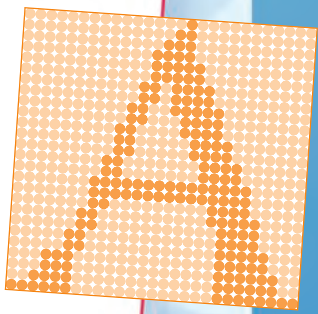
Chapter

6 Grids and Graphs

Dear Student,

A computer screen is a rectangular array of hundreds of thousands of little dots (called "pixels"). By lighting certain ones and leaving others dark, the computer creates a picture. The pixels are so small that we do not see dots, but just see the pattern that they make.



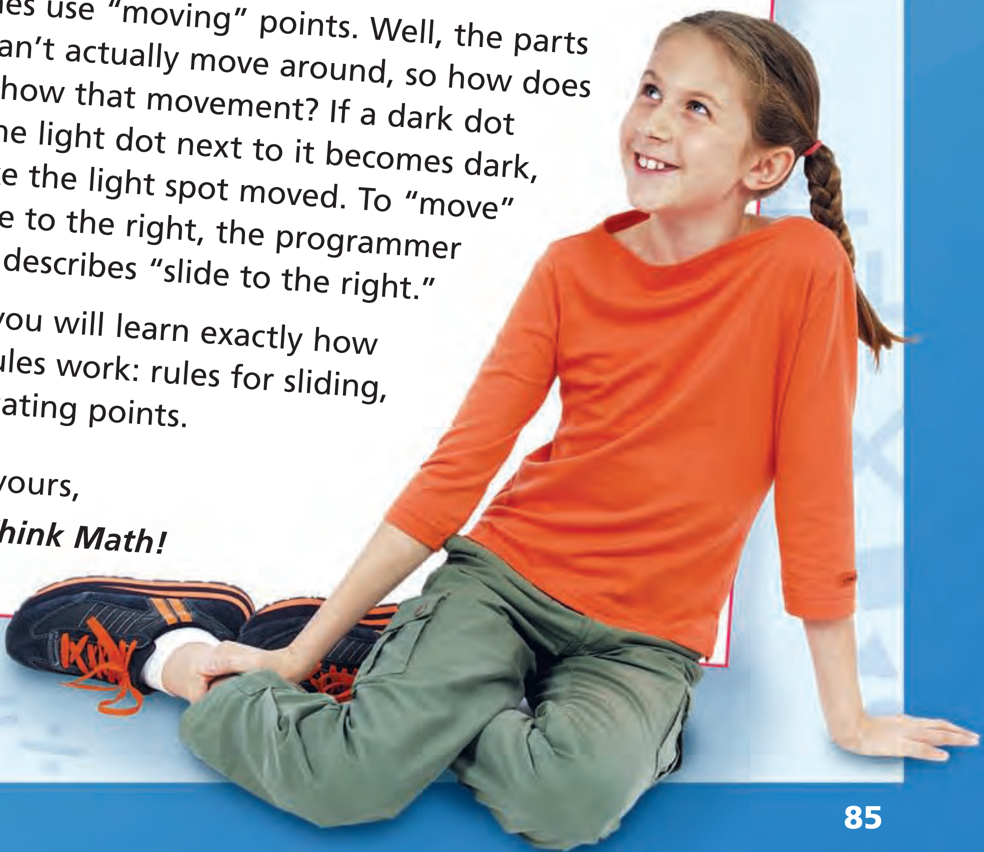
When we type, a set of instructions (a program) that is already in the computer, tells the computer exactly which pixels to change to make each letter appear on the screen. The designer of that set of instructions refers to each pixel by giving the coordinates of its location. The coordinates are two numbers: one to say how far left or right that point is from a special point called the origin, and another to say how far up or down that point is.

Computer games use "moving" points. Well, the parts of the screen can't actually move around, so how does the computer show that movement? If a dark dot lights up and the light dot next to it becomes dark, then it looks like the light spot moved. To "move" an entire picture to the right, the programmer uses a rule that describes "slide to the right."

In this chapter, you will learn exactly how some of these rules work: rules for sliding, reflecting, or rotating points.

Mathematically yours,

The authors of ***Think Math!***



THE WORLD ALMANAC FOR KIDS

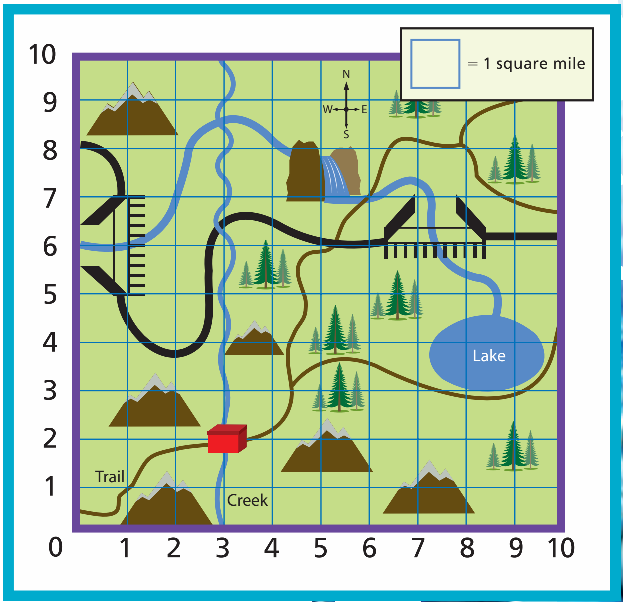
Treasure Hunting

Treasure hunters search for buried or underwater artifacts to learn about past cultures, to find valuable treasures, and to enjoy the adventure of discovery.

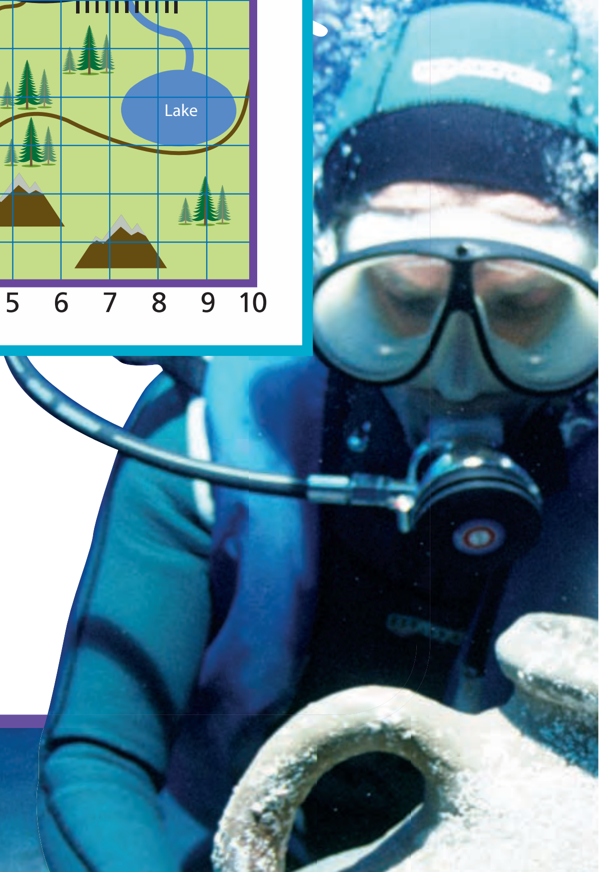
People who love adventures play a game called *geocaching*. The goal is to find hidden "caches," which are containers with objects placed by other players.

FACT·ACTIVITY 1

Try geocaching! Begin at the first cache located at (3,2) where the trail and the creek intersect.



1. A clue in the first cache says: The coordinates of where you stand are *(x,y)*. Your next cache is at *(x + 3, y + 3)*. Go! What are the coordinates of the second cache?
2. A map in the second cache points you to the third cache, a big rock under the waterfall. The coordinates of the third cache are (5,7). Explain how you get there from the second cache.
3. The third cache clue says: Connect the points of the 3 caches and reflect the figure over the creek. Go 2 miles south of the point where the figures meet to find the buried treasure. Where is the buried treasure?



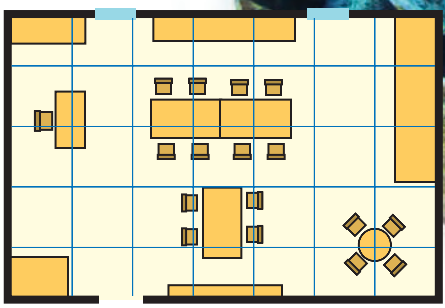
FACT • ACTIVITY 2

The *Senora de Atocha*, a Spanish ship, was lost at sea in 1622 near Key West, Florida. More than 350 years later, Mel Fisher, a treasure hunter, rediscovered the *Atocha's* cargo of gold and silver. Suppose Mel Fisher discovered these numbers of coins at different locations near Key West: 16, 31, 44, 25, 32, 15, 20, 17, 19, 22, 43, 29, 34, 17, and 11.

1. What is the range of the number of coins found at the fifteen locations? the mean? the median?
2. Explain how you might use these numbers to predict the number of coins at a sixteenth location.

CHAPTER PROJECT

Your class should be divided into 4 groups. On grid paper, each group draws a map of your classroom. Draw and label the *x*-axis and *y*-axis with positive and negative numbers so the origin is near the middle of the room.



- Design a treasure hunt for another group. Select the starting point and 3 more points. The fourth point will be the location of your treasure. Each point must be a location on your map where you can hide the coordinates for the next location. Write the coordinates for the 4 locations on a slip of paper and place each one in a sealed envelope.
- Hide 3 of the envelopes. Place a small treasure such as a candy bar in the last location. Exchange maps and the first envelope with another group.
- Time each group's treasure hunt. Make a graph of the times. Describe and analyze your results.

ALMANAC Fact

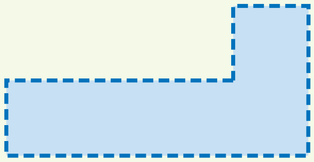
More than 40 tons of gold and silver worth about $400 million was recovered from the wreck of the *Senora de Atocha*.

Chapter 6

Lesson 2

EXPLORE
Introducing Translations

Katy is writing a computer game. This is the figure that will move across the screen.



1. Cut out the  at the bottom of Activity Master 35: Blank Grid with an L and place it over the  on the activity master.

2. Record the coordinates of its vertices on a piece of scratch paper.

3. From here, move the  3 spaces to the right (east). Record the new coordinates of its vertices on scratch paper. Describe how the coordinates have changed.

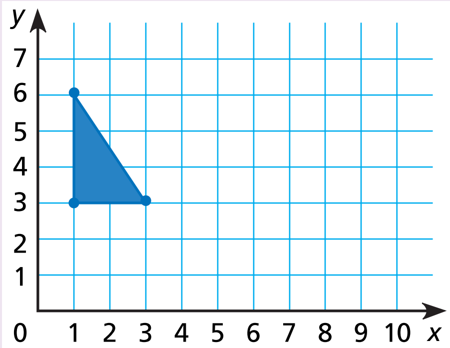
4. Move the  2 spaces down (south). Record the new coordinates of its vertices on scratch paper. Describe how the coordinates have changed.

Chapter 6

Lesson 2

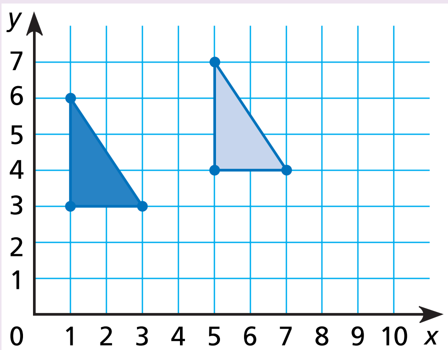
REVIEW MODEL
Translating a Figure

When you **translate** a figure you slide it in any direction without turning it or changing it in any other way.



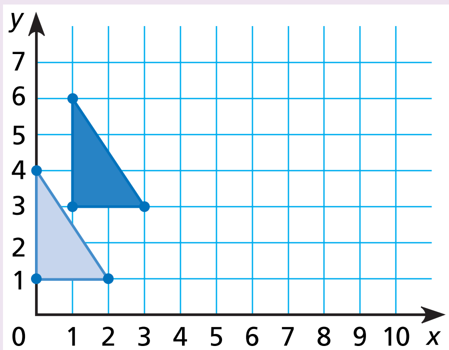
- Increasing the *y*-coordinate of a point moves the point up.
- Increasing the *x*-coordinate of a point moves the point to the right.
- If you move all the vertices of a figure the same amount and in the same direction, and connect the new vertices the same way the originals were connected, you have translated the entire figure.

Example 1 Translate the blue triangle 4 spaces to the right and 1 space up. What are the coordinates of the new triangle?



The coordinates of the new triangle are (5,4), (7,4), (5,7).

Example 2 Translate the blue triangle 1 space to the left and 2 spaces down. What are the coordinates of the new triangle?

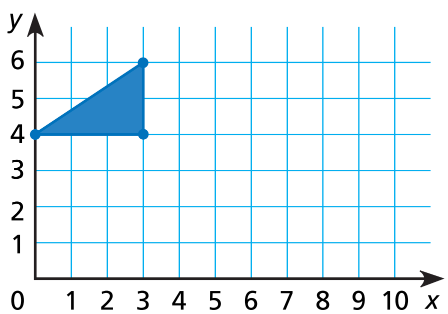


The coordinates of the new triangle are (0,1), (2,1), (0,4).

Check for Understanding

1. Copy the triangle onto the grid on AM34. Translate the triangle 2 spaces to the right and 3 spaces down. Draw the new triangle.

2. What are the coordinates of the vertices of the new triangle?



Chapter 6

Lesson 3

EXPLORE
Reflecting to Create Symmetrical Figures

Part A Create a symmetric design from an asymmetric one.

1. Choose one of the four figures on Activity Master 36: Asymmetrical Figures, color it or shade it, and then carefully cut it out.

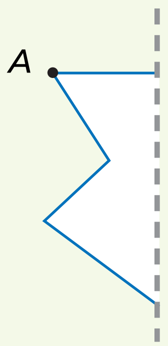
2. On a piece of plain, unlined paper, carefully trace around your figure.

3. Trace the figure a second time so that you have made a symmetric design.

Part B Look at this figure and imagine it reflected across the dotted vertical line.

1. Think about where the reflection of Point *A* will be located.

2. Be prepared to tell how you came up with that prediction.

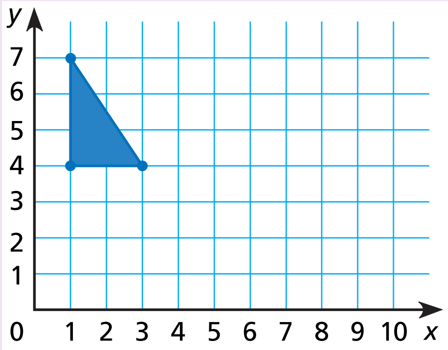


REVIEW MODEL
Reflecting a Figure

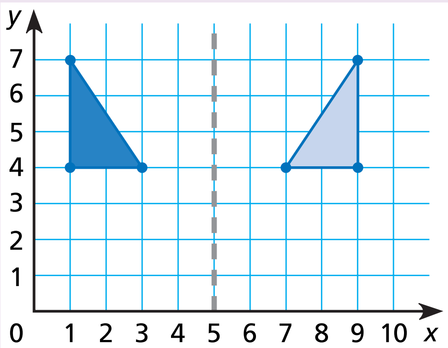
When you **reflect** a figure across a line you make a mirror image of the figure as if the line were a mirror.

- Each vertex in the reflection is the same distance from the 'mirror' line as the corresponding vertex in the original figure.

This blue triangle has vertices at (1,4), (3,4), (1,7).

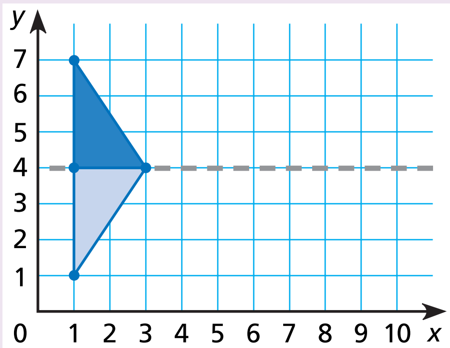


Example 1 Reflect the blue triangle over the dotted vertical line. How far are the two right-angle vertices from the mirror line?



Both right-angle vertices are 4 units from the mirror line.

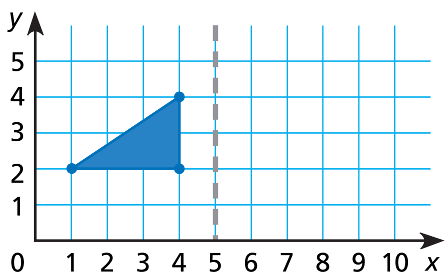
Example 2 Reflect the blue triangle over the dotted horizontal line. How far are the two right-angle vertices from the mirror line?



Both right-angle vertices are on the mirror line.

Check for Understanding

1. Copy the triangle onto the grid on AM34. Reflect the triangle over the dotted line. Draw the new triangle.
2. How far are the two right-angle vertices from the mirror line?

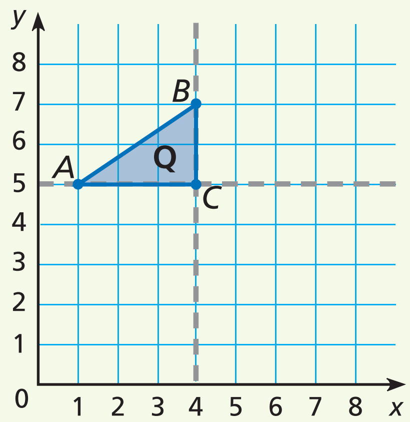


Chapter 6

Lesson 4

EXPLORE
Reflections of Reflections

Katy is making a background design for her computer game from copies of this figure. She knows how to reflect an image across an imaginary line on the computer screen. Help her experiment to find out what happens when she reflects a reflection.



1 On Activity Master 39: Reflections, list the coordinates of the vertices of Figure Q in column Q.

2 Draw Figure R by reflecting Figure Q across the dotted vertical line. Fill in the column for Figure R.

3 Draw Figure S by reflecting Figure R across the dotted horizontal line. Fill the in column for Figure S.

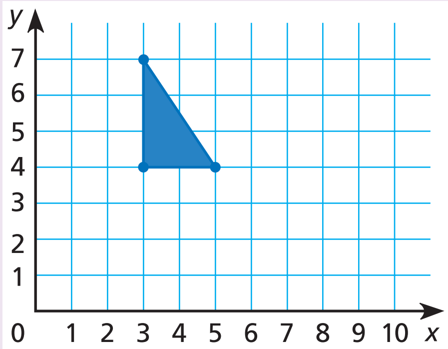
4 Can you think of a way to transform Figure Q into Figure S in just one step?

REVIEW MODEL
Rotating a Figure

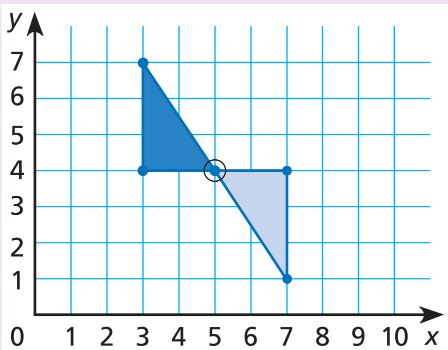
To rotate, or turn, a figure around a point, you must say which point to rotate around, and how much to turn.

The result will look different depending on what point the figure turns around and on how big the angle of the rotation is.

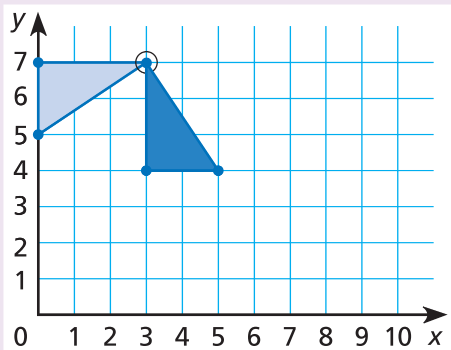
This blue triangle has vertices at (3,4), (5,4), (3,7).



Example 1 Rotate the blue triangle 180° around the point (5,4).



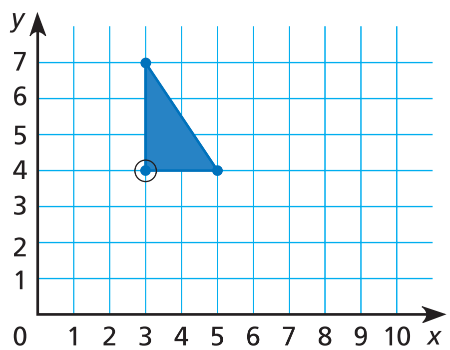
Example 2 Rotate the blue triangle 90° clockwise around the point (3,7).



Check for Understanding

1. Copy the triangle onto the grid on AM34. Rotate the triangle 180° around the point (3,4). Draw the new triangle.

2. How far is each vertex of the first triangle you drew from the point (3,4)? How far is each vertex of the rotated image from that point?



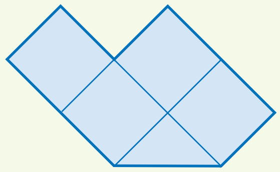
Chapter 6

Lesson 5

EXPLORE
Combining Transformations

Katy knows how to make the computer reflect, rotate, or translate a figure on the screen. By combining transformations, she can move figures in many ways. She wondered, though, if there might be times when she could use only one transformation instead of two, and get the same result.

Katy's figure



1. Use the following combinations of transformations of Katy's figure on Activity Master 40: Drawing Transformations. Think about whether there might be one transformation that will give the same result.

   A Reflect the figure across the first dotted line and reflect the result over the second dotted line.

   B Reflect the figure across the vertical dotted line and reflect the result over the horizontal dotted line.

   C Translate the figure up two spaces and reflect the result across the dotted vertical line.

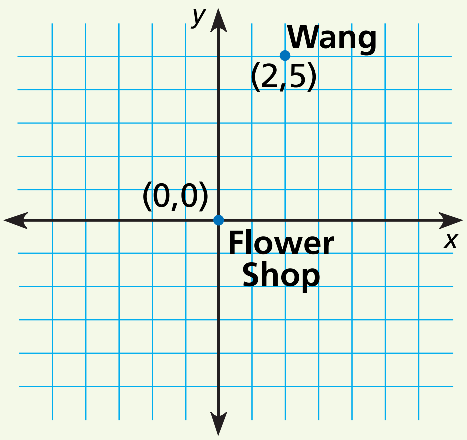
2. Is there a single translation, reflection, or rotation of any of the original figures that would give the same results as the transformations you just did? HINT: Cutting out the figure at the bottom of Activity Master 40 and moving it on the grids may help.

Chapter 6

Lesson 6

EXPLORE
Where Does Mr. Smith Live?

Marilyn drives a delivery truck for the Fancy Flower Shop. The directions that she was given were not very good!



1. On Monday, Marilyn had to deliver some flowers to Mr. Smith. The directions just said, "Drive straight 5 blocks from the Flower Shop (0,0). Turn, then drive for 2 more blocks." Marilyn followed the directions, but ended up at Ms. Wang's house, instead. What went wrong?

2. Where might Mr. Smith's house be? On Activity Master 44: Marilyn's Map, mark all of the points that her directions could lead her.

3. Marilyn knew she needed better directions to Mr. Smith's. What could make the directions more useful?

Chapter 6
Lesson 6

REVIEW MODEL
Graphing on the Coordinate Plane

A coordinate plane is formed by two intersecting and perpendicular number lines. The point where they intersect is the origin, or (0,0).

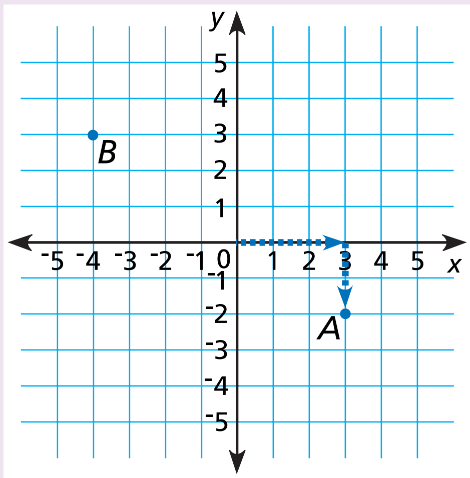
- The numbers on the *x*-axis are positive to the right of the origin and negative to the left of it.
- The numbers on the *y*-axis are positive above the origin and negative below.

Example 1 To graph the point (3,$^{-}$2) and label it

- Start at the origin.
- Move right 3 spaces.
- Move down 2 spaces.
- Plot the point and label it *A*.

Example 2 To name the coordinates for Point *B*

- Start at the origin.
- Move left 4 spaces.
- Move up 3 spaces.
- The coordinates for Point B are ($^{-}$4,3)



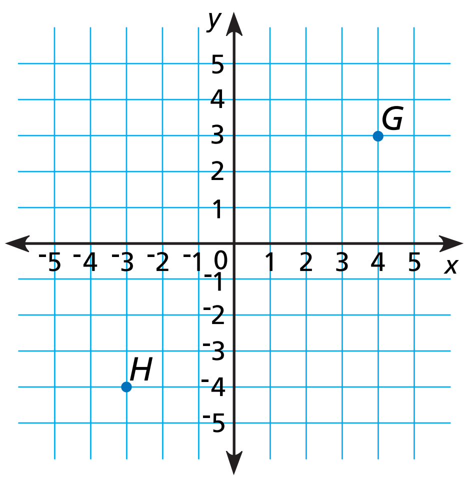
Check for Understanding

1. Explain how you would graph the point (3,$^{-}$2).

2. Explain how you would graph the point ($^{-}$3,0).

3. What are the coordinates of Point *G*?

4. What are the coordinates of Point *H*?



Chapter 6
Lesson 9

EXPLORE
Cafeteria Food

In an effort to provide the most popular and the most healthy food in the school cafeteria, your class is going to conduct a survey of students' food preferences.

These are the options being considered to replace the chips and cookies.

- Carrots and Dip
- Pretzels
- Fruit Cup
- Red Apples
- 1% Milk

1. Create a survey that will inform your recommendation to the cafeteria. You can use Activity Master 50: Survey Data as a guide.

2. Survey your classmates.

3. What will you recommend? Why?

Chapter 6

Lesson 9

REVIEW MODEL
Scatter Plots and Line Plots

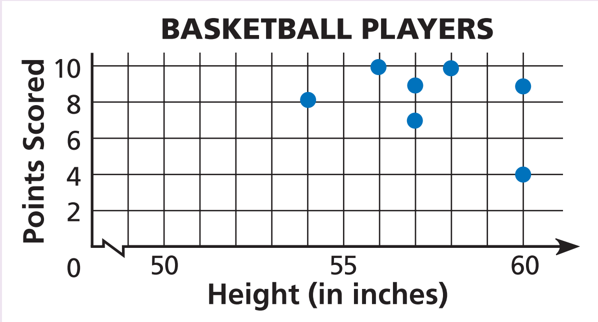
Scatter plots and line plots are two types of frequency graphs. Frequency graphs give a visual picture of data and often reveal patterns and relationships in the data.

In a **scatter plot**, the values of two variables are used as coordinates of each point in the graph.

Example 1 The table shows the heights and number of points scored for 7 basketball players. Make a scatter plot of the data.

BASKETBALL PLAYERS							
Height (in inches)	54	57	57	56	60	60	58
Points Scored	8	9	7	10	4	9	10

- Find the minimum and maximum values for both variables (height and points scored). Use these values to help you label the horizontal and vertical axes of a grid.
- Plot the data. Title the graph.

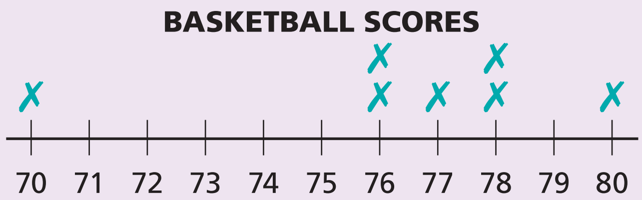


In a **line plot,** the frequency of data is shown along a number line.

Example 2 The table shows the scores for the first 7 basketball games of the season. Make a line plot of the data.

BASKETBALL SCORES							
Game	1	2	3	4	5	6	7
Score	78	76	78	76	80	77	70

- Draw a horizontal line.
- Find the minimum and maximum values for the data. Use these values to help you label the number line.
- Plot the data. Title the graph.



Check for Understanding

1. Make a scatter plot for this data.

STUDY TIME FOR MATH QUIZ							
Minutes Studying	15	20	17	5	20	12	18
Math Score	80	80	85	75	95	12	90

2. Make a line plot for this data.

PENCILS IN OUR DESKS							
Student	Al	Ed	Lou	Roe	Jay	Cal	Ana
Pencils in Desk	4	5	8	3	4	3	2

Chapter 6

Lesson 10

EXPLORE
Leveling Books

What is the difference between a 1st grade book and a 5th grade book?

1. Choose a reading book from a 1st grade class and a book from your class.

2. Open each book to a random page and choose a sentence at random. Count the number of words in that sentence and record it in the table on Activity Master 51: Book Level Data.

3. Do the same for 12 more sentences from each book, making sure that you are always picking difference sentences. You should have a total of 13 numbers in the table for each book.

4. Use the tables to make a graph of the data for each book.

5. Find the **minimum, maximum, median, mode** and **range** of the number of words per sentence for the 1st grade book and then for the 5th grade book.

6. Which, if any, of these data seem useful in describing the differences between the reading levels for 1st grade and 5th grade books?

Chapter 6
Lesson 11

EXPLORE
Mean Towers

Part A Build 6 towers of cubes as indicated in the table.

Tower Number	1	2	3	4	5	6
Number of Cubes	2	3	6	5	3	5

1. Record the moves you make as you now rearrange the cubes so that each tower is the same height. If you move more than one cube all at once from one tower to another, that is considered just "one move." Try to use as few moves as you can.

   Examples of ways to record moves:
   - 2 cubes from tower 3 to tower 1
   - 2: 3 → 1

2. When all the towers have been adjusted to the same height, that height is called the **mean height** of the towers. What is the **mean height** for these towers?

Part B Build 4 towers of cubes as indicated in this table.

Tower Number	1	2	3	4
Number of Cubes	8	5	4	7

Without actually moving any cubes figure out what the mean height of these towers would be.

Chapter 6
Lesson 11

REVIEW MODEL
Describing Data

There are many ways to describe data. Mode, median, and mean are three ways you might choose a “typical” value for a set of data.

- The **mode** is the number or item that occurs most often in a set of data. There may be one mode, more than one mode, or no mode.
- The **median** is the middle value when the data are listed from minimum to maximum values. When there are two middle numbers, the median is the number halfway between the two middle values.
- The **mean** is the value you get from “evening out” all of the data values. You can find the mean by finding the sum of the values and then dividing the sum by the number of values.

Example

The table shows the number of pets some children have.

NUMBER OF PETS WE HAVE										
Child	Alex	Barb	Carl	Dean	Edie	Fran	Gail	Hal	Iris	Jake
Number of Pets	6	2	1	6	1	2	4	6	7	5

Find the mode, median, and mean for the data.

Ordering the data makes it easier to find the mode and the median for a set of data: 1, 1, 2, 2, 4, 5, 6, 6, 6, 7

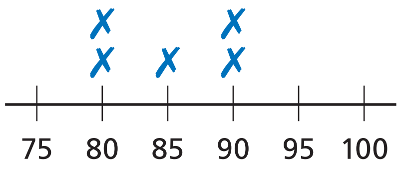
- The number 6 occurs more than any other number, so 6 is the mode.
- The numbers 4 and 5 are the two middle numbers, so the median is 4.5.
- The sum of the numbers is 40 $(1 + 1 + 2 + 2 + 4 + 5 + 6 + 6 + 6 + 7)$ and $40 \div 10 = 4$, so 4 is the mean.

Check for Understanding

1 Find the mode, median, and mean for the number of days of rain in a week.

DAYS OF RAIN										
Week	1	2	3	4	5	6	7	8	9	0
Number of Days	5	6	0	0	4	5	1	0	5	4

2 Find the median and mean for Jay’s test scores.



JAY’S TEST SCORES

Chapter 6

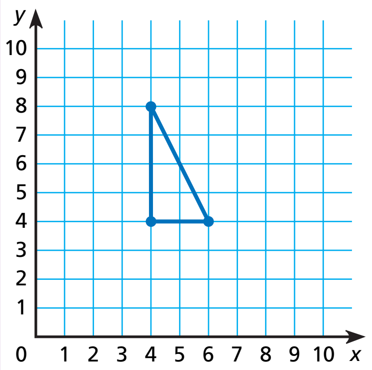
Lesson 12

REVIEW MODEL

Problem Solving Strategy

Act It Out

Jessie is making a design for a greeting card. She started with this triangle on a grid. She rotated the triangle 90° clockwise around the point (4,4) three times. Draw what the design looked like.



Strategy: Act it Out

Read to Understand

What do you know from reading the problem?

Jessie is rotating a triangle 90° clockwise around the point (4,4) three times to make a design.

What do you need to find out?

What the design looked like.

Plan

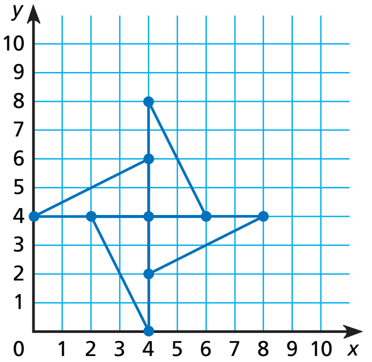
How can you solve this problem?

You use the strategy *act it out*.

Solve

How can you act it out?

You can cut out a shape to match the size and shape of the triangle on the grid. Then you can place the cut-out triangle on the triangle on the grid and see how the triangle looks as you rotate the cut-out triangle 90° clockwise around (4,4) three times.



Check

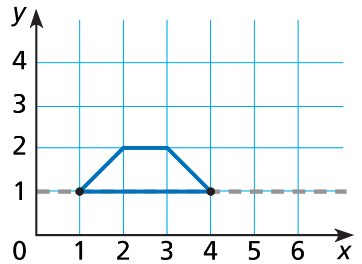
Look back at the problem. Did you solve the problem that was given? Does your solution make sense?

Problem Solving Practice

Act it out to solve.

1. Bert started with this shape and translated it one space up and one space right. What are the vertices of the new shape?

2. Kristin reflected the shape over the dotted horizontal line. Draw what her design looked like.



Problem Solving Strategies

- **✔ Act It Out**
- ✔ Draw a Picture
- ✔ Guess and Check
- ✔ Look for a Pattern
- ✔ Make a Graph
- ✔ Make a Model
- ✔ Make an Organized List
- ✔ Make a Table
- ✔ Solve a Simpler Problem
- ✔ Use Logical Reasoning
- ✔ Work Backward
- ✔ Write an Equation

Mixed Strategy Practice

Use any strategy to solve. Explain.

3. Ted read for 4 minute on Monday, 8 minutes on Tuesday, 16 minutes on Wednesday, and 32 minutes on Thursday. If the pattern continues, how many minutes will he read on Saturday?

4. At the snack bar, Marco bought a drink for $1.25, a sandwich for $3.45, and a bag of peanuts for $0.75 each. He was given $4.55 in change. How much money did he give the cashier at the snack bar?

5. The theater holds 542 people. If the theater was full for all 21 performances of a play. How many people attended the play?

6. The soccer team practices for 2 hours each Monday and for 1 hour and 30 minutes each Wednesday and Friday. For how many hours will the team practice in 8 weeks?

For 7–9, use the sign.

7. How much did it cost for the Blackwell family to buy two adult tickets and two tickets for their 12-year-old twins?

8. Mrs. Brewster paid $64 for a senior ticket for herself and for children's tickets for her grandchildren. How many tickets did she buy for her grandchildren?

9. Next year the amusement park will raise the price of all tickets by $2. How much will it cost a family to buy 2 adult tickets and 3 children's tickets next year?

Chapter 6 Vocabulary

Choose the best vocabulary term from Word List A for each sentence.

1. The __?__ is the value that occurs more often than any other in a set of data.
2. In the data set 2, 3, 9, 10, 72, the number 72 is a(n) __?__.
3. A translation is another word for __?__.
4. A line segment has two __?__.
5. A diagonal of a square is a(n) __?__ of the square.
6. A(n) __?__ is a graph that can show whether two types of data are related.
7. The horizontal axis is called the __?__.
8. The __?__ of a set of data is the middle term when all the terms are listed in order.
9. The horizontal and vertical axes divide the coordinate plane into four __?__.
10. A reflection is a type of __?__.

Complete each analogy using the best term from Word List B.

11. Most is to __?__ as least is to minimum.
12. Point is to rotation as __?__ is to reflection.

Word List A

- average
- axes
- coordinates
- endpoints
- line of symmetry
- mean
- median
- mode
- ordered pair
- outlier
- quadrants
- reflection
- scatter plot
- slide
- transformation
- translation
- turn
- *x*-axis
- *y*-axis

Word List B

- flip
- line
- maximum
- range

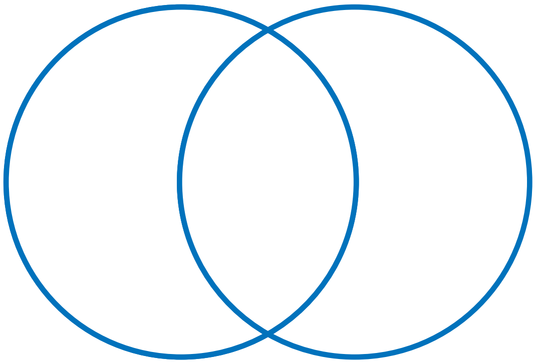
Talk Math

Discuss with a partner what you have just learned about grids and graphs. Use the vocabulary terms *origin*, *vertical coordinate*, and *horizontal coordinate*.

13. How can you graph an ordered pair on a coordinate grid?
14. A square is graphed on a coordinate grid. How could you graph a translation of the square that is 3 spaces left and 2 spaces up?
15. A line segment is reflected across the horizontal axis. From there, it is reflected across the vertical axis. How can you describe the result as one transformation?

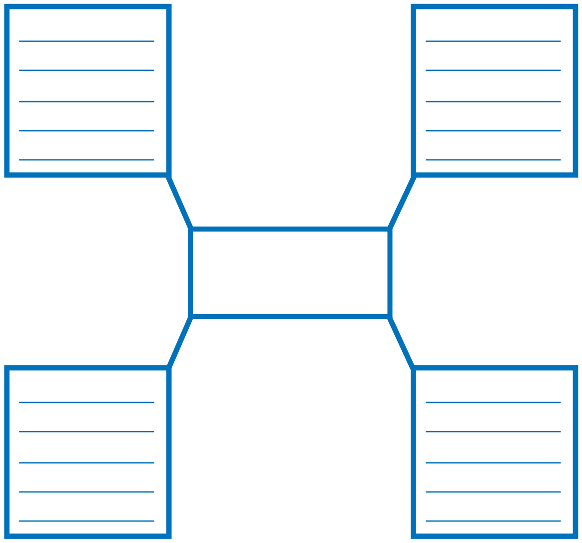
Venn Diagram

16 **Create a Venn diagram for the words vertical and horizontal. Use vocabulary terms *horizontal axis, horizontal coordinate, ordered pair, origin, quadrants, vertical axis, vertical coordinate, x-axis,* and *y-axis.***



Word Web

17 **Create a Word Web using the term *slide.* Use what you know and what you have learned about transformations.**



What's in a Word?

TRANSFORMATION, TRANSLATION Sometimes people confuse these words because they start and end the same way. The prefix *trans-* means "to change." So, the first part of each word means the same thing. And the last part *-tion* makes each word a noun.

The middle part of *transformation* is *-form-*. A form is a shape. In math, a transformation is what happens when a shape is changed somehow—moved to a new position, turned over, rotated. The middle part of translation comes from a Latin word that means "to carry." So, a math translation is changing by carrying, or moving, to a new position—but not changing in any other way. In math, a translation is a type of transformation.

GO ONLINE **Technology**
Multimedia Math Glossary
www.harcourtschool.com/thinkmath

GAME

Area Claim

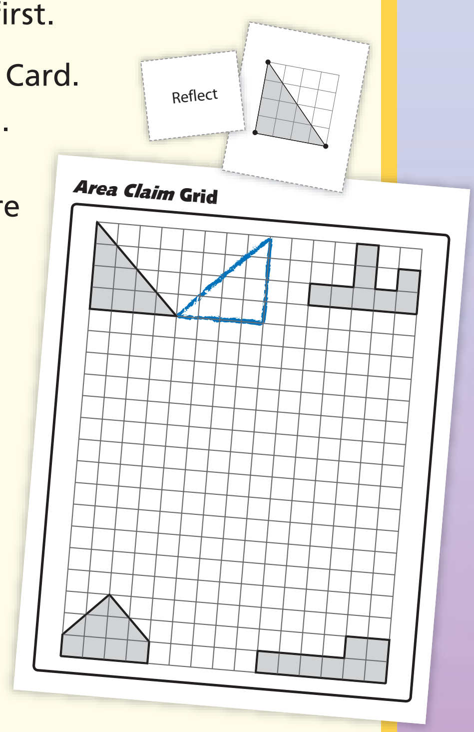
Game Purpose
To practice identifying translations, reflections, and rotations

Materials
- Activity Master 41: *Area Claim* Grid
- Activity Master 42: *Area Claim* Figure Cards
- Activity Master 43: *Area Claim* Transformation Cards

How To Play The Game

1. This game is for 2 players. Cut out the Figure Cards and the Transformation Cards. Put each set in a separate pile, face down on the table. Decide who will be first.

2. Player 1 takes a Figure Card and a Transformation Card.
   - Find the matching figure on the *Area Claim* Grid.
   - Use the transformation named on the card to copy the figure to a new position. The new figure cannot overlap any figure already on the grid.
   - Label the new figure with your initials. Put the used cards aside in discard piles.



3. Players take turns using a figure card and a transformation card to draw a figure on the grid.
   - You must start from a figure that is printed on the grid or from one of your own figures. You must not use the other player's figures.
   - Your turn ends after you have drawn a figure or if you cannot find space to draw a new figure.

4. When the cards run out, mix up the cards in the discard piles to start again. The game ends when you have gone through both piles twice. You can end the game early if you both agree that there is not enough space left to draw any of the figures.

5. When the game is over, both players find the total area of their figures. Whoever has the greater area wins.

GAME

Mean, Median, Mode

Game Purpose

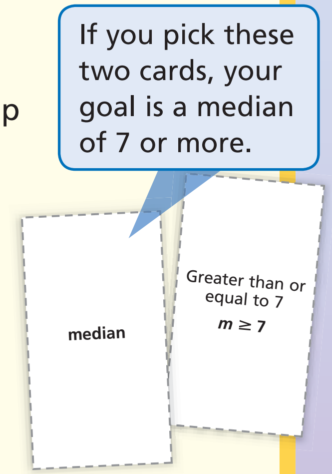
To practice using and interpreting mean, median, and mode

Materials

- Activity Master 52: Data Measure Cards
- Index cards

How To Play The Game

1. This game is for 2 or 3 players.
   - Make 4 sets of number cards, each set numbered 1–10.
   - Cut out the cards from Activity Master 52. Make a group of word cards and a group of inequality cards. Mix up the cards in each group, and place them face down in 2 piles.
   - Each player picks a word card and an inequality card. Don't show them to anyone—this is your secret goal. Any leftover cards are not used.



2. Mix up the number cards. Choose one player to pass out 2 number cards to each player. The rest of the cards are put face down in a pile.

3. Players take turns creating a data set.
   - Take the top number card. Now you will have 3 cards. Choose one of your cards and place it in the center.
   - When there are 12 cards in the data set, the dealer turns over the top number card and adds it to the data set.

4. Work together to find the mean, median, and mode of the data set. If your goal is met, you win!
   - There could be more than one mode. If any one of the modes matches a player's goal, that player wins.
   - This game could have one winner, more than one winner, or no winners at all!

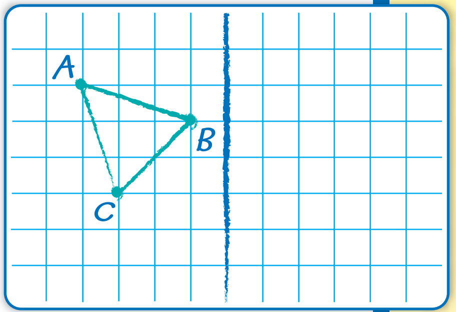
CHALLENGE

Reflections

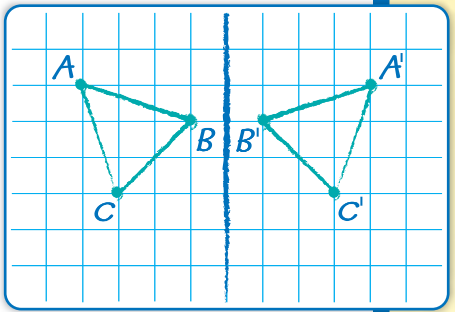
How much do you know about a figure and its reflection? To find out, follow the steps below and answer the questions.

You will need a sheet of grid paper.

Step 1 Draw a vertical line in the center of the grid paper. Then draw a triangle on the left side of the line. Label the vertices *A*, *B*, and *C*. Your drawing might look like this one.



Step 2 Draw the reflection of the triangle across the line. Label the vertices of the reflected image *A*', *B*', and *C*'.



A How could you show that the triangle and its image are congruent?

Step 3 Draw a line segment from each vertex of the original figure to its image. That is, draw line segments *AA*', *BB*', and *CC*'.

B What type of angle does each segment make with the line of reflection?

C What else do you notice about the segments? Are any of the line segments parallel? How do their lengths on each side of the line of reflection compare?

Step 4 Mark any point on the original figure that is not a vertex. Label it point *M*. Draw a line segment from point M to its reflection image, point *M*'.

D Are your answers to questions **B** and **C** also true for line segment *MM*'?