Chapter

7 Decimals

Dear Student,

In this chapter, "Decimals," you will see how our base-ten place-value system extends in both directions from the decimal point. Directly to the left of the decimal point, there may be ones, and then tens, hundreds, thousands, ten-thousands, hundred-thousands, millions, and so on. To the right of the decimal point, there may be tenths, then hundredths, thousandths, and so on. This number system is based on ten, because the value of each place is ten times the value of the place directly to its right. As we go down the list, each number is 10 times the previous number. Just as 400 is 10×40, and 40 is 10×4, so the number 0.4 is 10×0.04. Another way of saying the exact same thing is that each number is one tenth of the next number.

0.04
0.4
4.0
40.0
400.0

You will begin the chapter by using decimal notation to name numbers within smaller and smaller spaces on the number line. Can you name several numbers between 16 and 17?

By the end of the chapter, you will know how to compare numbers with digits on both sides of the decimal point and also how to add, subtract, and multiply decimal numbers. You will also be more familiar with decimal names for some common fractions.

Mathematically yours,
The authors of ***Think Math!***

THE WORLD ALMANAC FOR KIDS

Making Money

The U.S. Mint was founded in 1792. The Mint produced its first circulating coins in March 1793. Branches of the U.S. Mint in Denver, Colorado, and Philadelphia, Pennsylvania, currently make coins for circulation.

FACT•ACTIVITY 1

The tables below show the thickness of coins and the number of coins produced in a six month period.

Use the tables for 1–4.

1. Which coin is the thickest? Which is the thinnest?
2. Which has a thickness between that of the dime and the quarter? Explain using decimal values.
3. Margie says that the half dollar is 0.5 mm thicker than the quarter. Is she correct? Explain.
4. Which coin had more than 1 billion but less than 2 billion produced in the first 6 months of 2006?

Thickness of U.S. Coins

	Penny	Nickel	Dime	Quarter	Half Dollar	Golden Dollar
Thickness (in millimeters)	1.55	1.95	1.35	1.75	2.15	2.00

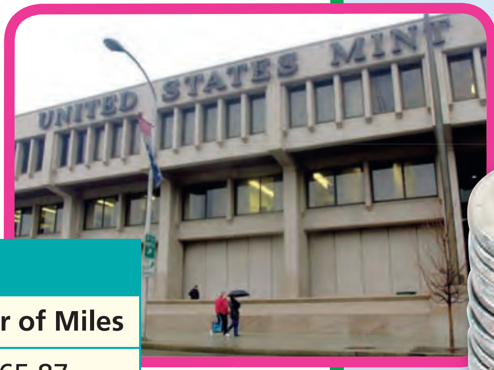
Coin Production: January 2006 – June 2006 (in millions)

	Penny	Nickel	Dime	Quarter	Half Dollar	Golden Dollar
Total Produced	6,171	1,122	2,126	2,025	4	8

FACT·ACTIVITY 2

You can take a tour to see how coins are made at the Philadelphia Mint.

The table shows the number of miles from the center of some cities to the Philadelphia Mint.



Distance by Car to Philadelphia Mint

Starting City	Number of Miles	Starting City	Number of Miles
Albany, NY	231.76	Dallas, TX	1,465.87
Atlanta, GA	787.54	Hartford, CT	211.52
Baltimore, MD	105.15	Richmond, VA	254.23
Buffalo, NY	404.89	Tampa, FL	1,059.33

Use the table.

1. For which distances are the decimal part of the number about $\frac{1}{2}$? Explain.
2. Which number has a decimal part that is about $\frac{3}{4}$?
3. Tom's family drives from Buffalo, NY, to the Philadelphia Mint and then on to Richmond, VA. How many miles was the entire trip?
4. If Tom's family drives about the same number of miles each day for 4 days, about how many miles did they drive per day?

CHAPTER PROJECT

Some people collect commemorative coins, which honor people or events in history.

- Design a new coin with a new denomination. Draw a picture of your design on a poster and include its specifications such as its thickness and size, and the number that you would like produced per month. Explain why your coin is unique and what the design represents.
- Compare your coin with two of your classmates' coins. Use number sentences in your descriptions.

ALMANAC Fact

In June 2006, about $37 billion worth of coins were in circulation.

EXPLORE
The Same 3 Digits

1. Write all the whole numbers using the digits 3, 6, and 8, using each digit once (and only once) in each number.

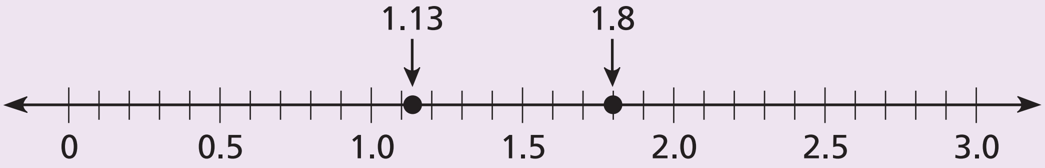
2. Use the digits 3, 6, and 8 to make numbers with one or two digits after the decimal point. Use each digit once (and only once) in each number. Make as many different numbers as you can.

3. How can you be sure you found all the numbers?

REVIEW MODEL
Compare and Order Decimals

You can use a number line to compare and order decimals.

Example 1 Compare 1.13 and 1.8.



- 1.13 is to the left of 1.8, so 1.13 is less than 1.8. $1.13 < 1.8$
- 1.8 is to the right of 1.13, so 1.8 is greater than 1.13. $1.8 > 1.13$

You can also use place value to compare and order decimals.

Example 2 Order 5.035, 5.08, and 5.009 from least to greatest.
First, write the numbers with the decimal points aligned.
Then, compare the digits left to right until they are different.

Step 1

Compare the ones and tenths.

5.035
↓↓
5.08 **same**
↓↓
5.009

Step 2

Compare the hundredths.

5.035
↓
5.08 **different**
↓ $0 < 3 < 8$
5.009

Step 3

Order the numbers.
$5.009 < 5.035 < 5.08$

Check for Understanding

Copy and complete. Write $<$, $>$, or $=$ for each ●.

1. 0.63 ● 0.625
2. 8.35 ● 83.5
3. 2.47 ● 2.470
4. 8.34 ● 8.305
5. Order from least to greatest.
   2.14, 2.04, 2.41
6. Order from greatest to least.
   0.452, 0.367, 1.4, 0.60

Chapter 7
Lesson 3

EXPLORE
Shifting Place Value

Use a calculator.
Read all the directions before you begin.

Step 1 Enter any 1-digit number.

Step 2 Multiply the number by 10. ■ × 10 =

Step 3 Read the number silently.

Step 4 Keep multiplying your number by 10 and silently reading each result.

Step 5 Stop when the calculator's window is filled with digits.

Step 6 Divide the number by 10. ■ ÷ 10 =

Step 7 Keep dividing by 10 and reading the number silently each time.

Step 8 Stop when you see 0.00 ■.

Your starting number

Were there any numbers you could not name?
Describe how your number changed.

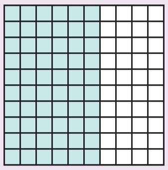
Chapter 7
Lesson 5

REVIEW MODEL
Connect Decimals to Fractions

You can use grids to help you connect decimals to fractions with denominators of 10 and 100.

Write a fraction or a mixed number and a decimal for each model.

1



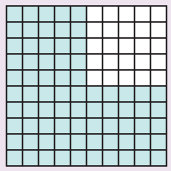
6 out of **10** equal parts are shaded.

$\frac{6}{10}$ is shaded.

0.6 is shaded.

$\frac{6}{10} = 0.6$

2



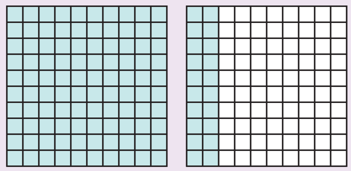
75 out of **100** equal parts are shaded.

$\frac{75}{100}$ is shaded.

0.75 is shaded.

$\frac{75}{100} = 0.75$

3



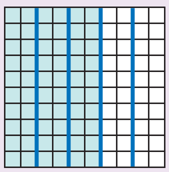
One whole plus **2** out of **10** parts are shaded.

$1\frac{2}{10}$ is shaded.

1.2 is shaded.

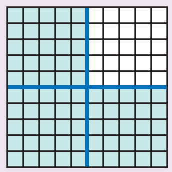
$1\frac{2}{10} = 1.2$

You can also use grids to help you connect decimals to familiar fractions with denominators that do not name tenths or hundredths.



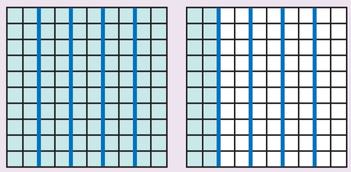
3 out of **5** equal parts are shaded.

$\frac{6}{10} = \frac{3}{5} = 0.6$



3 out of **4** equal parts are shaded.

$\frac{75}{100} = \frac{3}{4} = 0.75$



One whole plus **1** out of **5** parts are shaded.

$1\frac{2}{10} = 1\frac{1}{5} = 1.2$

Check for Understanding

Write a decimal for each fraction or mixed number.

1 $\frac{1}{4}$

2 $4\frac{2}{5}$

3 $7\frac{3}{100}$

4 $\frac{1}{2}$

5 $2\frac{4}{10}$

6 $20\frac{17}{100}$

7 $3\frac{3}{4}$

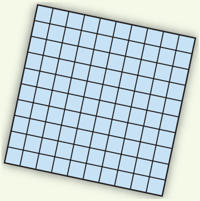
8 $10\frac{3}{5}$

Chapter 7

Lesson 6

EXPLORE
Decimal Notation for $\frac{1}{8}$

You previously used hundredths grids to find decimals that are equivalent to these fractions:



$\frac{1}{4}$ $\frac{3}{4}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$

Use Activity Master 61: Shading $\frac{1}{8}$ with this activity.

1 Shade $\frac{1}{8}$ of the grid. You may shade parts of squares.

2 How would you write the decimal for $\frac{1}{8}$?

3 How would you complete this equation?

$\frac{1}{8} = \frac{\blacksquare}{100} = \frac{\blacksquare}{1{,}000} = \blacksquare$

Chapter 7

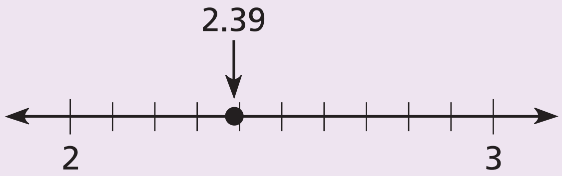
Lesson 7

REVIEW MODEL
Round Decimals

You can use a number line to help you round a decimal.

Example 1

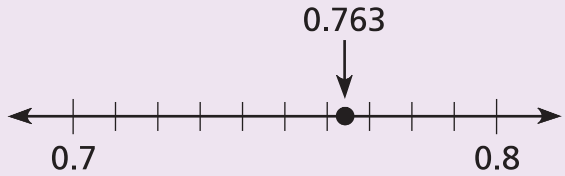
Round 2.39 to the nearest whole number.



- Find the two whole numbers that 2.39 is between: 2 and 3.
- Determine which of these numbers is closer to 2.39. 2.39 is closer to 2. So, 2.39 rounded to the nearest whole number is 2.

Example 2

Round 0.763 to the nearest tenth.



- Find the two tenths that 0.763 is between: 0.7 and 0.8.
- Determine which of these numbers is closer to 0.763. 0.763 is closer to 0.8. So, 0.763 rounded to the nearest tenth is 0.8.

When rounding to the nearest **whole number,** look at the **tenths** digit. 23.052 ⟶ 23

When rounding to the nearest **tenth,** look at the **hundredths** digit. 23.052 ⟶ 23.1

When rounding to the nearest **hundredth,** look at the **thousandths** digit. 23.052 ⟶ 23.05

Rounding Rules:
- Find the digit in the place to which you want to round.
- If the digit to the right is less than 5, round down.
- If the digit to the right is 5 or greater, round up.

Check for Understanding

Round to the nearest whole number.

1 45.8

2 102.35

3 3.5

4 78.28

Round to the nearest tenth.

5 3.45

6 45.08

7 117.88

8 20.582

Round to the nearest hundredth.

9 1.085

10 50.451

11 0.8091

Chapter 7

Lesson 8

REVIEW MODEL
Addition of Decimals

Use what you know about adding whole numbers to help you add decimals.

- Always add digits of like place value.
- Lining up decimal points helps you keep track of like place values.

Example 1 Add: 3.87 + 3.35

Step 1 Line up the decimal points.

$$\begin{array}{r} 3.87 \\ +\ 2.35 \\ \hline \end{array}$$

Step 2 Add as with whole numbers.

$$\begin{array}{r} 3.87 \\ +\ 2.35 \\ \hline 6\ 22 \end{array}$$

Step 3 Place the decimal point.

$$\begin{array}{r} 3.87 \\ +\ 2.35 \\ \hline 6.22 \end{array}$$

Check: Estimate the sum.

3.87 ⟶ 4 and 2.35 ⟶ 2; 4 + 2 = 6

6 is close to 6.22, so the answer is reasonable.

Example 2 Add 16.83 + 45.2

Step 1 Line up the decimal points.

$$\begin{array}{r} 16.83 \\ +\ 45.20 \\ \hline \end{array}$$

Place a zero.

Step 2 Add as with whole numbers.

$$\begin{array}{r} 16.83 \\ +\ 45.20 \\ \hline 62\ 03 \end{array}$$

Step 3 Place the decimal point.

$$\begin{array}{r} 16.83 \\ +\ 45.20 \\ \hline 62.03 \end{array}$$

Check: Estimate the sum.

16.83 ⟶ 17 and 45.2 ⟶ 45;
17 + 45 = 62

62 is close to 62.03, so the answer is reasonable.

More Examples

A 9.52 + 14.08

$$\begin{array}{r} 9.52 \\ +\ 14.08 \\ \hline 23.60 \end{array}$$

B 56.1 + 14.52

$$\begin{array}{r} 56.10 \\ +\ 14.52 \\ \hline 70.62 \end{array}$$

Place a zero.

C 25.802 + 51.28

$$\begin{array}{r} 25.802 \\ +\ 51.280 \\ \hline 77.082 \end{array}$$

Place a zero.

Check for Understanding

Find the sum. Estimate to check.

1. 3.95 + 0.56
2. 12.1 + 9.01
3. 57.81 + 12.65
4. 6.005 + 31.085
5. 4.95 + 3.5
6. 7.32 + 12.9
7. 100 + 50.5
8. 3.2 + 6.4 + 10.5

Chapter 7
Lesson 9

EXPLORE
Decimal Subtraction

1. Solve. You may use the number sentence $64 - 57 = 7$ or base-ten blocks.

$6 - 5 = \blacksquare$

$6.4 - 5 = \blacksquare$

$6 - 5.7 = \blacksquare$

$6.4 - 5.7 = \blacksquare$

$0.64 - 0.57 = \blacksquare$

---

2. Solve. You may use the number sentence $55 - 28 = 27$ or base-ten blocks.

$5 - 2 = \blacksquare$

$5.5 - 2 = \blacksquare$

$5 - 2.8 = \blacksquare$

$5.5 - 2.8 = \blacksquare$

$0.55 - 0.28 = \blacksquare$

Chapter 7
Lesson 9

REVIEW MODEL
Subtraction of Decimals

Use what you know about subtracting whole numbers to help you subtract decimals.

- Always subtract digits of like place value.
- Lining up decimal points helps you keep track of like place values.

Example 1 Subtract: 5.14 − 0.86

Step 1 Line up the decimal points.

$$\begin{array}{r} 5.14 \\ -\,0.86 \\ \hline \end{array}$$

Step 2 Subtract as with whole numbers.

$$\begin{array}{r} 5.14 \\ -\,0.86 \\ \hline 4\ 28 \end{array}$$

Step 3 Place the decimal point.

$$\begin{array}{r} 5.14 \\ -\,0.86 \\ \hline 4.28 \end{array}$$

Check: Estimate the difference.

5.14 ⟶ 5 and 0.86 ⟶ 1; 5 − 1 = 4

4 is close to 4.28, so the answer is reasonable.

Example 2 Subtract: 62.4 − 9.15

Step 1 Line up the decimal points.

$$\begin{array}{r} 62.40 \\ -\,9.15 \\ \hline \end{array}$$

Place a zero.

Step 2 Subtract as with whole numbers.

$$\begin{array}{r} 62.40 \\ -\,9.15 \\ \hline 53\ 25 \end{array}$$

Step 3 Place the decimal point.

$$\begin{array}{r} 62.40 \\ -\,9.15 \\ \hline 53.25 \end{array}$$

Check: Estimate the difference.

62.4 ⟶ 62 and 9.15 ⟶ 9; 62 − 9 = 53

53 is close to 53.25, so the answer is reasonable.

More Examples

A 7.15 − 2.08

$$\begin{array}{r} 7.15 \\ -\,2.08 \\ \hline 5.07 \end{array}$$

B 9 − 8.05

$$\begin{array}{r} 9.00 \\ -\,8.05 \\ \hline 0.95 \end{array}$$

← Place zeroes

C 8.52 − 5.1

$$\begin{array}{r} 8.52 \\ -\,5.10 \\ \hline 3.42 \end{array}$$

← Place a zero

Check for Understanding

Find the difference. Estimate to check.

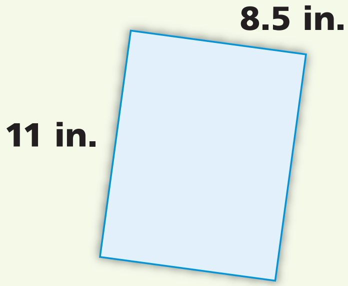
1. 1.5 − 0.7
2. 11.4 − 6.6
3. 7.095 − 5.64
4. 4.578 − 1.123
5. 14.1 − 8.65
6. 17 − 8.7
7. 21 − 15.06
8. 6.85 − 1.486

Chapter 7

Lesson 11

EXPLORE
Area of a Piece of Paper

A standard piece of paper measures $8\frac{1}{2}$ inches by 11 inches.



1. How would you find an estimate of the area in square inches?

2. Try to find the exact area of an $8\frac{1}{2}$-inch by 11-inch piece of paper without a calculator.

Chapter 7
Lesson 12

REVIEW MODEL
Problem Solving Strategy
Act it Out-Make a Model

Mr. Keys had five \$10 bills, two \$1 bills, five dimes, and three pennies. He spent \$14.63 to buy a gift and put \$26.50 in the bank. How much money does he have left? What bills and coins could he have?

Strategy: Act it Out-Make a Model

Read to Understand

What do you know from reading the problem?

Mr. Keys had some money: five \$10 bills, two \$1 bills, five dimes, and three pennies. He spent \$14.63 and put \$26.50 in the bank.

Plan

How can you solve this problem?

You can use bills and coins to *act out* the problem.

Solve

How can you *act it out?*

First, use bills and coins to show the amount of money Mr. Keys has. Record the amount (\$52.53).

Then, take away the money he spent and the money he saved. To take away the \$14.63 he spent, you have to first trade one \$10 bill for ten \$1 bills and trade one \$1 bill for ten dimes. Take away the \$14.63 he spent and the \$26.50 he put in the bank.

The money that is left (one \$10 bill, one \$1 bill, and four dimes) represents the bills and coins Mr. Keys has left. He has \$11.40 left.

Check

Look back at the problem. Did you answer the questions that were asked? Does the answer make sense? How could you check your answer?

You could use addition and subtraction to check your answer.

\$14.63	\$52.53
+ 26.50	− 41.13
\$41.13	\$11.40

Problem Solving Practice

Use the strategy *act it out-make a model* to solve.

1. Aaron, Brooke, Chris, Daniel, and Evan are waiting in line to buy ice cream. Brooke is in front of Daniel and after Chris. Aaron is between Chris and Brooke. Evan is after Daniel. Who is first in line?

2. Kristin and Theresa are playing a board game. In the first round, Kristin moves 6 spaces forward, 3 back, and 4 forward. Theresa moves 5 spaces forward, 2 back, and 5 forward. Who is ahead after the first round? How far ahead is she?

Problem Solving Strategies

- ✓ **Act It Out**
- ✓ Draw a Picture
- ✓ Guess and Check
- ✓ Look for a Pattern
- ✓ Make a Graph
- ✓ Make a Model
- ✓ Make an Organized List
- ✓ Make a Table
- ✓ Solve a Simpler Problem
- ✓ Use Logical Reasoning
- ✓ Work Backward
- ✓ Write an Equation

Mixed Strategy Practice

Use any strategy to solve. Explain.

3. Two numbers have a sum of 18 and a product of 56. What are the two numbers?

4. At the gymnastics vault competition, Robyn scored 9.925, Madison scored 9.950, and Belle scored 9.910. Which gymnast had the highest score?

5. Paul's checking account had a starting balance of $87.16. He wrote checks for $16.55 and $14.10. He made deposits of $20.50 and $19.50. What is his balance? Does Paul have enough money to write a check for $85.56?

6. Melissa can run a 100-meter dash in 10.49 seconds. Will her team beat the 400-meter record of 41.37 seconds if her three teammates also run 100 meters in 10.49 seconds?

For 7–9, use the table.

7. On which day did the temperature drop from morning to afternoon?

8. On which days was the afternoon temperature more than 10 degrees warmer than the morning temperature?

9. On which day was there the greatest difference between the morning and the afternoon temperatures? What was the difference?

Temperatures (in degrees F)

Day	Morning	Afternoon
Monday	67.2°	76.9°
Tuesday	59.3°	70.4°
Wednesday	60.4°	70.7°
Thursday	68.4°	56.8°
Friday	53.1°	65.4°

Chapter 7 Vocabulary

Choose the best vocabulary term from Word List A for each sentence.

1. Numbers with one or more digits to the right of the decimal point are called ___?___.
2. The digit 9 in the number 9,876,543,210 is in the ___?___ place.
3. When you multiply a whole number by ___?___, you write zeros at the end of the whole number.
4. The ___?___ of 8 in 4.89 is 8 tenths.
5. Sometimes ___?___ are used to estimate the value of a decimal number.
6. The ___?___ place is two places to the right of the decimal point.
7. You can ___?___ the number 429 to the nearest ten or hundred.
8. The mixed numbers $4\frac{3}{1,000}$ and $7\frac{3}{100}$ are examples of ___?___.

Word List A

- area
- benchmark fractions
- billions
- comparing
- decimals
- fractions that name tenths and hundredths
- hundredths
- like place values
- millions
- ordering
- place value
- powers of 10
- product
- round
- sum
- tenths

Complete each analogy using the best term from Word List B.

9. Ones is to hundreds as ___?___ is to tenths.
10. Mixed number is to fraction as decimal number is to a ___?___.

Word List B

- decimal part of a number
- hundredths
- tenths
- thousandths

Talk Math

Discuss with a partner what you have just learned about large and small numbers. Use the vocabulary term *place-value position.*

11. How can you compare two decimal numbers?
12. How can you use rounding to estimate the value of a decimal number?
13. How can you add two decimal numbers?

Word Line

14 **Create a word line for the terms *billions, hundredths, millions,* and *thousandths.***

Words:

Sequence:

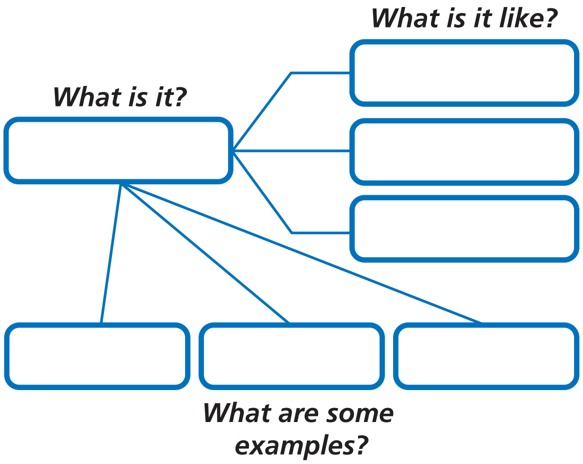
Word Definition Map

15 **Create a word definition map using the word *decimal.* Use what you know and what you have learned about large and small numbers.**

A What is it?

B What is it like?

C What are some examples?



What's in a Word?

HUNDREDS, HUNDREDTHS These two words are similar. But two letters—*th*—make a big difference between them. *Hundreds* means the numbers 100 to 999. *Hundredths* are 100 equal parts of one whole. When you write a decimal as a fraction, you can see how *hundreds* and *hundredths* are related. The number of zeros in 100 (one *hundred*) is the same as the number of zeros used in $\frac{1}{100}$ (one *hundredth*).

The place-value positions of *hundreds* and *hundredths* are not the same on opposite sides of the decimal point. Each place-value position is one-tenth the value of the position to its left. So, the *hundreds* position is the third place to the left of the decimal point. And the *hundredths* position is the second place to the right of the decimal point.

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GAME

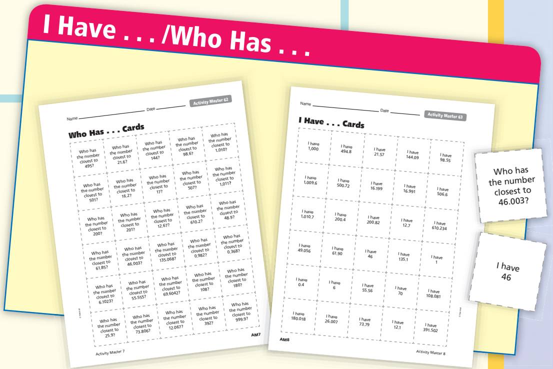
I Have . . ./Who Has . . .

Game Purpose

To practice estimation and comparison of decimals

Materials

- Activity Masters 62–63: *I Have . . ./Who Has . . .* cards
- Scissors
- Glue or tape



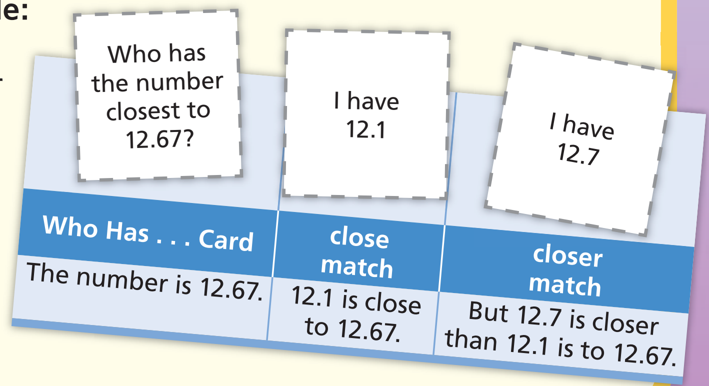
How To Play The Game

1. Play this game in a small group. Cut out all the *I Have . . .* cards and *Who Has . . .* cards. Make one set of 30 cards by gluing or taping two cards together so that one side shows an *I Have . . .* number and the other shows a *Who Has . . .* question.

2. Mix up the cards, and give an equal number to each player. Some players might have more cards than other players. Decide who will be first.

3. Player 1 reads the *Who Has . . .* question on one of his or her cards. Everyone else checks the *I Have . . .* side of their cards.
   - Raise your hand if you think your number is close.
   - If more than one player raises a hand, compare all the numbers to see which is the closest.

Example:



4. Whoever has the closest number turns over that card and reads the *Who Has . . .* question on the back.

5. Keep playing until all questions and answers have been matched.

GAME

Hit the Target

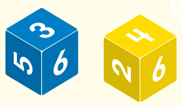
Game Purpose
To practice addition and subtraction of decimals

Materials
- 2 number cubes (1–6)

How To Play The Game

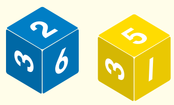
1. Play this game with a partner. Both players start with a score of 0. The goal is to reach a target number before your partner does.

2. Let one player toss both number cubes to make a target number.

   Example: You toss 

   Decide whether the target number will be 34 or 43.

3. Take turns.
   - Toss the number cubes, and make a decimal number between 1 and 10.

     Example: You toss 

     Decide whether your number will be 2.5 or 5.2.
   - Add the decimal number to your score. If your score is close to the target number, you can subtract the decimal number so that you do not go over the target number.

4. The game is over if:
   - you hit the target number exactly. You win!
   - your partner goes over the target number. You win!
   - no one has hit the target or gone over after 10 tosses. If that happens and your score is closer to the target number, you win!

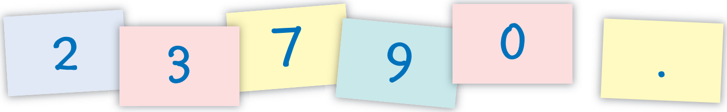
CHALLENGE

Rounded Numbers

This is a game for 2 players. A third student can act as a referee if needed.

GET READY

- Each player writes the digits 2, 3, 7, 9, and 0 on index cards. Write a decimal point on a sixth card.



RULES

- Use all 6 cards to make a number that matches each statement below. There might be more than one correct number for a statement.
- After you have made all 10 numbers, check each other's numbers. Ask a referee if you cannot agree that a number is correct. You get 1 point for each correct number. The player with more points wins!

Make a number that rounds to . . .

1. 327.1 when rounded to the nearest tenth.
2. 79.30 when rounded to the nearest hundredth.
3. 29 when rounded to the nearest whole number.
4. 790.3 when rounded to the nearest tenth.
5. 30.28 when rounded to the nearest hundredth.
6. 907 when rounded to the nearest whole number.
7. 73.2 when rounded to the nearest tenth.
8. 73.3 when rounded to the nearest tenth.
9. 9,702 when rounded to the nearest whole number.
10. 371 when rounded to the nearest whole number.