

Exploring Missing Factors

GROUP A: \$1 \$2 \$3 \$4 \$5 \$6 \$7 \$8 \$9

GROUP B: 10¢ 20¢ 30¢ 40¢ 50¢ 60¢ 70¢ 80¢ 90¢

GROUP C: 1¢ 2¢ 3¢ 4¢ 5¢ 6¢ 7¢ 8¢ 9¢

Use one stamp from each group (A, B, and C) in the puzzles below.

1

	A	B	C	
×	100			
9				1,242

2

	A	B	C	
×				
7				1,736

3

	A	B	C	
×				
5				1,830

4

	A	B	C	
×				
4				1,164

5

	A	B	C	
×				
11				2,915

6

	A	B	C	
×				
10				4,230

Connecting Multiplication and Division

Solve one problem in each pair to help you solve the other.

①

$$\begin{array}{r} 8 \overline{) 72} \\ \updownarrow \\ 4 \overline{) 72} \end{array}$$

②

$$\begin{array}{r} 25 \overline{) 800} \\ \updownarrow \\ 50 \overline{) 800} \end{array}$$

③

$$\begin{array}{r} 70 \overline{) 910} \\ \updownarrow \\ 35 \overline{) 910} \end{array}$$

④

$$\begin{array}{r} \overline{) 30} \\ \updownarrow \\ 80 \overline{) 1,200} \end{array}$$

⑤ Use the first two problems to help you with the third.

$$\begin{array}{l} 35 \overline{) 700} \\ 35 \overline{) 140} \end{array} \rightarrow 35 \overline{) 840}$$

Fill in the missing numbers in these problems. Look for shortcuts to reduce the work.

⑥

$$80 \overline{) 20} \rightarrow 79 \overline{) 20}$$

⑦

$$80 \overline{) 480} \rightarrow 79 \overline{) 6}$$

⑧ $474 + 1,580 = \underline{\hspace{2cm}}$

⑨

$$79 \overline{) 2,054}$$

Dividing Using Multiplication and the Area Model

Akiko was experimenting with numbers and came up with an idea.

She said:

Whenever you add two multiples of 7, the sum will **always** be a multiple of 7.

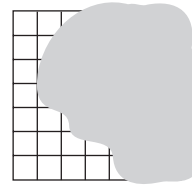
And if you add a multiple of 7 to a number that is **NOT** a multiple of 7, the sum is **never** a multiple of 7.

And she wrote:

Mult of 7 + Mult of 7 = Mult of 7

Mult of 7 + ~~Mult of 7~~ = ~~Mult of 7~~

- 1 Try some experiments to check her claims, and then explain what you found. An array picture with seven rows may help.



Extend Akiko’s idea. Here is an experiment. Try it.

What if neither number is a multiple of 7?

~~Mult of 7~~ + ~~Mult of 7~~ = ???

None of the numbers in Lists A and B is a multiple of 7.

List A	List B
3	4
10	11
17	18
24	25

- 2 Add any number from A to any number from B.

Is the sum a multiple of 7? _____

- 3 Add two numbers from the same list.

Is the sum a multiple of 7? _____

Recording the Steps in Division

Jonathan experimented with multiples of 3.

He said:

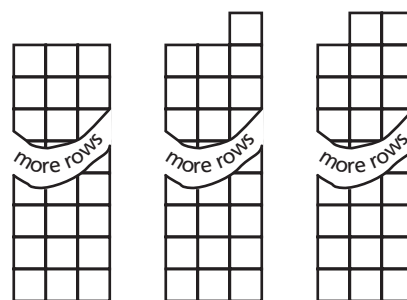
I made 3 lists:

- multiples of 3,
- numbers 1 greater than that,
- numbers 2 greater than that.

And he wrote:

List A Mult of 3	List B 1 + Mult of 3	List C 2 + Mult of 3
3	4	5
6	7	8
9	10	11
etc.	etc.	etc.

I drew array pictures like these because all the numbers in List A could be made with rows of 3 tiles and the numbers in List B could be made the same way but with one extra tile, and the numbers in List C needed two extra tiles.



Then I invented “Jonathan’s First Rule.” It says that if you add any two numbers from List B, their sum must **always** be in List C. I can prove it! Each number in List B has rows of 3 and one extra tile, so the sum of two such numbers must be rows of 3 with 2 extra tiles. Numbers like that are in List C.

Jonathan’s First Rule:
 $B + B = C$

1 Experiment with the numbers. What rules can you invent?

2 Explain one of your rules using the array pictures.

Dividing and Recording Division Efficiently

In some of these division problems, there will be something left over at the end — a remainder.

Some division problems have interesting patterns in the quotients. All the problems on this page involve dividing into a “power of 10” (1, or 10, or 10×10 , or $10 \times 10 \times 10$, or $10 \times 10 \times 10 \times 10$, and so on).

Find the quotients and remainders, and look for patterns.

- ① What do these problems have in common? What do the quotients have in common?

$$3 \overline{)1,000,000}$$

$$9 \overline{)1,000,000}$$

- ② What do these problems have in common? What do the quotients have in common?

$$22 \overline{)10,000,000}$$

$$55 \overline{)10,000,000}$$

$$66 \overline{)10,000,000}$$

- ③ What do you notice about this quotient?

$$\begin{array}{r} 142,857,142,857 \text{ r}1 \\ 7 \overline{)1,000,000,000,000} \end{array}$$

- ④ Without actually doing the division, try to predict this quotient. Problem 3 might help.

$$7 \overline{)1,000,000,000,000,000}$$

Using Multiplication to Check Division

To solve the puzzles on this page, you need to know three rules:

1 55×2 and 2×55 (and similar reversals) do not count as "different."

2 Multiplication problems can use only two numbers.

$$\cancel{55 \times 2 \times 13}$$

3 Use only whole numbers, please.

$$\cancel{3 \times 2\frac{1}{3}}$$

$$\cancel{3.5 \times 2}$$

Puzzle 1

The number 110 can be the answer to only four different whole-number multiplication problems.

1×110 , 2×55 , and 11×10 are three of them.

What is the fourth? _____

Puzzle 2

The numbers 210 and 1,155 can be the answers to many more multiplication problems. How many?

(Pick either 210 or 1,155.) List the problems.

Investigating Remainders

Try to predict the remainders without actually performing the division! In each set, pick three and check them with division.

1 $10 \overline{)84} \text{ r } \boxed{4}$

$10 \overline{)184} \text{ r } \boxed{}$

$10 \overline{)797} \text{ r } \boxed{}$

$10 \overline{)1,351} \text{ r } \boxed{}$

$10 \overline{)770} \text{ r } \boxed{}$

$10 \overline{)5} \text{ r } \boxed{}$

Tell why your rule works. _____

2 $2 \overline{)15} \text{ r } \boxed{}$

$2 \overline{)184} \text{ r } \boxed{}$

$2 \overline{)797} \text{ r } \boxed{}$

$2 \overline{)1,351} \text{ r } \boxed{}$

$2 \overline{)770} \text{ r } \boxed{}$

$2 \overline{)3} \text{ r } \boxed{}$

Tell why your rule works. _____

3 $5 \overline{)84} \text{ r } \boxed{}$

$5 \overline{)89} \text{ r } \boxed{}$

$5 \overline{)797} \text{ r } \boxed{}$

$5 \overline{)1,351} \text{ r } \boxed{}$

$5 \overline{)770} \text{ r } \boxed{}$

$5 \overline{)9} \text{ r } \boxed{}$

Tell why your rule works. _____

Interpreting Remainders in Word Problems

Write three word problems that involve division with a remainder. Be creative! Try to write something fun, or something you would want to solve.

1 In this one, it should make sense to ignore the remainder.

2 In this one, it should make sense to ignore the remainder.

3 In this one, it should make sense to use a fraction (or decimal) instead of a remainder.

Another Option for Interpreting Remainders

Write three different story problems that use the same numbers but a different option for dealing with the remainder.

- 1 Ignore the remainder.

$$\begin{array}{r} 40 \\ 10 \overline{) 403} \\ - 400 \\ \hline 3 \end{array}$$

- 2 Round the quotient up.

$$\begin{array}{r} 40 \\ 10 \overline{) 403} \\ - 400 \\ \hline 3 \end{array}$$

- 3 Use a fraction or decimal instead of a remainder.

$$\begin{array}{r} 40 \\ 10 \overline{) 403} \\ - 400 \\ \hline 3 \end{array}$$