Chapter

5 Recording Multi-Digit Multiplication

Dear Student,

In Chapter 2 you used area models and puzzles to make multiplication of 2-digit numbers easier. To multiply two large numbers, you broke each factor into two smaller numbers and multiplied the four smaller numbers like this.

	20	4
20	400	80
9	180	36

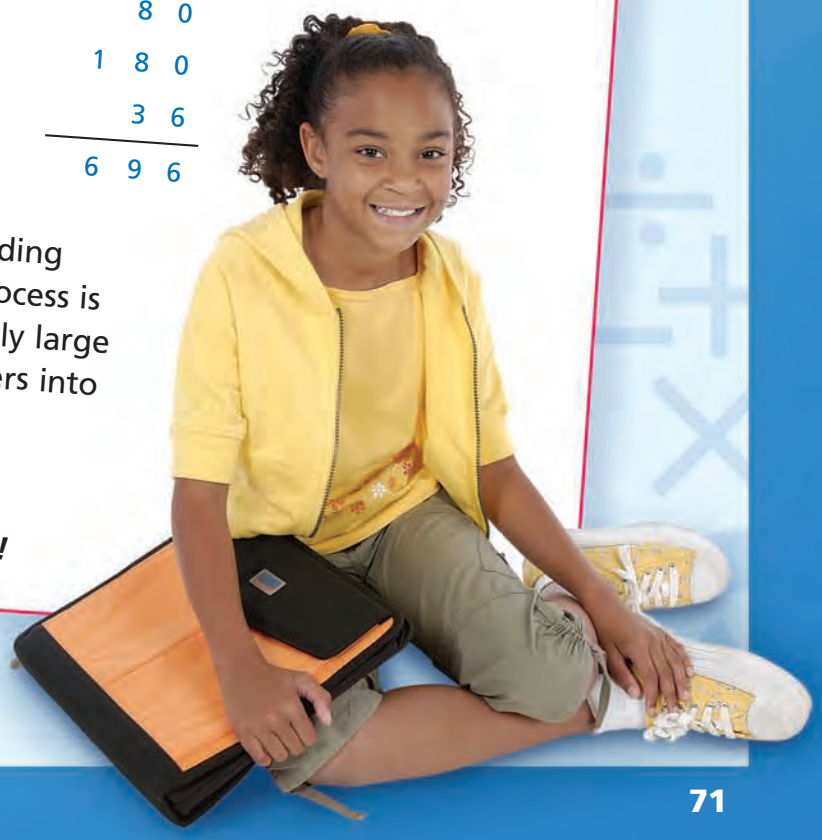
×	20	4	24
20	400	80	480
9	180	36	216
29	580	116	696

In this chapter, "Recording Multi-Digit Multiplication," you will multiply as you have before, but use a more compact way to record your work.

```
   2 4
 × 2 9
-------
   4 0 0
     8 0
 1 8 0
     3 6
-------
 6 9 6
```

You will also investigate other ways to record the step-by-step process that you use to multipy large numbers. Whichever recording method you prefer, the process is always the same: to multiply large numbers, break the numbers into smaller ones!

Mathematically yours,
The authors of ***Think Math!***



The Bear Facts

Admiralty Island is a thickly-forested and remote area in Alaska. Nine-tenths of the island is preserved as a national monument. All but two small sections of its 956,000 acres is designated as the *Kootznoowoo Wilderness*. *Kootznoowoo* means "Fortress of Bears" in the Native Alaskan language.

FACT·ACTIVITY 1

People travel to Admiralty Island to observe the wildlife. The Pack Creek Air Taxi Service provides transportation to the island. Its costs are listed below.

Pack Creek Air Taxi Service
Round trip transportation only.
Departure times to be arranged.

Must have your *own* permits and gear.
2 passengers: $350 per person
3 passengers: $240 per person
4 passengers: $185 per person
5 passengers: $152 per person

1. How much will the Air Taxi Service receive in fees if 2 people travel together? If 3 people travel together? If 4 people travel together?
2. Describe the pattern in your answers to Problem 1.
3. How much more will the Air Taxi Service receive if 5 passengers rather than 2 passengers travel together?
4. The cost of a tour during peak season (June, July, and August) is $49 per person. Find the cost of the tour for 4 people.
5. What is the total cost for air taxi service to Admiralty Island and the tour for 4 people?

FACT ACTIVITY 2

Alaska is home to almost all of the brown bears in the United States. The table shows the estimated brown bear population on some of the Alaskan islands.

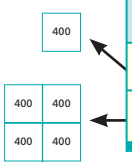
Approximate Brown Bear Population on Alaskan Islands

Alaskan Islands	Area (km^2)	Brown Bear Density (per 1,000 km^2)	Estimated Total Number of Bears
Chicagof	5,000	300	1,500
Baranof	4,000	200	800
Admiralty	4,300	400	

1. Look at the numbers in each row. Explain a way to calculate the total number of bears from the area and the density.
2. Write an expression to calculate the total number of bears on Admiralty Island. Estimate the population of brown bears.

CHAPTER PROJECT

Admiralty Island is also home to many bald eagles, with about one eagle per 400 acres. Suppose a larger, nearby island has about one eagle per 400 acres. Copy the diagrams below; then make a 3-square by 3-square diagram and a 4-square by 4-square. Extend the table to record the data represented by the diagrams.



Eagle Population

Area (square acres)	Number of Eagles
400	1
1,600	■

- Your diagrams and the numbers in the table should show the number of eagles in various numbers of acres.
- Describe the pattern of the numbers in the Number of Eagles column.
- Describe the pattern you see in the Area column.
- Extend the diagrams and the table further by adding two more rows.

ALMANAC Fact

About 1,700 brown bears, some more than 9 feet tall, live on Admiralty Island in Alaska. Bears outnumber the human population by 3 to 1.

Chapter 5

Lesson 3

EXPLORE
Exploring Records

1 As you solve this problem, think about the order in which you want to find the FOUR partial products. You may draw an area model if you wish.

$$\begin{array}{r} 87 \\ \times\ 54 \\ \hline \end{array}$$

2 Mari did the problem by finding two partial products, like this.

$$\begin{array}{r} 87 \\ \times\ 54 \\ \hline 4350 \\ 348 \\ \hline 4,698 \end{array}$$

A Where did the two partial products, 4,350 and 348 come from?

B If Mari uses her "two partial products" method, how could she write a record for this multiplication?

$$\begin{array}{r} 48 \\ \times\ 72 \\ \hline \end{array}$$

Chapter 5

Lesson 3

REVIEW MODEL
Strategies for Multiplying

You can use area models, puzzles, and vertical records to help you simplify a multiplication problem.

The example below shows these three strategies and how they can be used to find 35 × 24.

Example **Find 35 × 24.**

One Way

Area Model

	30	5
20	600	100
4	120	20

Puzzle

×	30	5	35
20	600	100	700
4	120	20	140
24	720	120	840

Vertical Record

35
× 24
600
120
100
20
840

A Shorter Way

Area Model

	35
20	700
4	140

Puzzle

×	35	
20	700	→ 700
4	140	→ 140
24	840	→ 840

Vertical Record

35
× 24
700
140
840

Check for Understanding

Find the product. Use area models, puzzles, or vertical records. Show your work.

1. 24 × 17
2. 28 × 16
3. 33 × 21
4. 45 × 25

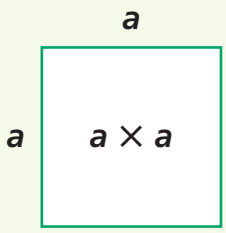
Chapter 5

Lesson 4

EXPLORE
Floor Tiling

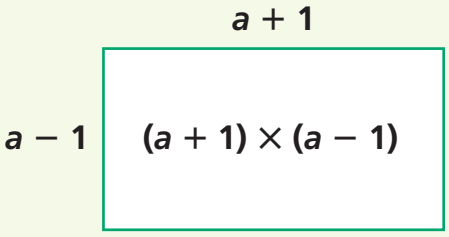
Marcus thought that the floor he wanted to cover with square tiles was a square.

He drew this sketch.



However, when Marcus measured the sides of the floor, he found out that the length of the room was a foot longer than he first thought, and the width of the room was a foot shorter.

He drew a new sketch.



1. Try some numbers for a to decide if Marcus would have ordered enough square tiles if he based his decision on his first sketch ($a \times a$).

2. What numbers did you try?

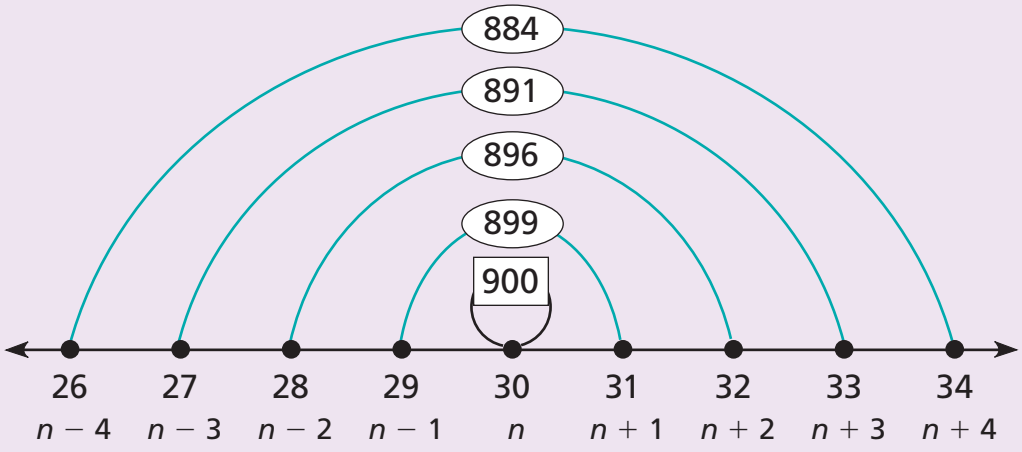
Chapter 5

Lesson 4

REVIEW MODEL
Using Patterns in Square Number Differences

You can use patterns in square number difference to help you multiply large numbers.

Look at the products for the pairs of factors that are 1, 2, 3, and 4 steps away from 30. Then look at the table to see how the expressions with numbers and the expressions with letters are related.



n^2 means "n squared" or $n \times$ n.

	Expressions with Numbers	Expressions with Letters
Square Number	$30 \times 30 \rightarrow 900$	$n \times n \rightarrow n^2$
• 1 step away	$29 \times 31 \rightarrow 900 - 1$, or 899	$(n - 1) \times (n + 1) \rightarrow n^2 - 1$
• 2 steps away	$28 \times 32 \rightarrow 900 - 4$, or 896	$(n - 2) \times (n + 2) \rightarrow n^2 - 4$
• 3 steps away	$27 \times 33 \rightarrow 900 - 9$, or 891	$(n - 3) \times (n + 3) \rightarrow n^2 - 9$
• 4 steps away	$26 \times 34 \rightarrow 900 - 16$, or 884	$(n - 4) \times (n + 4) \rightarrow n^2 - 16$

Check for Understanding

Use a pattern to help you write the products

1. 40×40
   39×41

2. 50×50
   48×52

3. 25×25
   22×28

4. $n = 10$
   $n \times n$
   $(n - 1) \times (n + 1)$

5. $a = 20$
   $a \times a$
   $(a - 2) \times (a + 2)$

6. $d = 12$
   $d \times d$
   $(d - 3) \times (d + 3)$

Chapter 5

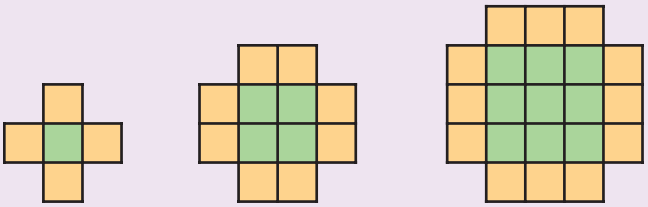
Lesson 6

REVIEW MODEL

Problem Solving Strategy

Make a Table

Kalista made these designs with green and orange square tiles.



Which of these expressions give the number of orange tiles needed for a design with $n \times n$ green tiles?

$2n + 2n$ | $4n$ | $2n + 2$

Strategy: Make a Table

Read to Understand

What do you know from reading the problem?

Kalista made a pattern of designs with green and orange tiles.

What do you need to find out?

which expressions show the number of orange tiles needed for a design with $n \times n$ green tiles

Plan

How can you solve this problem?

I can use the strategy *make a table* to give me an organized way to test the expressions for various values for n.

Solve

How can you *make a table* to solve the problem?

I can use the expressions as pattern indicators and see if they give numbers that match the number of orange tiles in the designs for various values for n.

So, $2n + 2n$ and $4n$ are correct expressions.

Expression	Values for n			Yes or No?
	1	2	3	
$2n + n$	4	8	12	Yes
$4n$	4	8	12	Yes
$2n + 2$	4	6	8	No

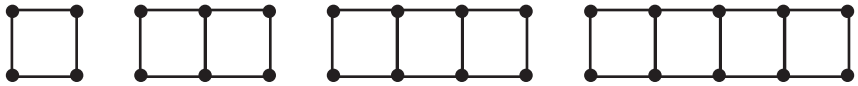
Check

Look back at the problem. Did you answer the questions that were asked? Does the answer make sense?

Problem Solving Practice

Make a table to solve.

1. Lauren used dot paper to make a pattern with squares.



Which expessions give the number of dots needed to make n squares?

$2 \times (n + 1)$ $2n + 1$ $2n + 2$

2. Josh saved two pennies on Day 1, four pennies on Day 2, and eight pennies on Day 3. If he continues to double the number of pennies he saves each day, how many pennies will he save on the tenth day?

Problem Solving Strategies

- ✔ Act It Out
- ✔ Draw a Picture
- ✔ Guess and Check
- ✔ Look for a Pattern
- ✔ Make a Graph
- ✔ Make a Model
- ✔ Make an Organized List
- ✔ **Make a Table**
- ✔ Solve a Simpler Problem
- ✔ Use Logical Reasoning
- ✔ Work Backward
- ✔ Write an Equation

Mixed Strategy Practice

Use any strategy to solve. Explain.

3. Tanner's dad builds 3-legged stools and 4-legged tables. He used 40 legs to build 4 more stools than tables. How many stools and tables did he build?

4. Jason reads for a half-hour on Monday. Each day he reads for 10 minutes more than the previous day. How long does Jason read on Friday?

5. There are 40 rows with 36 seats each and 20 rows with 25 seats each in the auditorium. How many seats are in the auditorium?

6. A factory produces 56 machines each week. How many machines does it produce in a year?

For 7–10, use the menu.

Today's Specials	
pizza slice	$2.50
garden salad	$3.75
burrito	$2.25
lemonade	$0.95

7. Marliss bought a slice of pizza and two burritos. How much did she spend?

8. Glenn has $4.00. Can he buy a garden salad and lemonade?

9. Sheli bought 5 burritos. She gave the clerk a $20-dollar bill. How much change should she receive?

10. The snack bar owner has lots of extra pizza slices and decides to sell them for half price. How much will 10 half-price slices of pizza cost?

Chapter 5 Vocabulary

Choose the best vocabulary term from Word List A for each sentence.

1. A(n) ___?___ is the part of the mathematical expression that can change.
2. In a(n) ___?___ for multiplication, the factors are outside the rectangle, and the partial products are inside it.
3. In ___?___, the factors are in the top portion, and the total product is at the bottom.
4. When two partial products are added, the result is called a ___?___.
5. In 4^3, the 3 is called the ___?___.
6. ___?___ is a kind of mathematical shorthand.

Word List A

- algebraic notation
- area model
- combined partial product
- exponent
- variable
- vertical records

Complete each analogy using the best term from Word List B.

7. A number-line model is to addition as a(n) ___?___ is to multiplication.
8. Addend is to addition as ___?___ is to multiplication.
9. Repeated addition is to multiplication symbol as repeated multiplication is to ___?___.

Word List B

- area model
- exponent
- factor

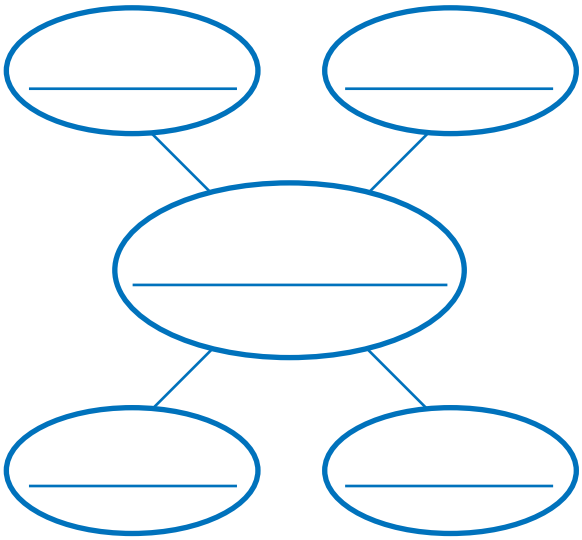
Talk Math

Use the vocabulary term *area model* to discuss with a partner what you have learned about multiplying 2-digit numbers.

10. How can you multiply 2-digit numbers?
11. How do the numbers in area models relate to the numbers in vertical records?
12. How can you square a 2-digit number?

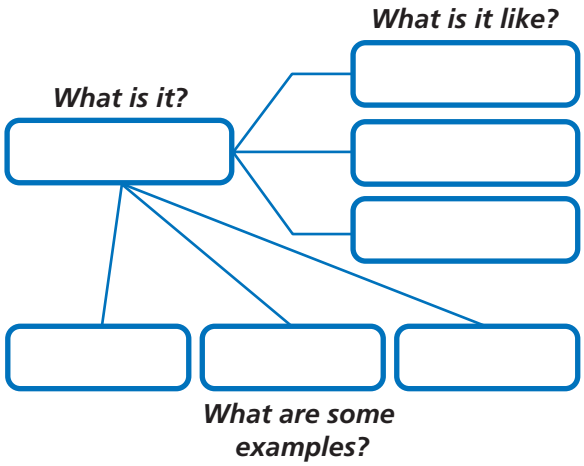
Concept Map

13 **Create a concept map for words associated with multiplication. Use the terms *exponent, area model, vertical records,* and *combined partial product.***



Word Definition Map

14 **Create a word definition map using the term *algebraic notation.***



What's in a Word?

VARIABLE The word *variable* is a combination of the words *vary* and *able*. Vary means "to change." *Able* means "to have enough power, skill, or resources to do something." So, a *variable* is something that can change. *Variable* has the same meaning when it is used as an adjective in everyday language. The weather is *variable* because the temperature or amount of sunshine can change.

In mathematics, *variable* is a noun. A variable stands for a missing number. In arithmetic, it appears as an answer blank or a shape. In algebra, it appears as a letter. The value for the letter can change for every number sentence in which it is used. That's why it is called a *variable*.

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GAME

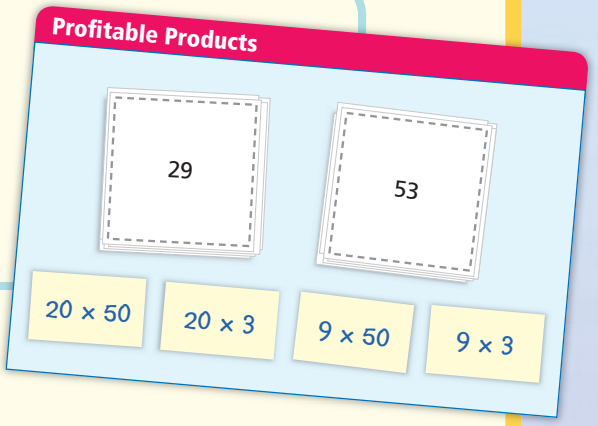
Profitable Products

Game Purpose

To practice multiplying 2-digit numbers

Materials

- Activity Masters 10–13: *Factor Search* Cards
- calculator



How To Play The Game

1. Play this game with a partner. Cut out all the *Factor Search Cards* that show 2-digit numbers. Decide who will play first.

2. Player 1 mixes up the cards and puts them face down in a pile. Player 2 turns over the top two cards.
   - Player 1 mentally computes one of the partial products and records it on paper. Since you earn more points for larger products, think before choosing.
   - Player 2 mentally computes a different partial product and records it on paper.
   - Take turns until all four partial products are listed.
   - Player 1 adds the partial products. Player 2 checks the product with a calculator. If the product is not correct, decide together which partial products are incorrect.

3. To find your score, add your partial products. Then add 100 points for each correct partial product.

4. Player 2 starts the next round by turning over the next two cards and choosing the first partial product. Play as many rounds as you can so that both players start a round an equal number of times. Then add up each player's score to find the winner.

GAME

Favorable Factors

Game Purpose

To form factors that give the greatest product

Materials

- Number cards 1–9
- Paper and pencil for each player

How To Play The Game

1. Play this game with a partner. The goal is to make the greater product.

2. Mix up the cards. Place them face down in a pile. Turn four number cards face up. Both players record all four numbers on their papers.
   - Secretly make two 2-digit factors from the four numbers.
   - Find the product of your two factors without a calculator.
   - Compare your products.
   - The player with the greater product scores 1 point.

   Example: These four cards are turned face up.

2	3	7	4

Player 1	Player 2
Makes factors 43 and 72. $43 \times 72 = 3{,}096$	Makes factors 37 and 24. $37 \times 24 = 888$
$3{,}096 > 886$ Player 1 has the greater product. So, Player 1 scores 1 point.	

3. Mix up the cards again. Play until someone gets 10 points. If time is called, the player with more points wins.

   You can change the game two ways:
   - Turn over five cards. Make a 3-digit factor and a 2-digit factor.
   - Change the goal to making the lesser product.

CHALLENGE

Choose Your Numbers

Test your estimation and mental math skills. Use numbers from the chart to solve Problems 1–7. Use a number only once for each problem. Solve each problem *without* using paper and pencil or a calculator.

9	3	19	4
6	15	30	2
20	7	12	5
8	50	10	40

1. Choose two numbers whose product is the greatest number less than 100.
2. Choose two numbers whose product is the least number greater than 100.
3. Choose three numbers whose product is exactly 1,000.
4. Choose three numbers whose product is between 5,000 and 6,000.
5. Choose three numbers whose product is an odd number greater than 1,000 but less than 2,000.
6. Choose three numbers whose product is an even number greater than 2,000 but less than 4,000.
7. Choose three numbers whose product is the greatest number less than 1,000.

After you have solved all the problems, follow these steps for each problem:

Step 1 Check your work with a calculator. Have you met the conditions of the problem? For example, if the problem asks for a product greater than 100, check to be sure that your product is greater than 100. If it is not, try again.

Step 2 If you think you have met all the conditions, check to see whether any other numbers would be a better solution. If not, you have solved the problem.