## Investigating Angles

## Another Experiment with Triangles

Step 1: Use a ruler to draw a large acute triangle (all acute angles) on a blank sheet of paper. Try to make all the sides different lengths. Label the angles on the inside of the triangle $\boldsymbol{X}, \boldsymbol{Y}$, and $\boldsymbol{Z}$, or use a different-colored dot in each corner of the triangle.


Step 2: Cut out your triangle and tear off the three corners.

(1) Imagine putting two angles together, for example, angles $X$ and $Z$. What are all the ways you can arrange two of the three angles?

$\qquad$
(2) Make each pair of angles. Is the combined angle acute, obtuse, or right?
$\qquad$
(3) Try to explain your result for Problem 2.

$\qquad$
$\qquad$
$\qquad$
(4) Try again, using a right triangle (one right angle and the others acute) and an obtuse triangle (one obtuse and the others acute). In each case, do steps 1 and 2 and then explain your results.
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Classifying Angles and Triangles

When there is more than one angle at a vertex, you can name angles using other points. For example, $\angle B A C$ has sides $A B$ and $A C$.


| Angle | Measure |
| :---: | :---: |
| $\angle B A C$ | about $\square^{\circ}$ |
| $\angle C A D$ | about $\square^{\circ}$ |
| $\angle B A D$ | about $\square^{\circ}$ |

2


| Angle | Measure |
| :---: | :---: |
| $\angle F E G$ | about $\square^{\circ}$ |
| $\angle G E H$ | about $\square^{\circ}$ |
| $\angle F E H$ | about $\square^{\circ}$ |

(3)


| Angle | Measure |
| :---: | :---: |
| $\angle J I K$ | about $\square^{\circ}$ |
| $\angle K I L$ | about $\square^{\circ}$ |
| $\angle J I L$ | about $\square^{\circ}$ |



Draw $\angle N M O$ to measure $60^{\circ}$.
$\qquad$
$\qquad$

## Constructing Triangles


(1) Use a cm ruler and a protractor to measure each side and angle.

| Name | Measure |  | Name <br> $\angle D$ | Measure |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\angle A$ | about | - |  | about |  |
| $\angle B$ | about | - | $\angle E$ | about | $\bigcirc$ |
| $\angle C$ | about | - | $\angle F$ | about | $\bigcirc$ |
| $\overline{A B}$ | about |  | $\overline{D E}$ | about |  |
| $\overline{B C}$ | about |  | $\overline{E F}$ | about |  |
| $\overline{A C}$ | about |  | $\overline{D F}$ | about |  |

2 Find two pairs of congruent sides and a pair of congruent angles.
$\qquad$
$\qquad$
(3) In the lesson, using two pairs of congruent sides and a pair of congruent angles, everyone made congruent triangles. What's different here that allows the triangles not to be congruent?
$\qquad$
$\qquad$
$\qquad$

## Constructing Similar Triangles

If you double, or triple, or quadruple the lengths of the sides of a figure, how many of the originals will fit inside the enlargement?

Here's a way to explore this question.
(1) Use tiles or graph paper to make squares of various sizes. How many of one size fit inside a square that has twice the side length? Does your answer depend on how large the smaller square is (as long as the larger has twice the side length)?
(2) What if the larger square's sides are triple the lengths of the smaller one's sides? How many of the smaller fit inside the larger? Does your answer depend on how large the smaller square is?
$\qquad$
(3) What if the larger square's sides are four times the length of the smaller one's sides? How many of the smaller fit inside the larger?

Can you predict the results for five times the side lengths? 10 times the side lengths? $\boldsymbol{n}$ times the side lengths?
$\qquad$
$\qquad$
(5) Try to find a way to check tripling the lengths of the sides for rectangles or triangles. Does it work?

## Angles Formed by Intersecting Lines

This figure shows $\triangle A B C$ split into two other triangles:
$\triangle A D B$ and $\triangle A D C$. Two of the angle measures are shown, and $\angle W$ and $\angle X$ are congruent. Write the angle measures.
(1) $\mathrm{m} \angle W=$
(2) $\mathrm{m} \angle X=$
(3) $\mathrm{m} \angle Y=$ $\qquad$
(4) $\mathrm{m} \angle Z=$ $\qquad$


No protractors, please!


Complete the number sentences using only angle measurements G, $\boldsymbol{H}, \mathbf{I}$, and $J$.
(5) $\mathrm{m} \angle$ $\qquad$ $+m \angle$ $\qquad$ $+m \angle$ $\qquad$ $=180^{\circ}$
(6) $m \angle$ $\qquad$ $+m \angle$ $\qquad$ $=180^{\circ}$
(7) $\mathrm{m} \angle$ $\qquad$ $+m \angle$ $\qquad$ $=\mathrm{m} \angle$ $\qquad$

## Angles Formed by a Line Intersecting Parallel Lines

Charles designed the rectangular flag below. To get his mother to help him make the flag, he wanted to tell her the sizes of all the pieces, including all the angle measures.

The center line is parallel to the vertical sides of the rectangle. The diagonal lines in the center form a parallelogram (opposite sides are parallel.)
Two angle measures have been given. All the rest can be found using what you know about opposite angles, Z's, straight angles, the sum of angles in triangles, and the angles of rectangles.
Find the measures of all the angles.


| Angle | Measure |
| :---: | :---: |
| $\angle \boldsymbol{A}$ | $\circ$ |
| $\angle \boldsymbol{B}$ | $\circ$ |
| $\angle \boldsymbol{C}$ | $\circ$ |
| $\angle \boldsymbol{D}$ | $\circ$ |
| $\angle \boldsymbol{E}$ | $\circ$ |
| $\angle \boldsymbol{F}$ | $\circ$ |
| $\angle \boldsymbol{G}$ | $\circ$ |
| $\angle \boldsymbol{H}$ | $\circ$ |


| Angle | Measure |
| :---: | :---: |
| $\angle \boldsymbol{I}$ | ${ }^{\circ}$ |
| $\angle \boldsymbol{J}$ | $\circ$ |
| $\angle \boldsymbol{K}$ | $\circ$ |
| $\angle \boldsymbol{L}$ | $\circ$ |
| $\angle \boldsymbol{M}$ | ${ }^{\circ}$ |
| $\angle \boldsymbol{N}$ | $\circ$ |
| $\angle \mathbf{O}$ | $\circ$ |
| $\angle \boldsymbol{P}$ | $\circ$ |

## Comparing and Classifying Quadrilaterals

Here are some quadrilaterals, along with their most specific names.

rectangle

square

kite


In the diagram below, quadrilaterals $A$ and $D$ have each been placed inside an oval. Label the sections and place the remaining quadrilaterals so that each is correctly named by all the ovals and sections it's inside.


## Investigating Quadrilaterals

(1) Without using a protractor, find the missing angle measurements.


## The lines above form triangles, quadrilateral, and pentagons (five-sided polygons).

2 Identify two pentagons that do not "cave in" on themselves and record their vertices. (Pentagon ACHED doesn't count, because vertex $D$ makes it cave in.)
(3) Find the sums of the measures of the angles inside the convex pentagons you identified. (Only include the angles at the vertices.) What do you notice?
(4) Fill in the table for polygons that do not cave in on themselves. Find a pattern and predict the sums of the measures of the angles inside a hexagon (six sides) that does not cave in on itself.

| Number of Sides | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- |
| Sum of the Angle Measures |  |  |  |  |

