one to show $\frac{10}{12}$.

Draw three columns of dots, each exactly like the first one to show $\frac{15}{18}$.



$$\frac{5}{6} = \frac{10}{12} = \frac{15}{18}$$

The numerator shows the number of shaded dots.

The denominator shows the total number of dots.

You can also multiply or divide to write equivalent fractions.

Example 2 Multiply the numerator and the denominator by the same number to write fractions equivalent to $\frac{3}{5}$.

$$\frac{3}{5} = \frac{6}{10} = \frac{12}{20}$$

Multiply both the numerator and denominator of $\frac{3}{5}$ by 2.

Multiply both the numerator and denominator of $\frac{3}{5}$ by 4.

Example 3 Divide the numerator and the denominator by the same number to write fractions equivalent to $\frac{6}{12}$.

$$\frac{6}{12} = \frac{2}{4} = \frac{3}{6}$$

Divide both the numerator and denominator of $\frac{6}{12}$ by 3.

Divide both the numerator and the denominator of $\frac{6}{12}$ by 2.

Check for Understanding

Write two equivalent fractions for each fraction. Explain how you found the equivalent fractions.

1. $\frac{1}{6}$
2. $\frac{4}{16}$
3. $\frac{3}{10}$
4. $\frac{3}{8}$
5. $\frac{3}{4}$
6. $\frac{15}{20}$

Chapter 4

Lesson 6

REVIEW MODEL
Comparing Fractions

There are many different strategies you can use to compare fractions. For some pairs of fractions these strategies will work.

A **If the denominators are the same, compare the numerators.**

$\frac{12}{16} > \frac{10}{16}$

Bigger fraction has bigger numerator.

B If the numerators are the same, compare the denominators.

$\frac{3}{8} > \frac{3}{12}$

Smaller fraction has bigger denominator.

C Compare each fraction to $\frac{1}{2}$. If one is bigger than $\frac{1}{2}$ and other is smaller, it's easy to compare.

$\frac{4}{6} > \frac{3}{8}$

bigger than $\frac{1}{2}$

smaller than $\frac{1}{2}$

D Figure out which fraction is closer to 1. If both are less than 1, then the fraction closer to 1 is bigger.

$\frac{15}{16} > \frac{3}{9}$

$\frac{15}{16}$ is closer to 1.

If the strategies above do not help you, try this strategy.

E Write an equivalent pair of fractions with a common denominator. Then, compare the numerators.

15 is a common denominator.

$\frac{3}{5} \blacksquare \frac{2}{3}$

$\frac{9}{15} < \frac{10}{15}$

$\frac{10}{15}$ is equivalent to $\frac{2}{3}$.

$\frac{9}{15}$ is equivalent to $\frac{3}{5}$.

so $\frac{3}{5} < \frac{2}{3}$

12 is a common denominator.

$\frac{5}{6} \blacksquare \frac{3}{4}$

$\frac{10}{12} > \frac{9}{12}$

$\frac{9}{12}$ is equivalent to $\frac{3}{4}$.

$\frac{10}{12}$ is equivalent to $\frac{5}{6}$.

so $\frac{5}{6} > \frac{3}{4}$

Check for Understanding

Copy and compare. Write <, >, or = between each pair of fractions.

1. $\frac{15}{16} \bullet \frac{12}{15}$
2. $\frac{3}{8} \bullet \frac{1}{2}$
3. $\frac{7}{8} \bullet \frac{9}{12}$
4. $\frac{3}{16} \bullet \frac{3}{18}$
5. $\frac{3}{4} \bullet \frac{6}{8}$
6. $\frac{5}{12} \bullet \frac{7}{12}$
7. $\frac{2}{3} \bullet \frac{4}{5}$
8. $\frac{6}{9} \bullet \frac{4}{6}$

Chapter 4

Lesson 8

EXPLORE
Fractions Greater Than 1

Solve the problems.

Michaela made a number line showing her age and the ages of her two brothers. Michaela is $10\frac{1}{2}$, her younger brother is 8, and her older brother is $11\frac{1}{2}$.

1. Draw a number line to show Michaela and her brothers' ages.

Justin's mother baked some cookies and then cut each in half. She told him that he could only eat three of the halves before supper.

2. How many cookies was Justin allowed to eat before supper?

3. If Justin ate the three cookie halves before supper and two more cookie halves for dessert, how many cookies did he eat?

Chapter 4

Lesson 8

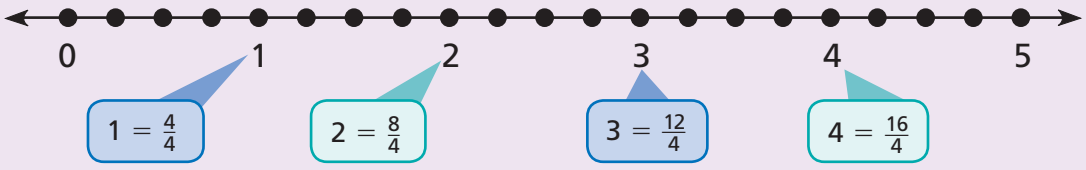
REVIEW MODEL
Mixed Numbers and Improper Fractions

Mixed numbers and improper fractions are two different forms of numbers that are greater than 1 but are not whole numbers.

You can convert back and forth between the two forms.

You can use a number line to help you convert improper fractions to mixed numbers.

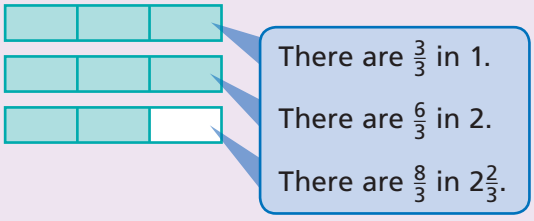
Example 1 Find a mixed number for $\frac{15}{4}$.



So, $\frac{15}{4}$ is between 3 and 4. You can write $\frac{15}{4}$ as $3\frac{3}{4}$.

You can use sketches to help you convert mixed numbers to improper fractions.

Example 2 Find an improper fraction for $2\frac{2}{3}$.



So, you can rewrite $2\frac{2}{3}$ as $\frac{8}{3}$.

Check for Understanding

Write an improper number for each.

1. $2\frac{1}{6}$
2. $1\frac{1}{5}$
3. $2\frac{3}{4}$
4. $3\frac{4}{7}$
5. $1\frac{15}{16}$
6. $3\frac{5}{6}$

Write a mixed number for each.

7. $\frac{7}{5}$
8. $\frac{13}{10}$
9. $\frac{11}{4}$
10. $\frac{9}{2}$
11. $\frac{11}{3}$
12. $\frac{23}{16}$

Chapter 4

Lesson 10

REVIEW MODEL

Problem Solving Strategy

Draw a Picture

Last week Caleb and Isabelle each earned the same amount of money doing yard work. Caleb spent $\frac{3}{4}$ of his money and Isabelle spent $\frac{2}{3}$ of her money. Who spent more money?

Strategy: Draw a Picture

Read to Understand

What do you know from reading the problem?

Both people earned the same amount of money. Caleb spent $\frac{3}{4}$ of his money and Isabelle spent $\frac{2}{3}$ of hers.

What do you need to find out?

who spent more money

Plan

How can you solve this problem?

I can use the strategy *draw a picture* to help me find out who spent more money.

Solve

How can you *draw a picture* to solve the problem?

I can make two dot sketches and shade the dots for the fractions. Then I can compare the number of shaded dots to see who spent more.

Caleb $\frac{3}{4}$

Isabelle $\frac{2}{3}$

$\frac{3}{4} > \frac{2}{3}$

So, Caleb spent more money than Isabelle.

Check

Look back at the problem. Did you answer the questions that were asked? Does the answer make sense?

Problem Solving Practice

Draw a picture to solve.

1. Ashley and Andy each order a small pizza. Ashley eats $\frac{2}{3}$ of hers. Andy eats $\frac{3}{5}$ of his. Who ate more pizza?

2. Katie put 2 dozen flowers in a vase. $\frac{1}{2}$ of the flowers were daisies and $\frac{3}{8}$ of the flowers were roses. How many daisies and how many roses were in the vase?

Problem Solving Strategies

- ✔ Act It Out
- ✔ **Draw a Picture**
- ✔ Guess and Check
- ✔ Look for a Pattern
- ✔ Make a Graph
- ✔ Make a Model
- ✔ Make an Organized List
- ✔ Make a Table
- ✔ Solve a Simpler Problem
- ✔ Use Logical Reasoning
- ✔ Work Backward
- ✔ Write an Equation

Mixed Strategy Practice

Use any strategy to solve. Then explain what strategy you used and how you solved the problem on a separate piece of paper.

3. Drew's goal is to practice the piano for 3 hours each week. He practiced for 40 minutes on Monday and 35 minutes on Tuesday. How much longer does he have to practice this week to meet his goal?

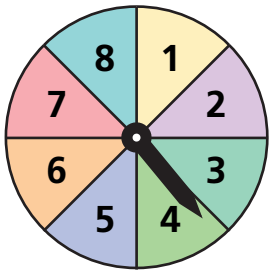
4. Kyle paid $10.50 for one adult ticket and one student ticket to a play. The adult ticket cost $2.50 more than the student ticket. How much did each ticket cost?

5. Ms. Blackwell deposited two checks in her checking account, one for $250 and one for $100. Later that day she withdrew $90 at an ATM. At the end of the day the balance in her account was $580. What was her balance at the beginning of the day?

6. Raul can choose from these pizza crusts and toppings.
Crusts: thick, thin
Toppings: mushrooms, sausage, peppers, onions, pepperoni, ham.

How many different choices of crust and one topping does he have?

Use the spinner for 7–10. Suppose you spin the pointer on this spinner. Write *impossible*, *unlikely*, *likely*, or certain to describe each event.



7. Landing on a square number is ___?___.

8. Landing on a multiple of 3 greater than 6 is ___?___.

9. Landing on a number greater than 2×1, but less than 2×5 is ___?___.

10. Landing on an even number or an odd number is ___?___.

Chapter 4 Vocabulary

Choose the best vocabulary term from Word List A for each sentence.

1. The ___?___ is the bottom number of a fraction.
2. A(n) ___?___ has both a fraction and a whole number.
3. The fractions $\frac{1}{4}$ and $\frac{3}{4}$ have a(n) ___?___.
4. A(n) ___?___ for $5\frac{1}{2}$ is $5\frac{6}{12}$.
5. When you put a number through a fraction machine, the machine gives you a(n) ___?___.
6. Equivalent fractions are ___?___ each other.
7. A(n) ___?___ has a numerator that is greater than the denominator.
8. A(n) ___?___ is an array that you can use to show equivalent fractions.

Word List A

- common denominator
- denominator
- dot sketch
- equal to
- equivalent
- equivalent mixed number
- greater than
- improper fraction
- input
- less than
- mixed number
- numerator
- output
- part of a whole
- simplest form

Complete each analogy using the best term from Word List B.

9. Equal to is to $=$ as ___?___ is to $>$.
10. Top is to bottom as ___?___ is to denominator.
11. Fraction is to equivalent fraction as ___?___ is to equivalent mixed number.

Word List B

- greater than
- improper fraction
- less than
- numerator
- mixed number

Talk Math

Discuss with a partner what you have just learned about comparing fractions. Use the vocabulary terms *numerator* and *denominator*.

12. How can you tell when two unlike fractions are equivalent?
13. How can you tell when a fraction is in simplest form?
14. How can you tell when a fraction is an improper fraction?

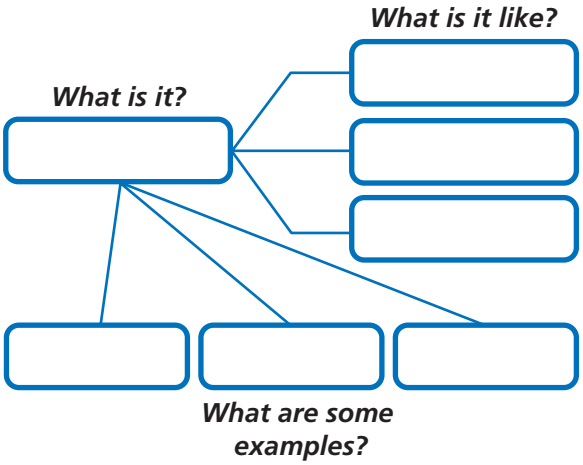
Word Definition Map

15 **Create a word definition map for the term *equivalent*.**

A What is it?

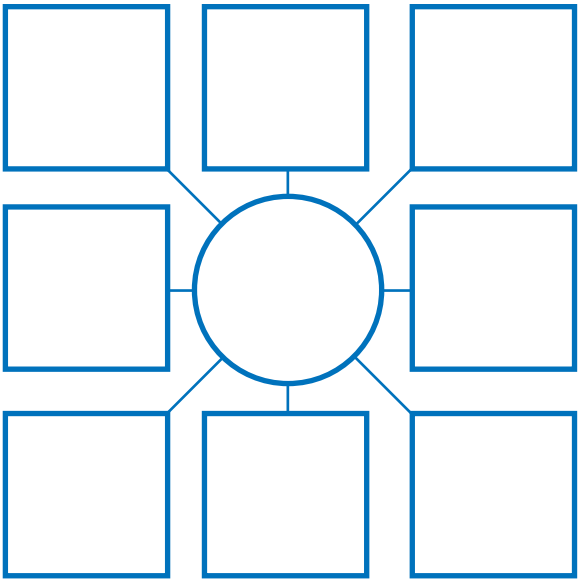
B What is it like?

C What are some examples?



Concept Map

16 **Use what you know and what you have learned about fractions to create a concept map for terms related to *fraction*.**



COMMON DENOMINATOR A *common denominator* is a trait or theme that different people or things have in common. A *common denominator* among the different members of a chess club is that they all like to play chess. A *common denominator* among tigers is that they have stripes. In math, a *common denominator* is the bottom number of a fraction that is the same for several different fractions.

GO ONLINE **Technology**
Multimedia Math Glossary
www.harcourtschool.com/thinkmath

GAME

Fraction Action

Game Purpose
To practice comparing fractions

Materials
- Activity Master 27: Fraction Cards 1
- scissors

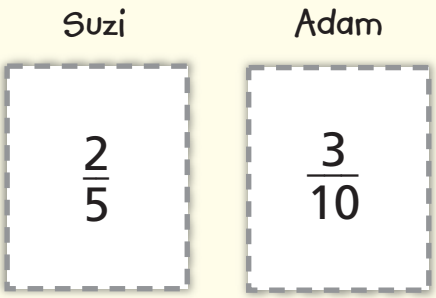
How To Play The Game

1. Play the game with a partner. Cut out the fraction cards on Activity Master 27. Decide together whether you want the bigger or the smaller fraction to be the winner in each round.

2. One player mixes up the cards and gives 8 cards to each player. Put your cards face down in a pile.

3. Both players turn over their top card and compare the fractions. The player with the bigger (or smaller) fraction takes both cards and sets them aside.

Example: The bigger fraction wins. These are the first 2 cards.



Suzi's fraction is bigger, so she takes both cards.

4. After both players have turned over their 8 cards, they mix up the cards they have won and keep playing.

5. There are two ways to win.
   - You can win if you collect all 16 cards.
   - You can win if you have more cards than your partner when time is called.

GAME

Fractiontration

Game Purpose

To practice identifying equivalent fractions

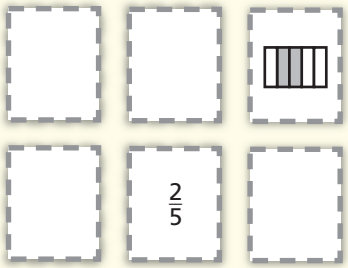
Materials

- Activity Master 27–28: *Fraction Cards*
- Activity Master 29–30: *Fraction Model Cards*
- scissors

How To Play The Game

1. Play this game with a partner. Cut out all of the cards from Activity Masters 27–30.
   - Mix up all the cards, and place them face down in a pile.
   - Take the top 20 cards from the stack. Place them face down in 4 rows of 5 cards.
2. The goal is to find equivalent fraction pairs. Look for 2 cards that have:
   - equivalent written fractions.
   - equivalent shaded models.
   - a written fraction and a shaded model that are equivalent.
3. Take turns. One player turns over 2 cards.
   - If they show equivalent fractions, take them. Replace those cards with 2 cards from the pile, face down.
   - If they do not match, return the 2 cards to their places, face down.
   - Then it's the other player's turn.

Example:



You turn over these two cards.

The written fraction and the shaded model show equivalent fractions.

So, you take those cards and replace them with 2 cards from the pile.

Now it's your partner's turn.

4. The winner is the player who has more cards at the end of the game.

CHALLENGE

Fraction Maze

Can you find a path through the maze? Begin at "Start" and end at "Finish". Here are the rules you must follow!

- **You can only move from left to right.**
- **You can only move only from top to bottom.**
- **You can move horizontally, vertically, or diagonally.**
- **You can only move from a smaller fraction to a larger one.**

The two orange fractions, $\frac{7}{28}$ and $\frac{1}{2}$, are hints for you.

Start

$\frac{1}{20}$	$\frac{1}{18}$	$\frac{5}{19}$	$\frac{3}{15}$	$\frac{3}{8}$	$\frac{7}{12}$	$\frac{3}{15}$	$\frac{1}{10}$	$\frac{6}{15}$	$\frac{1}{20}$
$\frac{6}{21}$	$\frac{1}{2}$	$\frac{3}{15}$	$\frac{1}{2}$	$\frac{9}{20}$	$\frac{1}{10}$	$\frac{1}{2}$	$\frac{7}{28}$	$\frac{1}{18}$	$\frac{9}{20}$
$\frac{1}{2}$	$\frac{9}{20}$	$\frac{7}{28}$	$\frac{1}{20}$	$\frac{2}{3}$	$\frac{6}{21}$	$\frac{1}{10}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{7}{12}$
$\frac{6}{15}$	$\frac{7}{12}$	$\frac{2}{3}$	$\frac{5}{19}$	$\frac{6}{21}$	$\frac{3}{8}$	$\frac{1}{20}$	$\frac{7}{9}$	$\frac{1}{6}$	$\frac{5}{19}$
$\frac{9}{20}$	$\frac{3}{15}$	$\frac{1}{2}$	$\frac{7}{9}$	$\frac{7}{12}$	$\frac{2}{9}$	$\frac{6}{15}$	$\frac{5}{40}$	$\frac{3}{8}$	$\frac{1}{10}$
$\frac{7}{9}$	$\frac{1}{10}$	$\frac{6}{15}$	$\frac{1}{20}$	$\frac{2}{3}$	$\frac{5}{19}$	$\frac{7}{12}$	$\frac{9}{20}$	$\frac{1}{5}$	$\frac{4}{15}$
$\frac{6}{21}$	$\frac{5}{40}$	$\frac{3}{8}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{6}{15}$	$\frac{1}{18}$	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{9}{20}$
$\frac{1}{6}$	$\frac{1}{20}$	$\frac{9}{20}$	$\frac{1}{10}$	$\frac{3}{8}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{10}$	$\frac{7}{12}$	$\frac{4}{15}$
$\frac{7}{12}$	$\frac{9}{20}$	$\frac{2}{9}$	$\frac{1}{18}$	$\frac{7}{9}$	$\frac{1}{5}$	$\frac{4}{15}$	$\frac{7}{28}$	$\frac{2}{3}$	$\frac{1}{20}$
$\frac{5}{19}$	$\frac{1}{10}$	$\frac{3}{15}$	$\frac{2}{3}$	$\frac{6}{21}$	$\frac{1}{10}$	$\frac{3}{15}$	$\frac{1}{6}$	$\frac{3}{8}$	$\frac{7}{9}$

Finish