Chapter

8 Developing a Division Algorithm

Dear Student,

You already know a lot about multiplication, like what is 3 × 7, and how to find products with larger factors, like 35 × 74. And you know how to use one multiplication problem to solve a related one. For example, you can use:

	35	×	74	=	2,590
to solve:	70	×	74	=	■

You also know connections between multiplication and division. Can you complete these related sentences?

35	×	74	=	2,590
2,590	÷	35	=	■
■	×	■	=	■
■	÷	■	=	■

There are other ways to connect multiplication and division. For example, you can use:

	35	×	74	=	2,590
to solve:	70	×	■	=	2,590

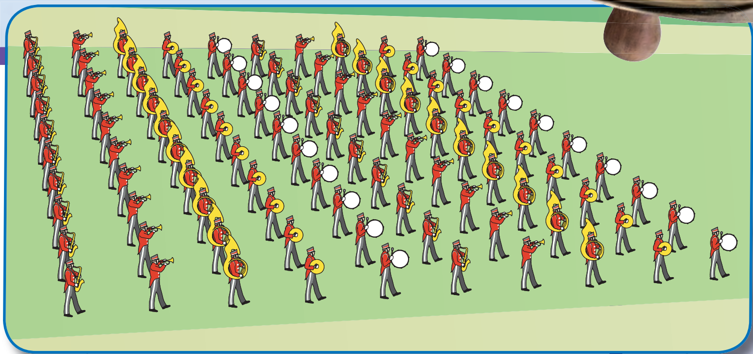
In this chapter, you will learn how to use what you already know—especially the connection between multiplication and division—to solve more difficult division problems, for example, 3,818 ÷ 46. You will also learn more about remainders.

Mathematically yours,
The authors of ***Think Math!***

THE WORLD ALMANAC FOR KIDS

Marching Bands

Have you ever seen a marching band in a parade or at a sporting event? Some elementary schools have marching bands, but you can see larger marching bands in high schools and colleges.



FACT·ACTIVITY 1

1. Suppose the University of Texas Longhorn Marching Band has 350 members. At the beginning of a performance, band members march in rows of 10. Describe what the missing factor in the equation tells you. $10 \times \blacksquare = 350$
2. Find the missing factor in Problem 1. Then, write a related division sentence.
3. If the marchers were arranged in rows of 5 instead of rows of 10, how many rows of marchers will there be? Explain how your answer is related to the answer to Problem 1.
4. Use an area model to show how the 350 Texas Longhorn marchers can be arranged in 25 rows. How many marchers will be in each row?

The Texas Longhorn Marching Band has played Presidential Inaugural events including the inauguration of Presidents George H. W. Bush and George W. Bush.

FACT • ACTIVITY 2

If you study a band instrument, it is likely to be one of three types: woodwind, brass, or percussion.

Use area models for 1–3.

1. Suppose the OMTAAMB band has 320 brass players, 128 woodwind players, and 112 percussion players. Group each section of the band so that there are 16 members in each row. How many rows will each section of the marching band have?

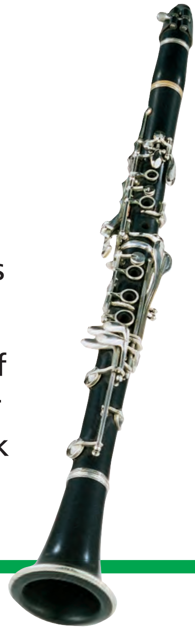
   Brass: $320 \div 16 = \blacksquare$

   Woodwind: $128 \div 16 = \blacksquare$

   Percussion: $112 \div 16 = \blacksquare$

2. If two of the drummers cannot march, what happens to the arrangement of percussion marchers?

3. Write a division sentence which shows the number of complete rows and the number of marchers left over if there are only 110 percussion marchers. Then check your answer using multiplication.



CHAPTER PROJECT

Pretend you are the band director for a marching band. Decide on the number of people in your band and how many are in each section (brass, percussion, and woodwind). Some bands also have flag twirlers, called color guards. Decide if they will be a part of your band.

Then, develop 3 different marching arrangements (formations) for the different sections of your band. Your first formation should be an attempt to have the band march into the stadium in equal rows. Use area models on grid paper, multiplication, and related division sentences for help if you wish. (Hint: The sections can be "rectangles" of different sizes.) Decide what you will do if there are "remainders."

Present your formations to the class. Use equations to explain the number of rows and the number of people in each row for each section.

ALMANAC Fact

The One More Time Around Again Marching Band (OMTAAMB) is the world's largest marching band. There are about 560 members in the band from far away places including Japan and New Zealand. The band played at the 2000 Olympic Games in Sydney, Australia.

Chapter 8

Lesson 1

EXPLORE
Shipping Stamps

Sheila's Shipping Company ships packages. The company uses special shipping stamps that look like this.

10¢	20¢	30¢	40¢	50¢	60¢	70¢	80¢	90¢
1¢	2¢	3¢	4¢	5¢	6¢	7¢	8¢	9¢

The clerks quickly discovered that any amount of postage under $1.00 could be made with no more than one stamp of each color.

Sheila's Shipping Company shipped 4 packages, each with the same amount of postage. If the total postage was $1.52, how much postage was on each package?

Remember that each package has only one stamp of each color.

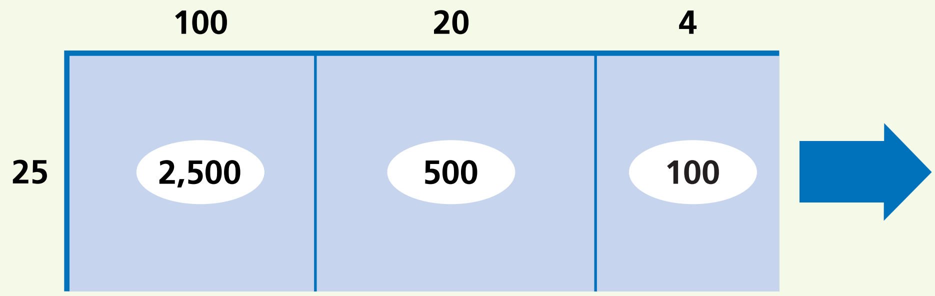
Here is a diagram to help you think about the problem.

Number of packages:	1	■	■	■	Postage on each package
Number of packages:	4	■	■	$1.52	Total Postage

Chapter 8
Lesson 3

EXPLORE
Multiplying and Un-Multiplying

You already know that you can figure out a large product by multiplying one part at a time and then adding the results.

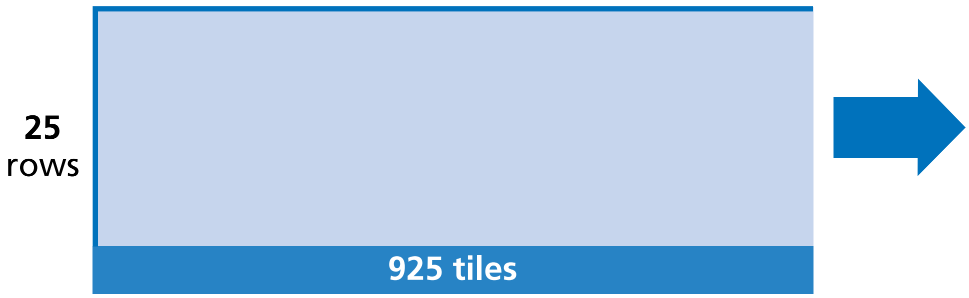


$$\begin{array}{r} 124 \\ \times\ 25 \\ \hline 2{,}500 \\ 500 \\ 100 \\ \hline 3{,}100 \end{array}$$

Use what you know to solve this problem.

A tile layer set 925 square tiles in 25 equal rows. How many tiles were in each row?

Draw the area model on a separate piece of paper. Use it to help you solve this problem any way you like.



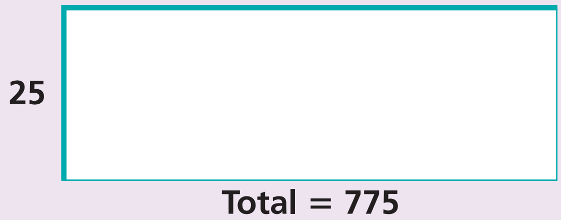
$$25\overline{)\,925}$$

Chapter 8
Lesson 3

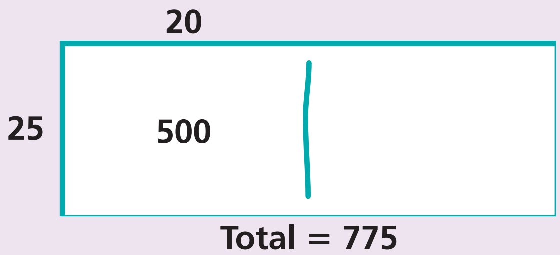
REVIEW MODEL
Dividing Using an Area Model

You can use an area model to divide a large number. The model below shows how to divide 775 ÷ 25.

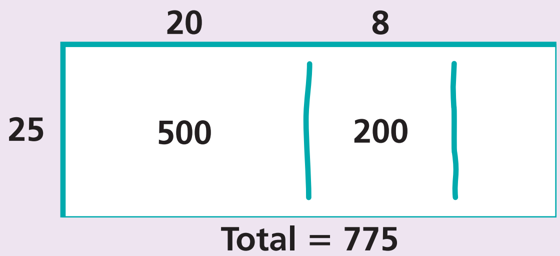
Step 1 Draw the area model.



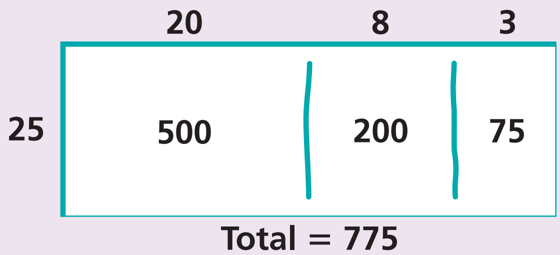
Step 2 In a box, write a multiple of 25 (the greater, the better!) that is less than or equal to 775. Carve out that part.



Step 3 Subtract to find out how much of the 775 is left; 775 − 500 = 275. Now, find a multiple of 25 that is less than or equal to 275. Carve out that part. Above the model, write how wide that part must be.



Step 4 Subtract to find out how much of the 275 is left; 275 − 200 = 75. Find a multiple of 25 that is less than or equal to 75. Above the model, write how wide that part must be.
20 + 8 + 3 = 31
So, 775 ÷ 25 = 31



Check for Understanding

Use an area model to divide.

1. 375 ÷ 25
2. 625 ÷ 25
3. 950 ÷ 25
4. 800 ÷ 25

Chapter 8
Lesson 4

REVIEW MODEL
Finding Multiples

You can use mental math strategies to make a table of multiples. Here is one way to find multiples of 12.

First, find 12 times 1, 2, 4, and 8.

×	**1**	**2**	3	**4**	5	6	7	**8**	9	10
12	**12**	**24**		**48**				**96**		

$12 \times 1 = 12$

Double what you wrote in the "× 1" box.

Double what you wrote in the "× 2" box.

Double what you wrote in the "× 4" box.

Now, find 12×10. Use the product to find 12×5.

×	1	2	3	4	**5**	6	7	8	9	**10**
12	12	24		48	**60**					**120**

Halve 12×10.

Write a "0" after 12.

Finally, use addition to find the remaining products.

×	1	2	**3**	4	5	**6**	**7**	8	**9**	10
12	12	24	**36**	48	60	**72**	**84**	96	**108**	120

$3 = 1 + 2$. So, add $12 + 24$.

$6 = 5 + 1$. So, add $60 + 12$.

$7 = 5 + 2$. So, add $60 + 24$.

$9 = 5 + 4$. So, add $60 + 48$.

Check for Understanding

Copy and complete the table of multiples of 16.

×	1	2	3	4	5	6	7	8	9	10
16	■	■	■	■	■	■	■	■	■	■

Chapter 8

Lesson 7

EXPLORE
What Remainders Are Possible?

When you divide 84 by 3 the way you did in class, the record ends with "0 left over."

$$\begin{array}{r l} 8 & \\ 2\,0 & \\ 3\overline{)8\,4} & \text{Total} \\ \underline{-\,6\,0} & \\ 2\,4 & \text{Left} \\ \underline{-\,2\,4} & \\ 0 & \text{Left} \end{array}$$

When you divide 97 by 3 the way you did in class, the record ends with "1 left over."

$$\begin{array}{r l} 2 & \\ 3\,0 & \\ 3\overline{)9\,7} & \text{Total} \\ \underline{-\,9\,0} & \\ 7 & \text{Left} \\ \underline{-\quad 6} & \\ 1 & \text{Left} \end{array}$$

1. What other leftovers can a division by 3 record end with?

   Experiment with some numbers, or find another way to support your answer.

2. What leftovers can a division by 12 record can end with?

   Be ready to give a reason that supports your answer.

Chapter 8

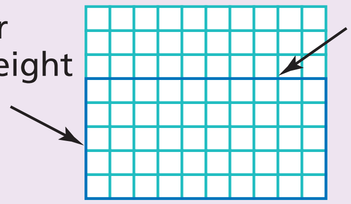
Lesson 7

REVIEW MODEL
Dividing Using a Grid

You can use a grid to find both the whole-number quotient and the remainder in a division problem. The following steps show how to use a grid to find 42 ÷ 5.

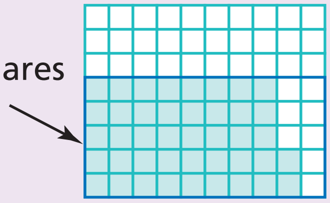
Step 1

Estimate the quotient. On grid paper, sketch a rectangle with a width greater than your estimate, and a height equal to the divisor.



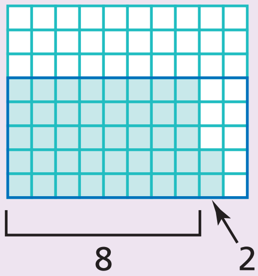
Step 2

Begin filling in columns, moving left to right. Count squares as you go. Stop when the number of squares you have filled equals the dividend.



Step 3

The whole-number part of the quotient is the number of columns you have filled. The remainder is the number of boxes you have filled in the last unfilled column.



So, $42 \div 5 = 8$ r2 or $8\frac{2}{5}$.

Check for Understanding

Find the whole-number quotient and, if present, the remainder. You can use a grid to help you.

1. $28 \div 5$
2. $58 \div 9$
3. $55 \div 7$
4. $78 \div 6$
5. $93 \div 4$
6. $79 \div 8$

Chapter 8
Lesson 8

EXPLORE
What Do I Do About the Remainder?

For each problem, divide and then decide what to do about the remainder.

1 Sharing pennies.

First graders collected pennies in a jar all year to give to charity. They wanted to divide the money evenly among a dozen favorite charities. When they had collected \$9.17, how much money, in pennies, would each charity have received?

2 Sharing dollars.

The first graders realized that a dozen charities was too many. They narrowed the number to only eight, and asked parents to help them raise more money. When the students had collected exactly \$100, they divided it evenly among the eight charities. How much did each charity get?

Chapter 8

Lesson 9

REVIEW MODEL
Interpreting Remainders

When you divide with whole numbers, there is sometimes a remainder. What you do about the remainder depends on the situation in the problem.

Ignore the Remainder

Emma made punch with 12 ounces of cranberry juice, 64 ounces of orange juice, and 32 ounces of pineapple juice. How many 8-ounce servings did she make?

$$\begin{array}{r} 12 \\ 64 \\ +\ 32 \\ \hline 108 \end{array} \qquad \begin{array}{r} 13 \\ 8\overline{)108} \\ -\ 8 \\ \hline 28 \\ -\ 24 \\ \hline 4 \end{array}$$

She made 13 eight-ounce servings.

Ignore the remainder because the 4 ounces that are left over are not enough to make a serving.

Include the Remainder as a Fraction or Decimal

James has a 270-foot piece of rope that he will cut into 12 equal lengths to make jump ropes. How long will each jump rope be?

$$\begin{array}{r} 22\tfrac{1}{2},\ \text{or } 22.5 \\ 12\overline{)270} \\ -\ 24 \\ \hline 30 \\ -\ 24 \\ \hline 6 \end{array}$$

Each jump rope will be $22\frac{1}{2}$, or 22.5, feet long.

Write the remainder as a fraction or a decimal because it makes sense to have a part of a foot. $\frac{6}{12} = \frac{1}{2} = 0.5$

Round the Quotient Up

There are 1,245 students and parents who signed up for the annual school picnic. Each table will seat 8 people. How many tables will they need?

$$\begin{array}{r} 31 \\ 8\overline{)250} \\ -\ 24 \\ \hline 10 \\ -\ \ 8 \\ \hline 2 \end{array}$$

They will need 32 tables.

Round up the quotient to the next whole number because 31 tables will not be enough to seat everyone.

Check for Understanding

Solve each problem. Explain how you decided what to do about the remainder.

1. A total of 124 players are riding a bus to the soccer game. If 25 players can ride in each bus, how many buses are needed?

2. The bakery sells boxes of a dozen muffins. They have 256 muffins ready to put into boxes. How many boxes can they fill?

3. Todd saved the same amount of money each week for 52 weeks. At the end of 52 weeks he had saved $754. How much did he save each week?

Chapter 8
Lesson 10

REVIEW MODEL
Problem Solving Strategy
Draw a Picture

Ashley rode her bike 13 miles due east from her house to the art museum. From there she rode 4 miles south to the mall, 2 miles west to the swimming pool, 4 miles north to the library, and then home. If she rode 6 miles per hour, how long did it take her to ride home from the library?

Strategy: Draw a Picture

Read to Understand

What do you know from reading the problem?

the speed, distances, directions, and destinations to which Ashley rode on her bike

What do you need to find out?

how long it took Ashley to ride from the library to her home

Plan

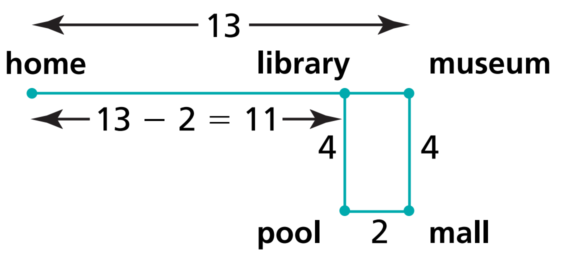
How can you solve this problem?

It's hard to tell where the library is in relation to Ashley's home, so draw a picture of her ride.

Solve

How can you draw a picture of the problem?

The drawing shows Ashley's ride. The library is 13 miles − 2 miles, or 11 miles from her house. Since she rode 6 miles per hour, you can divide 11 by 6 to find how long it took her to ride home from the library. $11 \div 6 = 1$ r5 hr, or $1\frac{5}{6}$ hr or 1 hr 50 min.



Check

Look back at the original problem. Did you answer the question that was asked? Does your answer make sense?

Problem Solving Strategies

- ✔ Act It Out
- ✔ **Draw a Picture**
- ✔ Guess and Check
- ✔ Look for a Pattern
- ✔ Make a Graph
- ✔ Make a Model
- ✔ Make an Organized List
- ✔ Make a Table
- ✔ Solve a Simpler Problem
- ✔ Use Logical Reasoning
- ✔ Work Backward
- ✔ Write an Equation

Problem Solving Practice

Use the strategy *draw a picture* to solve.

1. Sixteen teams are entered in a soccer tournament. A team is eliminated if it loses a game. The Jaguars won the tournament. How many games did they win?

2. A game board is in the shape of a hexagon. A line is drawn from each corner of the board to every other corner. How many lines are there on the game board?

Mixed Strategy Practice

Use any strategy to solve. Explain.

3. Martina has a nickel, a dime, a quarter, and a half dollar. How many different values can she make using combinations of one or more coins?

4. The sum of Andy's and Mandy's ages is 22. The product of their ages is 96. If Andy is older than Mandy, how old is Andy?

5. Brett wants to save $750 for his vacation. When he has saved 3 times as much as he has saved already, he will need only $27 more. How much has he already saved?

6. Amy started on the 8th floor of a skyscraper. She went up 13 floors, down 19 floors, up 7 floors, and down 5 floors. When she finished, how many floors from her starting point was she? Was she above or below?

7. A drawer contains 8 red socks and 8 white socks. How many socks must you remove to be sure you have two of the same color?

8. Jeff plans to build a 95-yard-long fence. He will put a post every 5 yards. Each post costs $8. How much will it cost him to buy the posts?

9. The first digit of Mr. Valvano's age is an even number. The second digit is one-third the first. How old is he?

10. There are 88 keys on a piano. Fifty-two of the keys are white. The rest are black. How many more white keys are there than black keys?

Chapter 8 Vocabulary

Choose the best vocabulary term from Word List A for each sentence.

1. To __?__ a number is to break it into smaller parts that total the given number.
2. When deciding how to report a remainder, you need to make the most sensible or __?__ choice.
3. In division, the __?__ is the number that is left over when one number does not divide into another evenly.
4. In a division problem, the number you divide into is called the __?__.
5. The result of multiplying two or more numbers together is a(n) __?__.
6. Multiplication and division are __?__ operations because each undoes the other.
7. In a division problem, the number you divide by is called the __?__.
8. An approximation and a(n) __?__ are both guesses that are near an exact answer.
9. In division, adding 1 to the quotient and dropping the remainder is called __?__.
10. __?__ are numbers used to estimate the quotient of a division problem.

Word List A

approximation
compatible numbers
dividend
division
divisor
estimation
factor
inverse
output
partition
product
quotient
reasonable
remainder
rounding up the quotient

Complete each analogy using the best term from Word List B.

11. Addend is to sum as __?__ is to product.
12. Subtraction is to difference as division is to __?__.

Word List B

dividend
divisor
factor
quotient

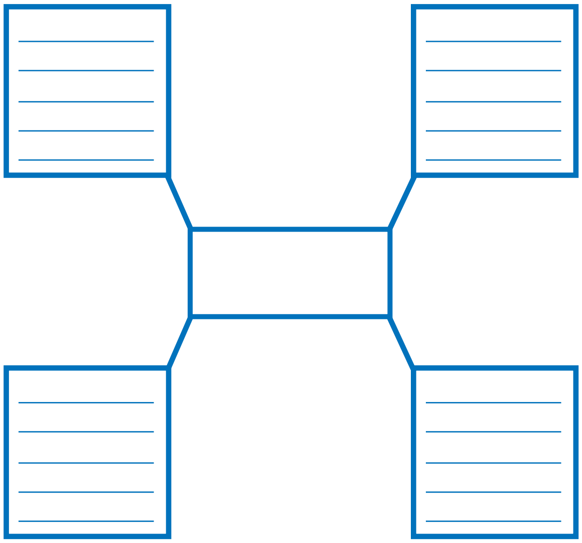
Talk Math

Discuss with a partner what you have just learned about division. Use the vocabulary terms *dividend, divisor,* and *quotient.*

13. How can you use compatible numbers to find a partial quotient?
14. How do you know whether a division problem has a remainder?
15. How can you use multiplication to check a division problem?

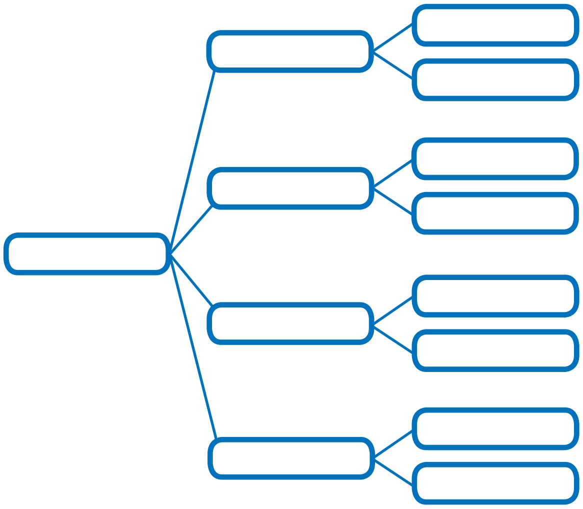
Word Web

16 **Create a word web for the word *divide.* Use what you know and what you have learned about division.**



Tree Diagram

17 **Create a tree diagram using the word *operation.* Use what you know and what you have learned about *addition, subtraction, multiplication,* and *division.***



What's in a Word?

PRODUCT The word *product* has two different meanings. In everyday conversation, a *product* is the result of work. The *product* of an athletic equipment company could be a soccer ball or a tennis racket. In mathematics, a *product* is the result of multiplying two or more numbers together. In both cases, a *product* is the result of some action, but the actions are different.

GO ONLINE **Technology**
Multimedia Math Glossary
www.harcourtschool.com/thinkmath

GAME

200 Zoom

Game Purpose
To practice multiplication and addition

Materials
- 4 sets of number cards (1 to 12)

200 Zoom

How To Play The Game

1. Play this game with up to 3 other students. The goal is to score as many points as possible without going over 200. Decide who will go first.

2. The first player mixes up all the cards, and places them face down in a pile. The same player turns over the top card and says the "Zoom" number for that round. A "Zoom" number is the number on a card plus 10.

   Example: The *top* card is a 4.
   Zoom number = 10 + 4, or 14.

3. Take turns, moving clockwise from the first player. Take one card from the pile, and keep the number secret.
   - Secretly multiply your number by the *Zoom* number. That is your score.
   - You can increase your score by taking more cards from the pile. Put each of those cards face up in front of you. If you have more than one card, your score is the sum of the numbers on the cards times the "Zoom" number.

   Example: Zoom number: 14 Cards drawn: 3, 7, 1
   Score: $3 + 7 + 1 = 11$, $11 \times 14 = 154$
   - If your score is more than 200, you are out and must show all your cards.

4. Once everyone who is not out has all the cards they want, calculate and say your scores. Whoever is the closest to 200 without going over wins the round.

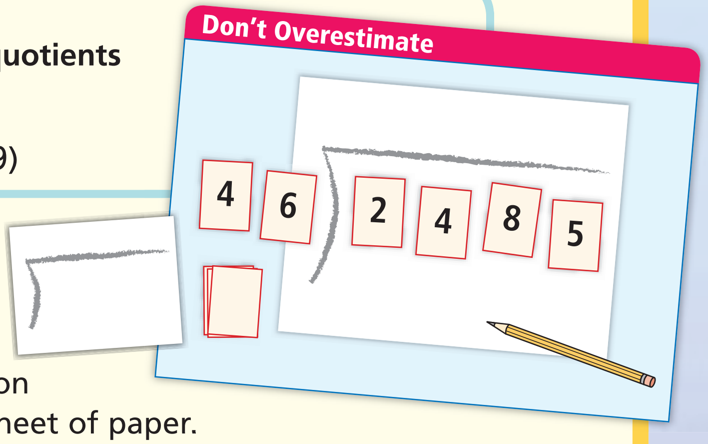
5. Play more rounds until all the cards have been taken. Whoever is the closest to 200 without going over wins the final round.

GAME

Don't Overestimate

Game Purpose
To practice estimating partial quotients

Materials
- 4 sets of number cards (1 to 9)

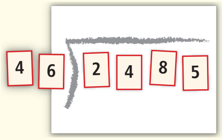


How To Play The Game

1. Play this game with a partner.
   - Make a game mat. Draw a division box that is almost as large as a sheet of paper.
   - Mix up the number cards. Place them face down in a pile.

2. The first player takes four cards. Place them on the mat side-by-side to make your dividend. Then take two more cards and place them side-by-side to make your divisor. Place the cards in the order in which they are picked.

   Example: Max picked: 2, 4, 8, 5, 4, 6.



3. The same player tries to estimate the quotient *without* going over the actual quotient.
   - The estimate can only have one non-zero digit.
   - Numbers such as 30, 70, 100, 400, 900 are allowed. Numbers such as 12, 150, and 410 are not.

4. After the first player names a number, the second player may challenge it by offering a different estimate if he or she thinks that:
   - a better estimate is possible; or
   - the first estimate is already too high.

   Otherwise, the second player passes.

5. Multiply each estimate by the divisor. If your product is less than the dividend, your product is your score. If your product is greater than the dividend, your score is 0.

6. Switch roles to complete the round. At the end of each round, calculate your scores. Whoever gets to 10,000 first wins!

CHALLENGE

Quotient Families

Sixteen division problems are grouped into four quotient families.

Blue Family	Yellow Family
818 ÷ 24	1,013 ÷ 36
618 ÷ 18	2,671 ÷ 62
1,404 ÷ 41	1,088 ÷ 19
1,193 ÷ 35	413 ÷ 24

Red Family	Green Family
529 ÷ 23	1,532 ÷ 14
1,225 ÷ 35	2,298 ÷ 21
361 ÷ 19	3,498 ÷ 32
1,681 ÷ 41	4,907 ÷ 45

1. Be a quotient detective. Look at each quotient family. How are the problems in each family alike?

2. Now use what you know about each quotient family to match each of these division problems with its quotient family.

A 5,017 ÷ 46

B 1,201 ÷ 46

C 1,573 ÷ 46

D 2,116 ÷ 46