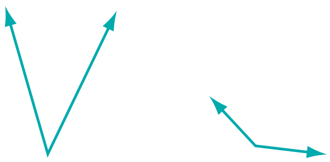
Chapter

9 Attributes of Two-Dimensional Figures

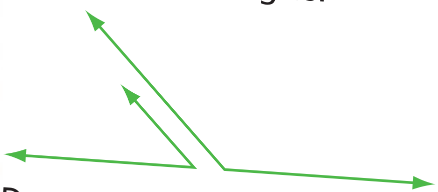
Dear Student,

You already know quite a bit about two-dimensional figures, such as quadrilaterals and triangles and their parts, such as sides and angles. In this chapter, you will extend this knowledge. You will concentrate on studying angles and on developing different methods of comparing and finding angle sizes. For example, what do you notice about these two angles?



Not only will you learn to measure angles, but you will also learn other strategies for finding the size of certain angles without measuring.

Here are two angles:



Do you know the sum of the measures of these two angles? You will know this answer (without measuring) and many more facts about angles and two-dimensional figures by the end of this chapter.

Mathematically yours,
The authors of ***Think Math!***

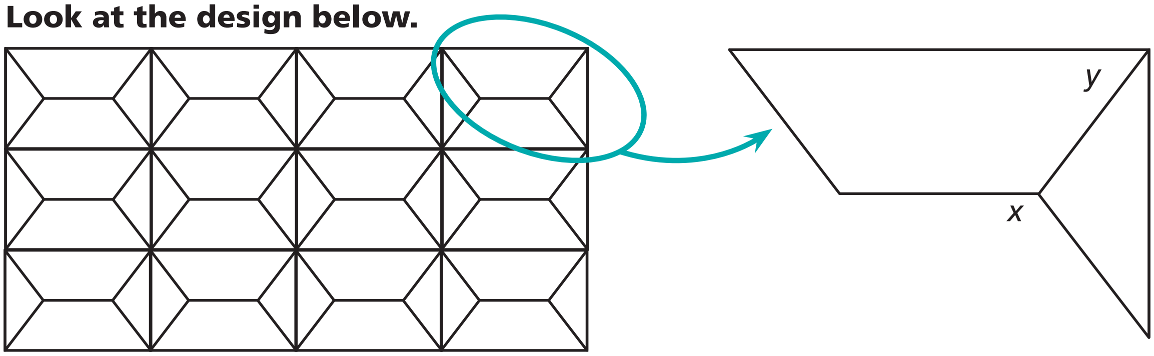
THE WORLD ALMANAC FOR KIDS

Patterns in Play

What could a soccer ball have in common with a brick wall? What about a quilt and a checker board? All of these objects are put together with a similar pattern called a tessellation. A tessellation is the tiling of a surface using a pattern of figures or polygons. Can you think of any other objects that have tessellations?

FACT · ACTIVITY 1

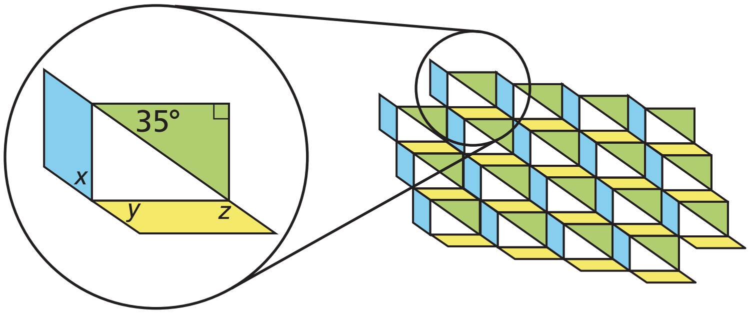
Look at the design below.



1. Use a protractor to measure angles *x* and *y*.
2. Is the triangle shown an acute, right, or obtuse triangle? Use a protractor to measure its angles.
3. Draw the triangle on a piece of paper so that the longest side measures 6 cm. About how long are the other 2 sides?

FACT·ACTIVITY 2

M.C. Escher is the father of modern tessellation art. The design below is an example of tessellation art. The white and green triangles are congruent.



Trace the enlargement of the single set of figures. Shade the sections blue, yellow, and green as shown. Use your drawing to answer the questions below.

1. Look at the blue quadrilateral. Which angle is congruent to *x?* Label the congruent angle *x.*
2. Find other angles that are congruent to *x.* Label them *x,* also.
3. Look at the yellow quadrilateral. Which angle is congruent to *y?* Label it *y.* Which angle is congruent to *z?* Label it *z.*

CHAPTER PROJECT

Make your own tessellating pattern. Draw a triangle or quadrilateral on an index card. Within the figure, draw 2 segments to divide your figure into 3 smaller figures (triangles or quadrilaterals).

Cut out the 3 figures and trace multiple copies of each one on a different color of construction paper. Cut out at least 10 pieces of each figure. Arrange and glue the pieces to a larger sheet of cardboard or poster board to form a tessellating pattern. Remember, there should be no gaps or open spaces between figures. Display your tessellation in your classroom.

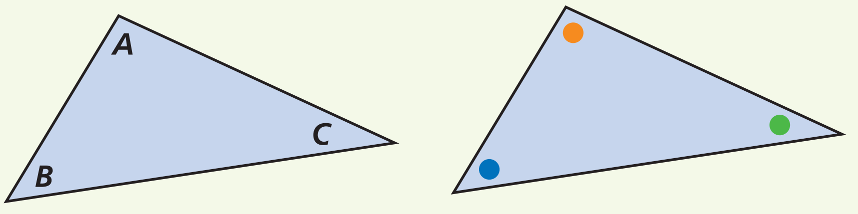
ALMANAC Fact

M.C. Escher created over 2,000 drawings and sketches and about 450 lithographs, woodcuts, and engravings.

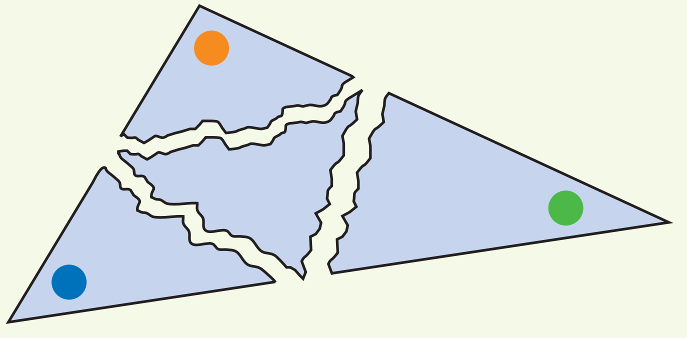
EXPLORE
An Experiment with a Triangle

Try this experiment and compare your results to your classmates' results.

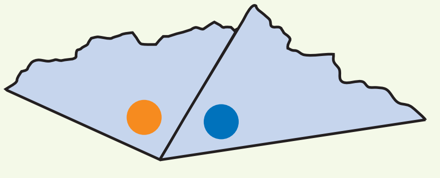
Step 1 Use a ruler to draw a large triangle on a blank sheet of paper. Try to make all the sides different lengths. Label the angles on the inside of the triangle ***A, B,*** and ***C,*** or use a different-color dot in each corner of the triangle.



Step 2 Cut out your triangle and tear off the three corners.



Step 3 Fit the three marked pieces together so that the dotted corners all meet at a single vertex. Tape them together as closely as possible without overlapping, like these two.



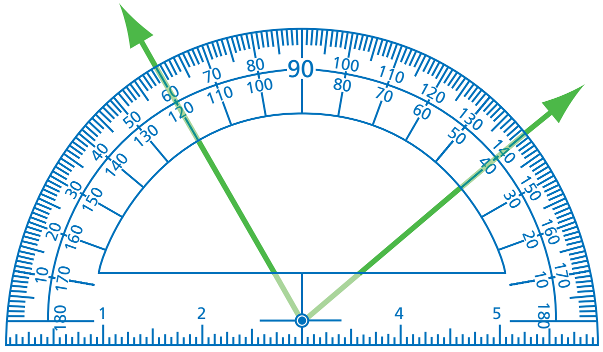
Sketch what you see and describe the result in words.

Chapter 9
Lesson 2

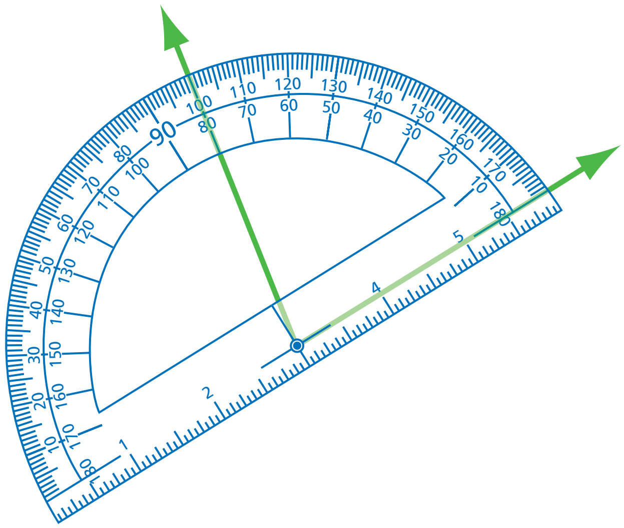
EXPLORE
How to Use a Protractor

Follow these steps to use a protractor to measure angles.

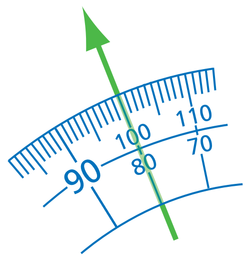
1. Match the circle in the center of the straight side of the protractor to the vertex of the angle you want to measure.



2. Match the zero mark on the protractor to one of the lines, or parts of lines, that form the angle.



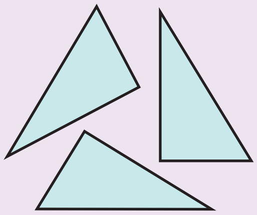
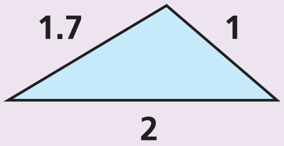
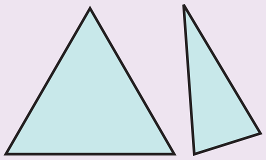
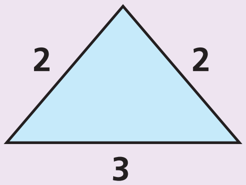
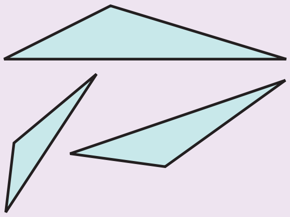
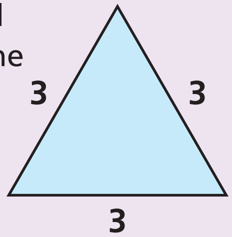
3. The other line, or part of a line, must cross the curved side of the protractor. Read the measurement from the curved side. For **acute angles**, use the smaller number. For **obtuse angles**, use the larger number.



Chapter 9
Lesson 2

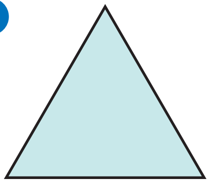
REVIEW MODEL
Classifying Triangles

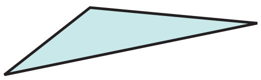
You can classify triangles by the measure of their angles or by the lengths of their sides.

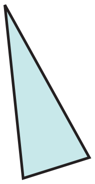
Classifying by Angles		Classifying by Sides	
A triangle is **right** if it has an angle that measures 90°.		A triangle is **scalene** if all of its sides have different lengths.	
A triangle is **acute** if all the angles measure less than 90°.		A triangle is **isosceles** if at least two of its sides have the same length. The two sides are congruent.	
A triangle is **obtuse** if one angle measures more than 90° and less than 180°.		A triangle is **equilateral** if all of its sides have the same length. All three sides are congruent.	

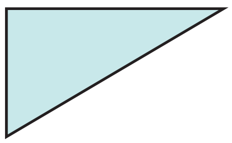
Check for Understanding

Classify each triangle. Write *right, acute,* or *obtuse.*

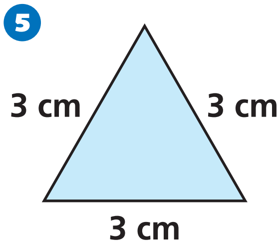
1 

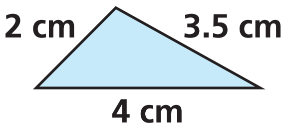
2 

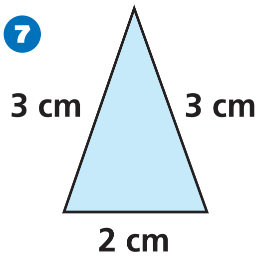
3 

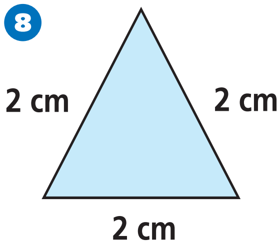
4 

Classify each triangle. Write *scalene, isosceles,* or *equilateral.*

5 

6 

7 

8 

REVIEW MODEL
Constructing Triangles

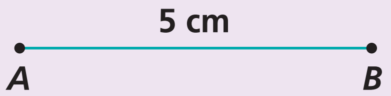
Side-Angle-Side

If you know the measurements of two sides of a triangle and the angle between them, you can construct the triangle.

For example, if $\triangle ABC$ has:

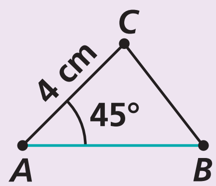
Length of $\overline{AB}$	5 cm
Length of $\overline{AC}$	4 cm
Measure of $\angle A$	45°

1. Use a ruler to draw either segment, using the correct length. Label the endpoints.



2. Use a protractor to draw the angle you know.

3. Extend or shorten the segment you drew until it is the correct length for the second side you know. Label the new endpoint and draw the third side.



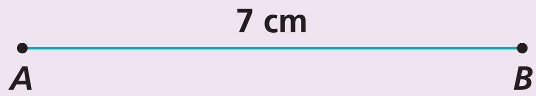
Angle-Side-Angle

If you know the measurements of two angles of a triangle and the side between them, you can construct the triangle.

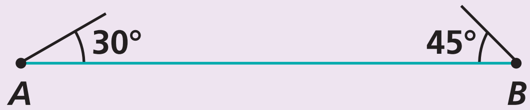
For example, if $\triangle ABC$ has:

Length of $\overline{AB}$	7 cm
Measure of $\angle A$	30°
Measure of $\angle B$	45°

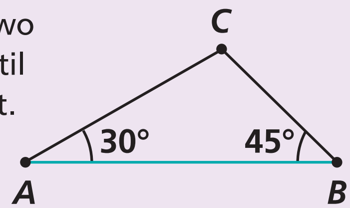
1. Use a ruler to draw a segment with the correct length. Label the endpoints.



2. Use a protractor to draw the two known angles at the endpoints.



3. Extend the two new sides until they intersect. Label the intersection as the third vertex.



Check for Understanding

Use a ruler and a protractor to construct a triangle for each group of measures.

1. $\triangle ABC$ has
length $\overline{AB}$: 6 cm
length $\overline{AC}$: 5 cm
measure of $\angle A$: 60°

2. $\triangle DEF$ has
length $\overline{DE}$: 7 cm
length $\overline{DF}$: 4 cm
measure of $\angle D$: 90°

3. $\triangle GHJ$ has
length $\overline{GH}$: 8 cm
measure of $\angle G$: 45°
measure of $\angle H$: 90°

4. $\triangle KLM$ has
length $\overline{KL}$: 10 cm
measure of $\angle K$: 30°
measure of $\angle L$: 60°

Chapter 9
Lesson 4

EXPLORE
Shrinking a Triangle

For this exploration, you need one of the Measure Me Activity Masters (AM70–AM73). Your goal is to make a triangle, △*ABC*, whose sides are half the size of the sides of △*ABC* on your Measure Me page, but which has the same angle measures.

DIRECTIONS:

A Measure the sides and angles of the triangle on your **Measure Me** page and record them. You will have 6 measurements (3 angles and 3 sides).

B Use your measurements to calculate the lengths of the half-sized triangle's sides:

The length of $\overline{XY}$ is half the length of $\overline{AB}$.

The length of $\overline{YZ}$ is half the length of $\overline{BC}$.

The length of $\overline{XZ}$ is half the length of $\overline{AC}$.

C On blank paper, draw $\overline{XY}$, using the measurement you wrote in Step B. Label its endpoints ***X*** and ***Y.*** (This works best if ***X*** is near the bottom left of your paper).

D You know how long $\overline{XZ}$ should be, but where does it go? At point ***X,*** measure the angle you need, and sketch the line that $\overline{XZ}$ is part of. Don't worry about that line's length.

E Now measure the correct distance from point ***X*** along your new line and label that point ***Z.***

F Use your ruler to connect points ***Y*** and ***Z.***

G Measure $\overline{YZ}$. Is it within one cm of the length you expected?

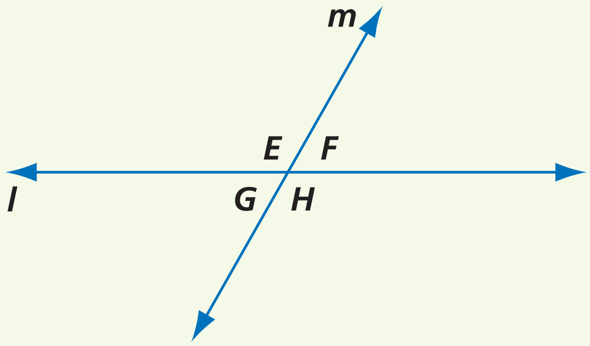
Chapter 9

Lesson 5

EXPLORE

Angles Formed by Intersecting Lines

In this activity, m∠*E*, m∠*F*, m∠*G*, and m∠*H* represent the measures of the angles formed by the intersecting lines.



1 Without using a protractor, what can you say about the following sums of angle measures?

A m∠*E* + m∠*F* = ■ **B** m∠*H* + m∠*F* = ■

2 Find **m∠*F*** with a protractor. **m∠*F*** is about ■

3 Without using a protractor, find **m∠*E*** and **m∠*H*.** Look back at your answer to Problem 1 if you are not sure.

m∠*E* is about ■ **m∠*H*** is about ■

4 What can you say about **m∠*E*** and **m∠*H*?**

5 Without using a protractor, what can you say about the following?

A m∠*G* + m∠*E* = ■ **B** m∠*G* + m∠*H* = ■

6 Which of the following is true? Explain your answer.

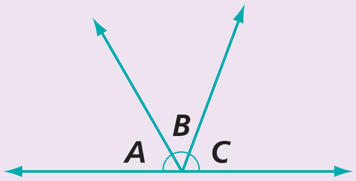
m∠*G* < m∠*F*
m∠*G* > m∠*F*
m∠*G* = m∠*F*

Chapter 9
Lesson 6

REVIEW MODEL
Angle Measures

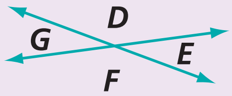
You can use what you know about straight angles, opposite angles, and Zs to figure out missing angle measures.

A straight angle forms a straight line and measures 180°.



$m\angle A + m\angle B + m\angle C = 180°$

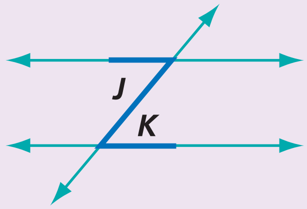
When two lines cross, the opposite angles have the same measure.



$m\angle D = m\angle F$

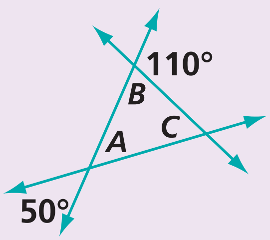
$m\angle E = m\angle G$

When two parallel lines have another line crossing them, the angles in the elbows of the Zs have the same measure.



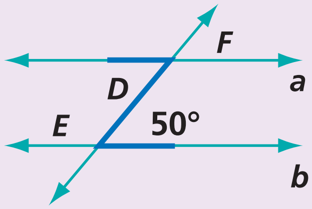
$m\angle J = m\angle K$

Example 1



- $\angle A$ measures 50°. It is opposite an angle that is 50°.
- $\angle B$ measures 70°. It forms a straight angle with an angle that is 110° and $180° - 110° = 70°$.
- $\angle C$ measures 60°. The sum of the measures of the angles in a triangle is 180° and $180° - (50° + 70°) = 60°$.

Example 2

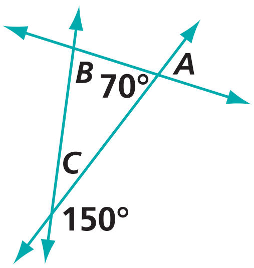


- $\angle D$ measures 50°. It has the same measure as the 50° angle that is in the other elbow of the Z.
- $\angle E$ measures 130°. It forms a straight angle with an angle that is 50° and $180° - 130° = 50°$.
- $\angle F$ is 50°. It is opposite an angle that is 50°.

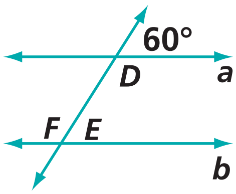
Check for Understanding

Without using a protractor, find the missing angle measures.

1



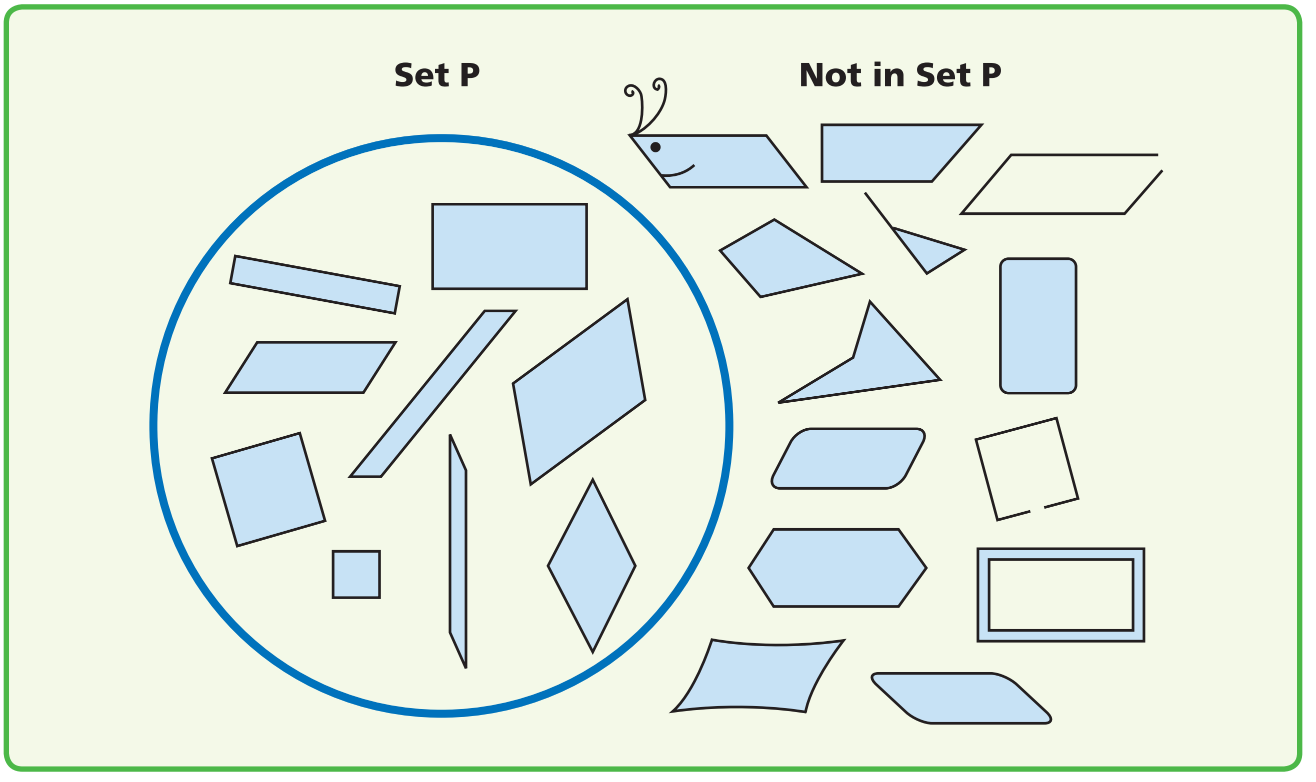
2 $a \parallel b$



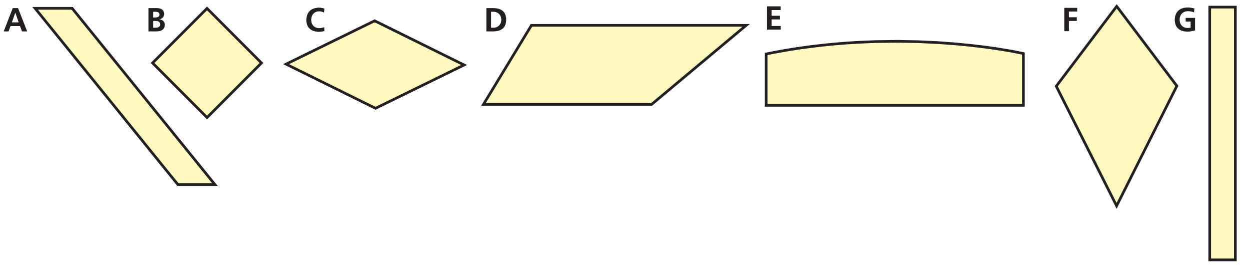
Chapter 9

Lesson 7

EXPLORE
Sorting Figures



1 Which of these figures belong in Set P?



2 Without using the names of any polygons, write a set of rules to sort any figure properly.

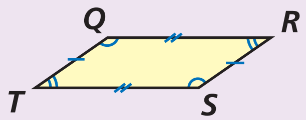
The rules will include, "To be set in P, a figure has to have four sides," but you will need other rules as well.

Chapter 9
Lesson 7

REVIEW MODEL
Classifying Quadrilaterals

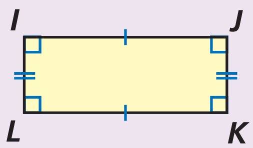
Here are four properties you can use to classify quadrilaterals: the number of congruent angles, the number of congruent sides, the number of pairs of parallel sides, and the number of right angles.

Look at all the names that apply to each quadrilateral below.



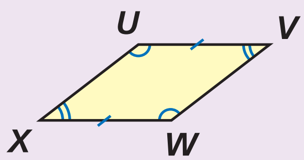
$\overline{QR} \parallel \overline{ST}$
$\overline{QT} \parallel \overline{RS}$

- Polygon
- Quadrilateral
- Parallelogram



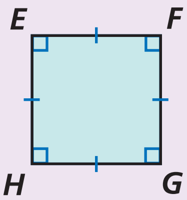
$\overline{IJ} \parallel \overline{KL}$
$\overline{IL} \parallel \overline{JK}$

- Polygon
- Quadrilateral
- Parallelogram
- Rectangle



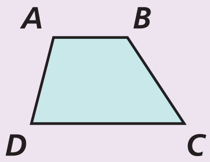
$\overline{UV} \parallel \overline{XW}$
$\overline{UX} \parallel \overline{XW}$

- Polygon
- Quadrilateral
- Parallelogram
- Rhombus



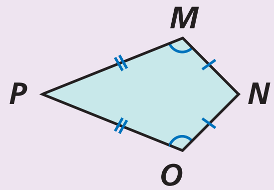
$\overline{EF} \parallel \overline{GH}$
$\overline{EH} \parallel \overline{FG}$

- Polygon
- Rectangle
- Quadrilateral
- Parallelogram
- Rhombus
- Square



$\overline{AB} \parallel \overline{DC}$

- Polygon
- Quadrilateral
- Trapezoid



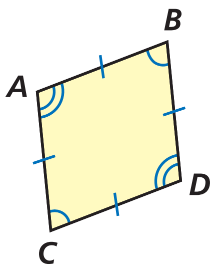
- Polygon
- Quadrilateral
- Kite

Check for Understanding

Choose all the names from the box that apply to each figure.

polygon quadrilateral parallelogram rectangle rhombus square kite trapezoid

1



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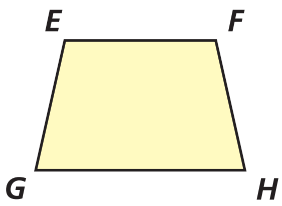

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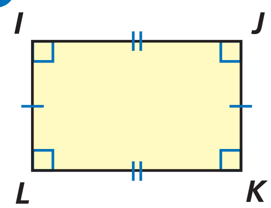
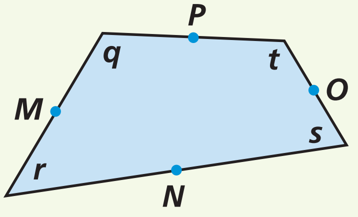


Chapter 9
Lesson 8

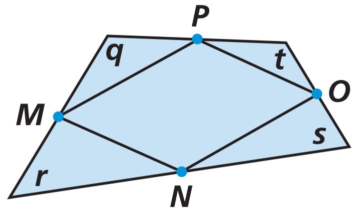
EXPLORE
An Experiment with a Quadrilateral

Remember the triangle experiment you did at the beginning of this chapter? Try this experiment with a quadrilateral and compare your results to your classmates' results.

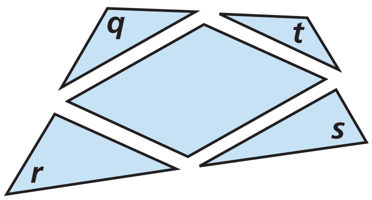
You will need a copy of one of the Activity Masters 80–84: Quadrilateral Experiment 1–10. Each activity master has two copies of the same quadrilateral on it. The angles of the quadrilateral are labeled *q, r, s,* and *t.* The midpoints of the sides are marked and labeled *M, N, O, P.* Here's an example, although your quadrilateral may look different from this one:



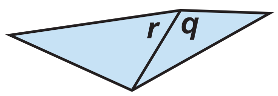
Step 1 Using a straightedge, neatly connect points *M*, *N*, *O*, and *P* this way:



Step 2 Cut out your quadrilateral. Then carefully cut along the lines you just made.



Step 3 Fit the corner pieces together so that the labeled angles all meet at a single vertex. Fit them together as closely as possible without overlapping, like these two.



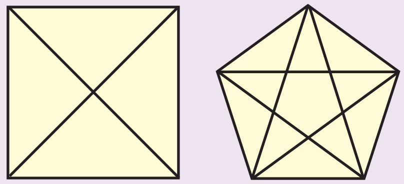
Sketch what you see and describe the result in words.

Chapter 9

Lesson 9

REVIEW MODEL
Problem Solving Strategy
Look for a Pattern

Andrew drew 2 line segments to connect the vertices of a square and 5 line segments to connect the vertices of a pentagon. How many segments will he draw to connect the vertices of a heptagon (7-sided polygon)?



Strategy: Look for a Pattern

Read to Understand

What do you know from reading the problem?

Andrew drew 2 segments to connect the vertices of a square and 5 segments to connect the vertices of a pentagon.

What do you need to find out?

the number of segments needed to connect the vertices of a heptagon

Plan

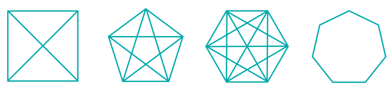
How can you solve this problem?

You can look for a pattern in the number of segments needed to connect the vertices of polygons that have fewer than 7 sides.

Solve

How can you look for a pattern to solve this problem?

Draw polygons with fewer than 7 sides and draw the segments connecting the vertices.



Look at the 6-sided figure. There are 3 segments (3 less than the number of sides) from each of the 6 vertices. BUT, this counts each segment twice, once from each vertex that it connects. So, we do not really have 18 segments but half that amount, or 9 segments.

Check this pattern for the other polygons above. For a polygon with n sides, the number of segments is always half the number of sides multiplied by the number of sides minus 3, or half of $n \times (n - 3)$. So, for a heptagon, the number of segments is half of 7×4, or 28. Half of 28 is 14. So, there are 14 segments connecting the vertices in a heptagon.

Check

Look back at the problem. Did you answer the question that was asked? Does the answer make sense?

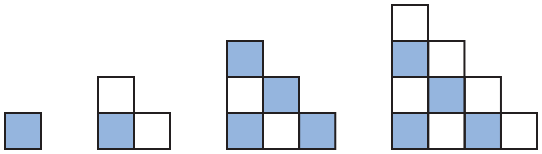
Problem Solving Practice

Problem Solving Strategies

- ✔ Act it Out
- ✔ Draw a Picture
- ✔ Guess and Check
- ✔ **Look for a Pattern**
- ✔ Make a Graph
- ✔ Make a Model
- ✔ Make an Organized List
- ✔ Make a Table
- ✔ Solve a Simpler Problem
- ✔ Use Logical Reasoning
- ✔ Work Backward
- ✔ Write an Equation

Use the strategy *look for a pattern* to solve.

1. Jacob is making a design using blue and white square tiles. How many tiles will he need to make the sixth figure in the pattern?



2. Rachel is making a quilt. She puts the squares in a repeating pattern of red, green, blue, yellow, white. What color will the 29th square be?

Mixed Strategy Practice

Use any strategy to solve. Explain.

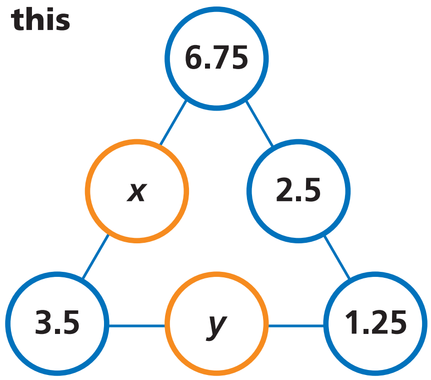
3. One row of carrots produces 6.5 pounds of carrots. One row of beets produces 5.1 pounds of beets. How many more pounds of carrots than beets would come from 3 rows of carrots and 3 rows of beets?

4. The drama club will present a play to 912 students in 4 different performances. If each performance has the same number of students, how many students will be at each performance?

5. Sophia read 45 pages on Sunday, 90 pages on Monday and 135 pages on Tuesday. If she continues this pattern, how many pages will she read on Friday?

6. Jason spent half of his money on a CD. After he spent half of what was left on lunch, he had $5.75 left. How much did he have to start?

For 7–10, use the diagram. The sum along each side of this triangle is the same.



7. What is the value of x?

8. What is the value of y?

9. If you multiply each number in a blue circle by 2, how will x and y change?

10. How can you check that your answer to 9 is correct?

Chapter 9 Vocabulary

Choose the best vocabulary term from Word List A for each sentence.

1. An angle that forms a square corner is called a(n) __?__.
2. A polygon with four sides is called a(n) __?__.
3. Two figures are __?__ if they have the same shape and the same or a different size.
4. A quadrilateral with two pairs of parallel sides, two pairs of congruent sides, and four right angles is a(n) __?__.
5. Figures with the same size and shape are __?__.
6. An angle with a measure less than a right angle is a(n) __?__.
7. A triangle that has three congruent sides is a(n) __?__.
8. A quadrilateral with four congruent sides and two pairs of parallel sides is a(n) __?__.
9. A quadrilateral with two pairs of parallel sides and two pairs of congruent sides is a(n) __?__.
10. Two lines that intersect at right angles are __?__.

Word List A

- acute angle
- concave
- congruent figures
- convex
- equilateral triangle
- isosceles triangle
- kite
- obtuse angle
- opposite angles
- parallel (||)
- parallelogram
- perpendicular
- quadrilateral
- rectangle
- rhombus
- right angle
- scalene triangle
- similar
- straight angle
- trapezoid
- vertex

Complete each analogy using the best term from Word List B.

11. Rectangle is to __?__ as equilateral triangle is to triangle.
12. Acute angle is to equilateral triangle as __?__ is to rectangle.

Word List B

- right angle
- congruent figures
- quadrilateral
- square

Talk Math

Discuss with a partner what you have just learned about attributes of two-dimensional figures. Use the vocabulary terms *acute angle, obtuse angle, right angle,* and *congruent.*

13. How can you tell whether a triangle is a scalene triangle?
14. How can you describe the angles formed by two intersecting lines?
15. How can you describe a parallelogram?

Degrees of Meaning Grid

16 **Create a degrees of meaning grid. Start at least two rows with the word *triangle* and at least two rows with the word *quadrilateral.* Use what you know and what you have learned about quadrilaterals and triangles.**

General	Less General	Specific

Analysis Chart

17 **Create an analysis chart. List various polygons. Show the greatest number of right angles, acute angles, obtuse angles, and pairs of parallel sides the polygons can have.**

What's in a Word?

KITE There is a small hawk called a *kite* because the word *kite* sounds a bit like the bird's cry. Toy *kites* are made of paper, lightweight wood, and string. Because toy *kites* hover and glide like the bird, they were named after it. The quadrilateral *kite* gets its name from the toy because its shape is like the shape of some toy *kites.*

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GAME

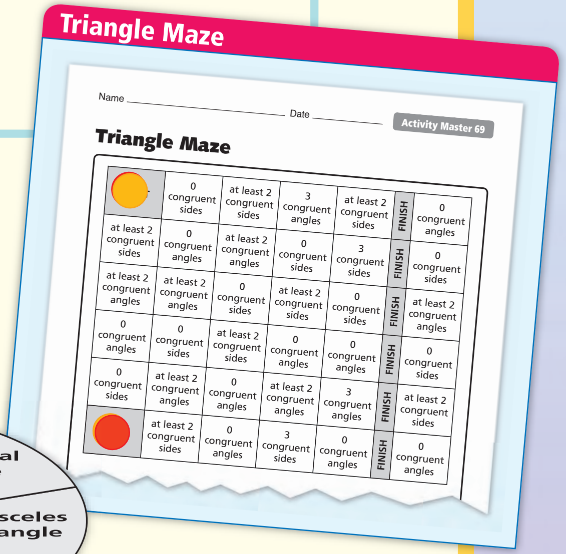
Triangle Maze

Game Purpose

To practice using the attributes of different types of triangles

Materials

- Activity Master 69: *Triangle Maze*
- Counters
- Pencil and paper clip


Triangle Maze

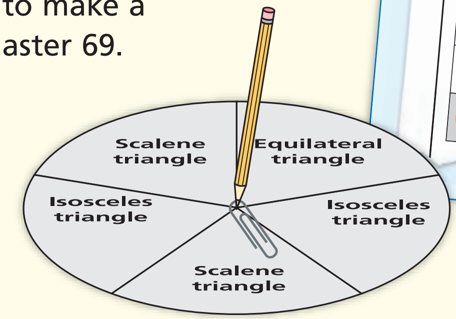
Name ______________________ Date __________ Activity Master 69

Triangle Maze

	0 congruent sides	at least 2 congruent sides	3 congruent angles	at least 2 congruent sides	FINISH	0 congruent angles
at least 2 congruent sides	0 congruent angles	at least 2 congruent angles	0 congruent sides	3 congruent sides	FINISH	0 congruent sides
at least 2 congruent angles	at least 2 congruent angles	0 congruent sides	at least 2 congruent sides	0 congruent sides	FINISH	at least 2 congruent angles
0 congruent angles	0 congruent sides	at least 2 congruent sides	0 congruent angles	0 congruent angles	FINISH	0 congruent sides
0 congruent sides	at least 2 congruent angles	0 congruent angles	at least 2 congruent angles	3 congruent angles	FINISH	at least 2 congruent sides
	at least 2 congruent sides	0 congruent angles	3 congruent sides	0 congruent angles	FINISH	0 congruent angles

How To Play The Game

1. Play this game with a partner. Use the pencil and paper clip to make a spinner on Activity Master 69.



2. Each player should use a different color counter. Put your counter on one of the START corners of the Triangle Maze. Players should start in different corners.

3. Take turns spinning the pointer.
   - After your spin, move your counter to a square that shares a side (not just a corner) with the square you are in AND describes the type of triangle shown by your spin.

     Hint: You can look at the examples of different types of triangles shown beside the spinner on Activity Master 69.
   - You *must* move your counter if you can, even if you have to move backward.

4. The first player to cross the finish line wins!

GAME

First to 360°

Game Purpose

To practice drawing angles of specific measures

Materials

- Activity Masters 74–76: *First to 360*° Game Boards
- Compass
- Small counters
- Protractor

How To Play The Game

1. This is a game for 2 players. Use Game Board 1. Each player uses a compass to draw a circle and draw in a radius.

2. Player 1 chooses an angle measurement on the game board and marks it with a counter. Then Player 1 uses the protractor to draw that angle in his or her circle.

3. Player 2 places a counter on any angle measurement on the game board that shares a side (not just a corner) with the one that is marked. Player 2 then draws the chosen angle in his or her circle.
   - Keep a record of each player's running total.
   - Sometimes a good strategy might be to put a counter on an angle measurement even if you cannot use it. That way, your opponent is blocked from using that measurement.

4. Take turns until one player's circle is completed. The angles must total exactly 360°. If they do, that player wins!

5. Play again using Game Board 2 and then Game Board 3.

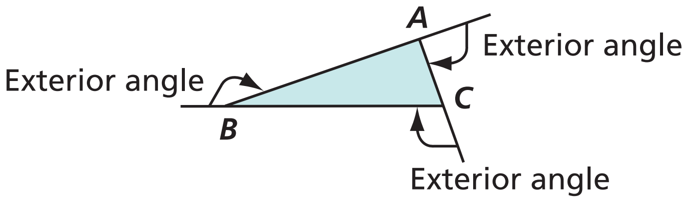
CHALLENGE

Exterior Angles

Every polygon has interior angles—the type of angles you have been studying. Every polygon also has exterior angles. In this activity, you will explore exterior angles.

You'll need a protractor and a straightedge.

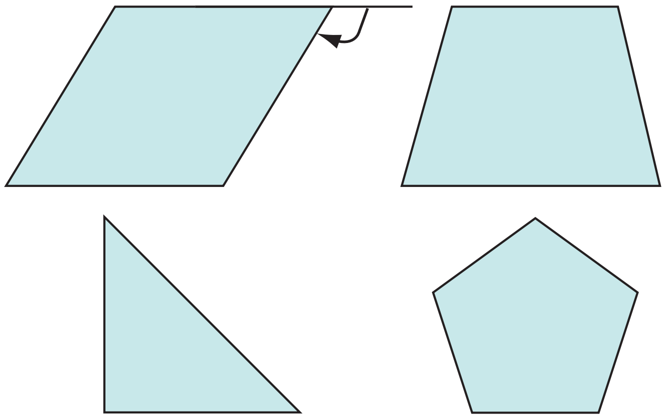
Exterior angles are formed by extending the sides of a polygon. A triangle has 3 exterior angles.



1. Use a protractor to measure the exterior angles at vertices *A*, *B*, and *C*.
   $m\angle A = \blacksquare^\circ$. $m\angle B = \blacksquare^\circ$. $m\angle C = \blacksquare^\circ$.

2. Add the measures of the angles.
   The measures of $m\angle A + m\angle B + m\angle C = \blacksquare^\circ$.

Now trace each of the four polygons below. Then use a straightedge to extend each side of the polygon. The first figure shows one exterior angle for you.



3. Measure the exterior angles of each figure and add them.

4. What pattern do you see in the sum of the exterior angles of each of the polygons?